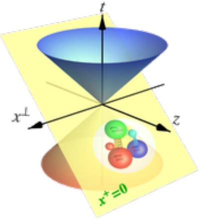
Quantum Stress on the Light Front

Xianghui Cao

University of Science and Technology of China



LFQCD Seminars

In collaboration with Prof. Yang Li and Prof. J.P. Vary

September 10, 2023

The last global unknown

- The energy-momentum tensor (EMT) characterizes the coupling between gravity and matter
- EMT for spin-$\frac{1}{2}$ hadrons:

$$\langle p', s' | T^{\mu\nu}(0) | p, s \rangle = \frac{1}{2M} \bar{u}_{s'}(p') \left[2P^\mu P^\nu A(q^2) + iP^{\{\mu}\sigma^{\nu\}\rho} q_\rho J(q^2) + \frac{1}{2}(q^\mu q^\nu - q^2 g^{\mu\nu}) D(q^2) \right] u_s(p).$$

where $P = (p' + p)/2$, $q = p' - p$.

- Gravitational form factors are connected with the intrinsic distributions of hadron.

$$\begin{aligned} A(q^2): &\quad \text{energy and mass distribution} && \rightarrow A(0) = 1 \\ J(q^2): &\quad \text{angular momentum distribution} && \rightarrow J(0) = \tfrac{1}{2} \\ D(q^2): &\quad \text{stress distribution} && D(0) \text{ is unconstrained} \end{aligned}$$

Mechanical properties

[Perevalova '16]

- Pressure distributions are Fourier transformation of D term

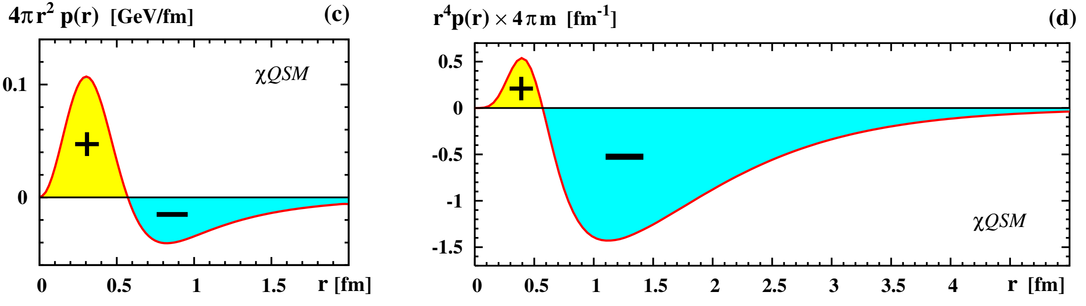
$$p(r) = -\frac{1}{6M} \int \frac{d^3q}{(2\pi)^3} e^{iq\cdot r} \boldsymbol{q}^2 D(-\boldsymbol{q}^2).$$

- D term indicates the stability of hadron

Local stability criteria: $\frac{2}{3}s(r) + p(r) > 0 \quad \to$ a negative $D(0)$

von Laue condition: $\int d^3 r p(r) = 0$

$$D(0) \sim \int d^3 r r^2 p(r) < 0?$$



[Goeke '07]

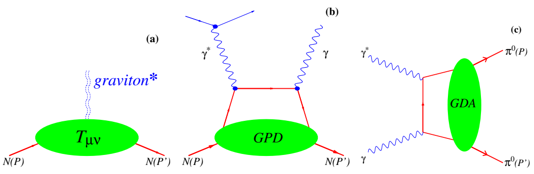
Experiment access

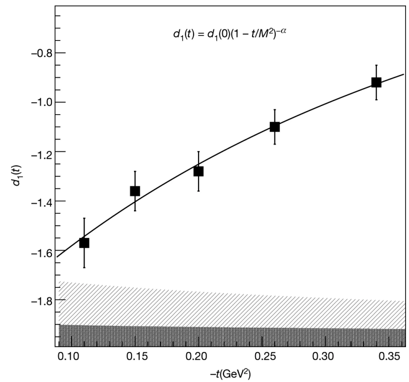
- Extract GFFs from Ji's sum rule

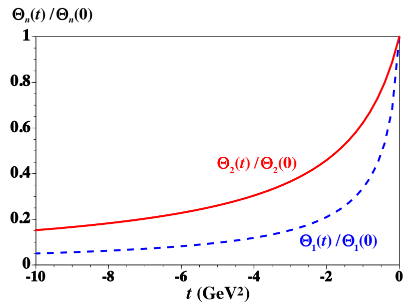
$$\int_{-1}^{1} dxxH^a(x,\xi,t) = A^a(t) + \xi^2 D^a(t), \quad \int_{-1}^{1} dxxE^a(x,\xi,t) = B^a(t) - \xi^2 D^a(t).$$

[Ji '96]

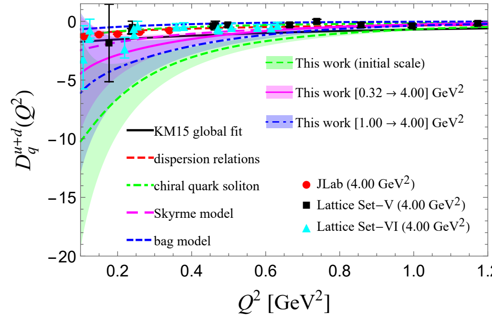
- JLAB: proton D term extracted from DVCS [Burkert '18]
- Bell II: pion GFFs extracted from $\gamma^*\gamma \to \pi^0\pi^0$ [Kumano '18]







Theoretical progress

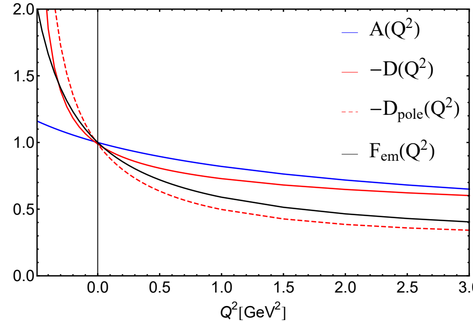


- perturbative QCD

  [Tong '22]
- light-front quark-diquark model

  [Chakrabarti '20]
- Dyson-Schwinger equation

  [Xing '23]



▶ Non-perturbative calculations based on quantum field theory are scarce. *D* term needs proper non-perturbative renormalization!

Scalar Yukawa model

$$\mathcal{L} = \partial_\mu \chi^\dagger \partial^\mu \chi - m^2 \chi^\dagger \chi + \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2}\mu^2 \varphi^2 + g_0 \chi^\dagger \chi \varphi + \delta m^2 \chi^\dagger \chi$$

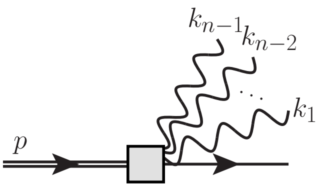
where $m = 0.94\text{GeV}$, $\mu = 0.14\text{GeV}$, $\alpha \equiv g^2/(16\pi m^2)$. g_0 and δm^2 are renormalization parameters.

- χ : mock nucleon, φ : mock pion
- quenched approximation, no nucleon-antinucleon loops $\qquad m^2 \gg \mu^2$

$$m^2_{\text{bare}} = m^2 - \delta m^2, \quad \mu^2_{\text{bare}} = \mu^2$$

- One nucleon sector:

$$|p\rangle = |\chi\rangle + |\chi\varphi\rangle + |\chi\varphi\varphi\rangle + |\chi\varphi\varphi\varphi\rangle + \cdots$$



- This model is solved up to $|\chi\varphi\varphi\varphi\rangle$ sector non-perturbatively [Li '15]
  - Fock sector dependent renormalization
  - converge up to $|\chi\varphi\varphi\rangle$ sector

Diagrammatic representation

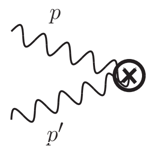
EMT in scalar Yukawa model:

$$\hat{T}^{\mu\nu} = \partial^{\{\mu}\chi^\dagger\partial^{\nu\}}\chi - g^{\mu\nu}\left[\partial_\sigma\chi^\dagger\partial^\sigma\chi - (m^2 - \delta m^2)\chi^\dagger\chi\right] - g^{\mu\nu}g_0\chi^\dagger\chi\varphi$$
$$+ \partial^\mu\varphi\partial^\nu\varphi - \frac{1}{2}g^{\mu\nu}\left(\partial^\rho\varphi\partial_\rho\varphi - \mu_0^2\varphi^2\right)$$

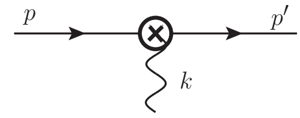
where $a^{\{\mu}v^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$.

(a) $(\frac{1}{2}q^2 - \delta m^2)g^{\mu\nu} + p^{\{\mu}p'^{\nu\}}$

(b) $\frac{1}{2}q^2 g^{\mu\nu} + p^{\{\mu}p'^{\nu\}}$



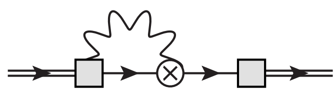
(c) $\frac{1}{4}q^2 g^{\mu\nu} - p^\mu p'^\nu$



(d) $-g_0 g^{\mu\nu}$

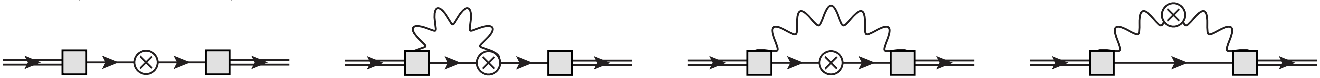
Diagrammatic representation for hadron matrix elements:

$$\langle\chi| - g^{\mu\nu}g_0\chi^\dagger\chi\varphi|\chi\varphi\rangle =$$

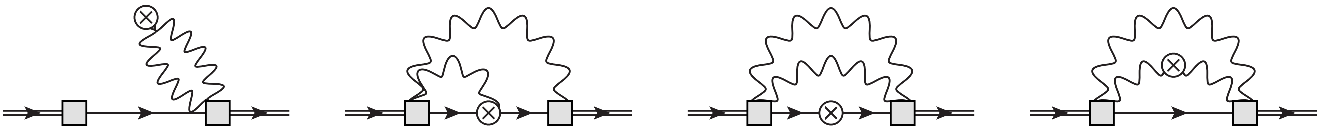


EMT renormalization

$$t^{\mu\nu} = \langle p'|\hat{T}^{\mu\nu}(0)|p\rangle =$$



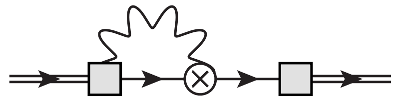
(a) $\delta m^2 = \delta m_3^2$ **(b)** $g_0 = g_{03}$ **(c)** $\delta m^2 = \delta m_2^2$ **(d)**



(e) **(f)** $g_0 = g_{02}$ **(g)** $\delta m^2 = 0$ **(h)**

Counterterms can cancel with divergent diagrams, e.g. diagram (a) and (b):

(a) $\delta m^2 = \delta m_3^2$



(b) $g_0 = g_{03}$

$$t_a^{\alpha\beta} = Z[(\frac{1}{2}q^2 - \delta m_3^2)g^{\alpha\beta} + p^{\{\alpha}p'^{\beta\}}] = Z[2P^\alpha P^\beta + (\frac{1}{2}q^2 - \delta m_3^2)g^{\alpha\beta} - \frac{1}{2}q^\alpha q^\beta]$$

$$t_b^{\alpha\beta} = -\sqrt{Z}g^{\alpha\beta}\int\frac{dx}{2x(1-x)}\int\frac{d^2k_\perp}{(2\pi)^3}g_{03}\psi_2(x,k_\perp) = g^{\alpha\beta}Z\delta m_3^2$$

GFFs on the light front

- In Drell-Yan frame $q^+ = 0$:

$$t^{\alpha\beta} = 2P^\alpha P^\beta A(q^2) + \frac{1}{2}(q^\alpha q^\beta - q^2 g^{\alpha\beta})D(q^2) + \frac{(q^2)^2\omega^\alpha\omega^\beta}{(P^+)^2}S_1(q^2) + \frac{1}{(P^+)^2}\epsilon^{\alpha\mu\nu\gamma}P_\mu q_\nu \omega_\gamma \epsilon^{\beta\rho\sigma\lambda}P_\rho q_\sigma \omega_\lambda S_2(q^2).$$

$S_{1,2}(q^2)$ are two spurious form factors originating from violation of the Lorentz symmetry.

- t^{++} and t^{+-} are free of the spurious contributions. In Breit frame ($\boldsymbol{P}_\perp = 0$):

$$t^{++} = 2(P^+)^2 A(-\boldsymbol{q}_\perp^2),$$
$$t^{+-} = 2(m^2 + \frac{1}{4}\boldsymbol{q}_\perp^2)A(-\boldsymbol{q}_\perp^2) + \boldsymbol{q}_\perp^2 D(-\boldsymbol{q}_\perp^2),$$
$$\mathrm{tr}t^{ij} = -\frac{1}{2}\boldsymbol{q}_\perp^2 D(-\boldsymbol{q}_\perp^2) + \boldsymbol{q}_\perp^2 S_2(-\boldsymbol{q}_\perp^2).$$

$$\Rightarrow A(-\boldsymbol{q}_\perp^2) = \frac{t^{++}}{2(P^+)^2}, \quad \boldsymbol{q}_\perp^2 D(-\boldsymbol{q}_\perp^2) = t^{+-} - \frac{m^2 + \frac{1}{4}\boldsymbol{q}_\perp^2}{(P^+)^2}t^{++}$$

Conservation laws

- Light-front Schrödinger equation,

$$\hat{P}^{\mu} |p\rangle = p^{\mu} |p\rangle ,$$

$$\Rightarrow p^{\mu} 2p^{+} \delta^{(3)}(p-p') = \langle p'|\hat{P}^{\mu}|p\rangle = \langle p'| \int d^3x \hat{T}^{+\mu}(x)|p\rangle$$
$$= \int d^3x e^{iq\cdot x} \langle p'|\hat{T}^{+\mu}(0)|p\rangle = \delta^{(3)}(p-p') \langle p'|\hat{T}^{+\mu}(0)|p\rangle .$$

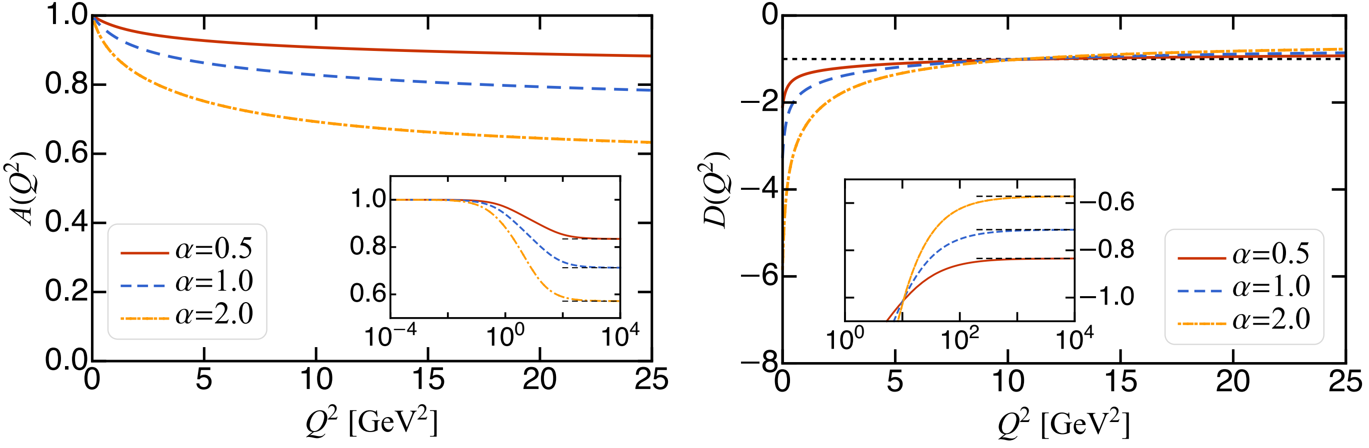
- Forward limit (q=0):

$$\hat{P}^{+} = \int d^3x \hat{T}^{++}(x), \Rightarrow \langle p|\hat{T}^{++}(0)|p\rangle = 2p^{+}p^{+}, \Rightarrow A(0) = 1,$$
$$\hat{P}^{-} = \int d^3x \hat{T}^{+-}(x), \Rightarrow \langle p|\hat{T}^{+-}(0)|p\rangle = 2p^{+}p^{-}, \Rightarrow \lim_{q_\perp \to 0} \boldsymbol{q}_\perp^2 D(-\boldsymbol{q}_\perp^2) = 0.$$

Here, $d^3x = \frac{1}{2} dx^{-} d^2x_\perp$.

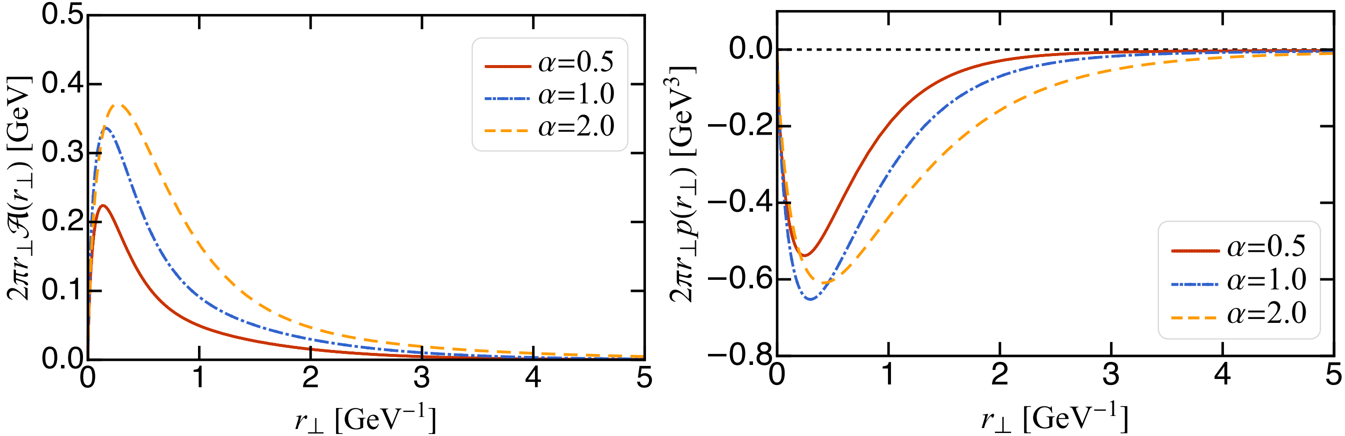
- Indeed, $D = D(0)$ is finite in our model.

Numerical results



- Compare $A(Q^2)$ and $D(Q^2)$ from perturbative regime to strong coupling regime, $Q^2 = -q^2 = \boldsymbol{q}_\perp^2$.
- For small α, $D(Q^2)$ is close to -1, the free scalar particle's result.
- In the forward limit: $A(0) = 1$, $D(0)$ is finite and less than -1.
- As α increases, $D(0)$ becomes more negative.
- For large Q^2, $A(Q^2 \to \infty) = Z$, $D(Q^2 \to \infty) = -Z$, the one-body Fock sector contribution.

Matter density and pressure



- Light-front distribution:

fit functions: $f(Q^2) = f(\infty) + \frac{a_1}{1+Q^2/\Lambda_1^2} + \frac{a_2}{1+Q^2/\Lambda_2^2}$

$$\mathcal{A}(r_\perp) = \int \frac{d^2q_\perp}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_\perp} A(-\boldsymbol{q}_\perp^2), \quad p(r_\perp) = -\frac{1}{6M}\int \frac{d^2q_\perp}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_\perp} \boldsymbol{q}_\perp^2 D(-\boldsymbol{q}_\perp^2)$$

- A point-like repulsive core at $r_\perp = 0$

$$\int \frac{d^2q_\perp}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_\perp} \boldsymbol{q}_\perp^2 \frac{1}{1+\boldsymbol{q}_\perp^2/\Lambda_1^2} = \frac{\Lambda_1^2}{2\pi}\delta^{(2)}(\boldsymbol{r}_\perp) - \frac{\Lambda_1^4}{2\pi}K_0(\Lambda_1 r_\perp)$$

Light-Front Wave Function Representation

A general light-front wave function (LFWD) representation for t^{++}: [Brodsky '00]

$$t^{++} = 2(P^+)^2 \sum_n \int \left[dx_i d^2 k_{i\perp}\right]_n \sum_j x_j \psi_n(\{x_i, \boldsymbol{k}_{i\perp}\})\psi_n(\{x_i, \boldsymbol{k}_{i,j\perp}\})$$

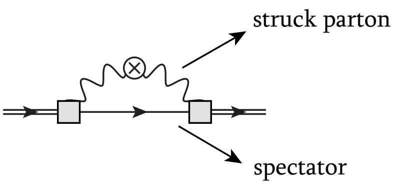
where

$$\int \left[dx_i d^2 k_{i\perp}\right]_n = \frac{1}{S_n} \prod_{i=1}^{n} \int \frac{dx_i}{2x_i} 2\delta(\sum_i x_i - 1) \int \frac{d^2 k_{i\perp}}{(2\pi)^3} (2\pi)^3 \delta^{(2)}(\sum_i \boldsymbol{k}_{i\perp})$$

$$\boldsymbol{k}_{i,j\perp} = \begin{cases} \boldsymbol{k}_{i\perp} - x_i \boldsymbol{q}_\perp, & \text{spectator: } i \neq j \\ \boldsymbol{k}_{i\perp} + (1 - x_i)\boldsymbol{q}_\perp, & \text{struck parton: } i = j \end{cases}$$

Transverse coordinate representation:

$$t^{++}(-\boldsymbol{q}_\perp^2) = (2P^+)^2 \sum_n \int \left[dx_i d^2 r_{i\perp}\right]_n \left|\widetilde{\psi}_n(\{x_i, \boldsymbol{r}_{i\perp}\})\right|^2 \sum_j x_j e^{i\boldsymbol{r}_{j\perp}\cdot\boldsymbol{q}_\perp}$$



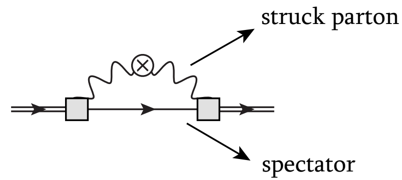
light-front wave function representation for t^{+-}:

$$t^{+-} = \sum_n 2 \int \left[dx_i d^2 k_{i\perp}\right]_n \sum_j \psi_n^*(\{x_i, \boldsymbol{k}^+_{i,j\perp}\}) \psi_n(\{x_i, \boldsymbol{k}^-_{i,j\perp}\}) \frac{\boldsymbol{k}^2_{j\perp} + m_j^2 - \frac{1}{4}\boldsymbol{q}^2_\perp}{x_j}$$

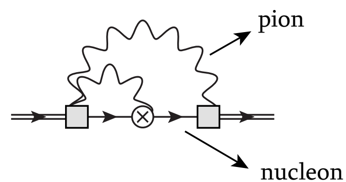
$$+ \sum_n 2 \int \left[dx_i d^2 k_{i\perp}\right]_n \psi_n^*(\{x_i, \boldsymbol{k}_{i\perp}\}) \psi_n(\{x_i, \boldsymbol{k}_{i,n\perp}\}) \left[M^2 - \sum_j \frac{\boldsymbol{k}^2_{j\perp} + m_j^2}{x_j}\right]$$

where

$$\boldsymbol{k}^{\pm}_{i,j\perp} = \begin{cases} \boldsymbol{k}_{i\perp} \pm \frac{1}{2} x_i \boldsymbol{q}_\perp, & \text{spectator: } i \neq j \\ \boldsymbol{k}_{i\perp} \mp \frac{1}{2}(1 - x_i)\boldsymbol{q}_\perp, & \text{struck parton: } i = j \end{cases}$$



$$\boldsymbol{k}_{i,n\perp} = \begin{cases} \boldsymbol{k}_{i\perp} - x_i \boldsymbol{q}_\perp, & \text{pion, i.e. } i \neq n \\ \boldsymbol{k}_{i\perp} + (1 - x_i)\boldsymbol{q}_\perp, & \text{nucleon, i.e. } i = n \end{cases}$$



Transverse coordinate representation:

$$t^{+-} = \sum_n 2 \int \left[dx_i d^2 r_{i\perp}\right]_n \widetilde{\psi}_n^*(\{x_i, \boldsymbol{r}_{i\perp}\}) \sum_j e^{i\boldsymbol{r}_{j\perp}\cdot\boldsymbol{q}_\perp} \frac{-\boldsymbol{\nabla}^2_{j\perp} + m_j^2 - \frac{1}{4}\boldsymbol{q}^2_\perp}{x_j} \widetilde{\psi}_n(\{x_i, \boldsymbol{r}_{i\perp}\})$$

$$- \sum_n 2 \int \left[dx_i d^2 r_{i\perp}\right]_n \widetilde{\psi}_n^*(\{x_i, \boldsymbol{r}_{i\perp}\}) \left[\sum_j \frac{-\boldsymbol{\nabla}^2_{j\perp} + m_j^2}{x_j} - M^2\right] \widetilde{\psi}_n(\{x_i, \boldsymbol{r}_{i\perp}\}) e^{i\boldsymbol{r}_n\cdot\boldsymbol{q}_\perp}$$

light-front wave function representation for $D(-\boldsymbol{q}_\perp^2)$:

$$D(-\boldsymbol{q}_\perp^2) = 2\sum_n \int \left[dx_i d^2 r_{i\perp}\right] \widetilde{\psi}_n^*(\{x_i, \boldsymbol{r}_{i\perp}\}) \times \sum_j \left\{ \frac{e^{i\boldsymbol{r}_{j\perp}\cdot\boldsymbol{q}_\perp} - e^{i\boldsymbol{r}_{n\perp}\cdot\boldsymbol{q}_\perp}}{\boldsymbol{q}_\perp^2} \frac{-\boldsymbol{\nabla}_{j\perp}^2 + m_j^2 - x_j^2 M^2}{x_j} - \frac{1+x_j^2}{4x_j} e^{i\boldsymbol{r}_{j\perp}\cdot\boldsymbol{q}_\perp} \right\} \widetilde{\psi}_n(\{x_i, \boldsymbol{r}_{i\perp}\}).$$

$D(0)$ is finite:

$$D(0) = -1 + 2\sum_n \int \left[dx_i d^2 r_{i\perp}\right]_n \widetilde{\psi}_n^*(\{x_i, \boldsymbol{r}_{i\perp}\}) \times \sum_j \frac{1}{x_j} \left\{ (r_n^2 - r_{j\perp}^2)(-\boldsymbol{\nabla}_{j\perp}^2 + m_j^2 - x_j^2 M^2) + \frac{1}{4}(x_j^2 - 1) \right\} \widetilde{\psi}_n(\{x_i, \boldsymbol{r}_{i\perp}\}).$$

Summary

- We calculate the GFFs of a strongly-coupled scalar nucleon using light-front Hamiltonian formalism.
- We extract matter distrubutions and pressure from form factors $A(-\boldsymbol{q}_\perp^2)$ and $D(-\boldsymbol{q}_\perp^2)$.
- We obtain a non-perturbative LFWF representation of the D-term, which can be used in phenomenological QCD models as well as to understand the nature of the stress inside hadrons

Thank You