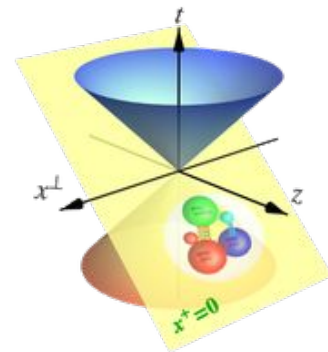




UNIVERSITÀ
DI TORINO

Unveiling the Proton Structure: collinear and TMD observables

Yiyu Zhou
University of Turin



Jefferson Lab Angular collaboration (JAM)

Theory

- Alberto Accardi (Christopher Newport University/Jefferson Lab)
- Daniel Adamiak (Jefferson Lab)
- Trey Anderson (William & Mary)
- Patrick Barry (Argonne National Lab)
- Ian Cloet (Argonne National Lab)
- Christopher Cocuzza (William & Mary)
- Adam Freese (Jefferson Lab)
- Leonard Gamberg (Penn State University Berks)
- Chueng-Ryong Ji (North Carolina State University)
- Yuri Kovchegov (Ohio State University)
- Wally Melnitchouk* (Jefferson Lab)
- Andreas Metz (Temple University)
- Eric Moffat (Argonne National Lab)
- Daniel Pitonyak (Lebanon Valley College)
- Alexei Prokudin (Penn State University Berks)
- Jianwei Qiu (Jefferson Lab)
- Nobuo Sato* (Jefferson Lab)
- Matthew Sievert (New Mexico State University)
- Anthony Thomas (University of Adelaide)
- Richard Whitehill (Old Dominion University)
- Yiyu Zhou (Turin)

Lattice QCD

- Martha Constantinou (Temple University)
- Joseph Karpie (Jefferson Lab)
- Kostas Orginos (William & Mary/Jefferson Lab)
- David Richards (Jefferson Lab)
- Savvas Zafeiropoulos (Aix Marseille University)

Experiment

- Harut Avakian (Jefferson Lab)
- Peter Bosted (Jefferson Lab)
- Jianping Chen (Jefferson Lab)
- Markus Diefenthaler (Jefferson Lab)
- Thia Keppel (Jefferson Lab)
- Sebastian Kuhn (Old Dominion University)
- Shujie Li (LBNL)
- Hanjie Liu (University of Massachusetts Amherst)
- Zein-Eddine Meziani (Argonne National Lab)
- Oscar Rondon (University of Virginia)
- Brad Sawatzky (Jefferson Lab)

Past collaborators

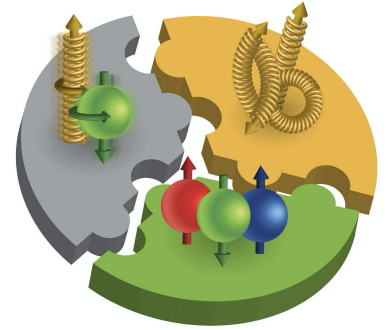
- Rabah Abdul Khalek
- Carlota Andres
- Jake Bringewatt
- Nina Cao
- Filippo Delcarro
- Jacob Ethier
- Nicholas Hunt-Smith
- Pedro Jimenez-Delgado



Proton spin decomposition

What is the decomposition of the proton spin?

- current extraction of $\Delta\Sigma$ is around 0.3 (contribution from quarks)
- spin can be extracted from parton distribution functions (PDFs)
- orbital angular momentum can be extracted from GPDs

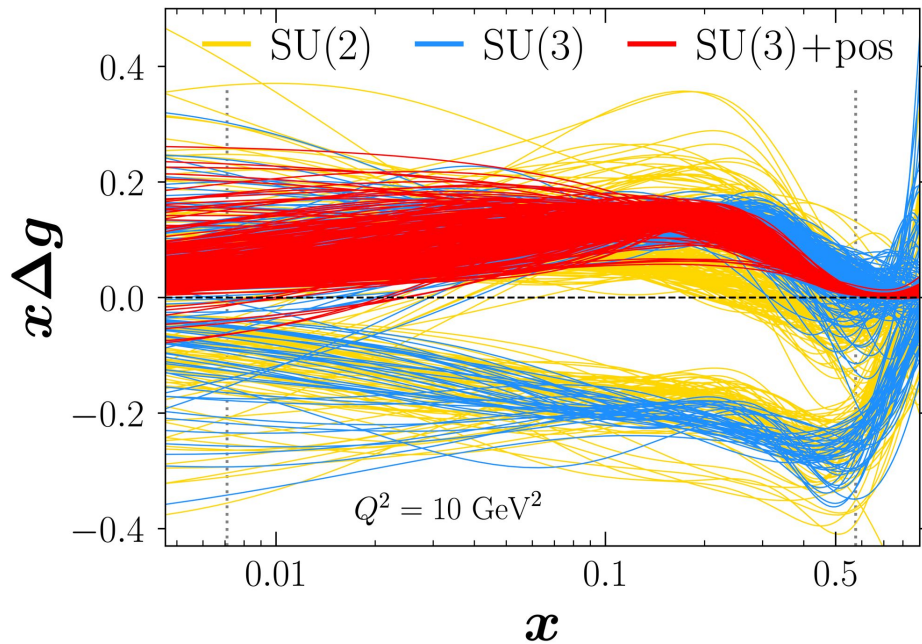


$$\frac{1}{2} = \boxed{\frac{1}{2} \Delta\Sigma} + \boxed{\Delta G} + L_{q+g}$$

0.15 $\int dx \Delta g = ?$

How well do we know the gluon polarization in the proton?

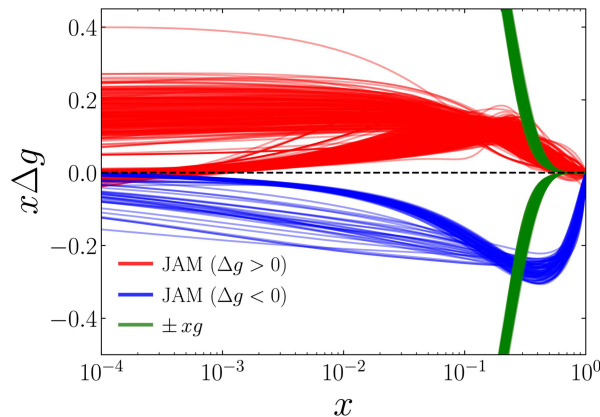
Y. Zhou, N. Sato, and W. Melnitchouk (Jefferson Lab Angular Momentum (JAM) Collaboration)
Phys. Rev. D **105**, 074022 – Published 25 April 2022

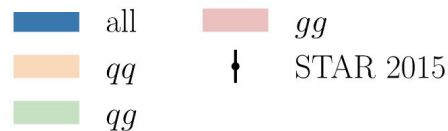
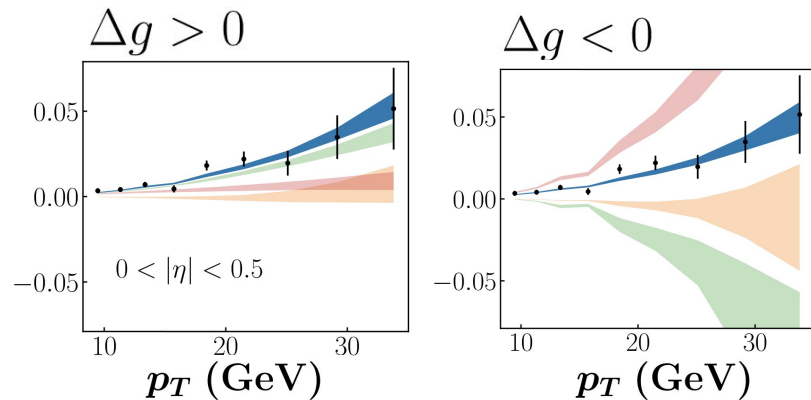
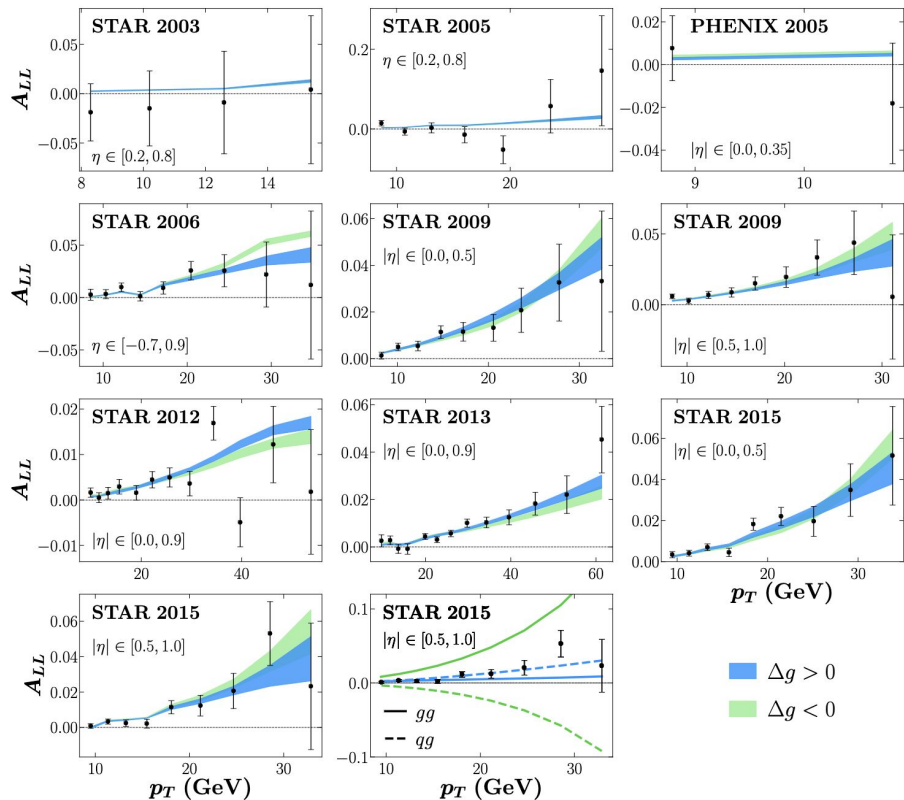


$$|\Delta g| \leq g$$

PDF positivity constraint

- Sign of Δg is not uniquely determined by existing experimental data (DIS $W^2 > 10 \text{ GeV}^2$)
- PDF positivity constraints + data strongly disfavors the negative Δg
- Negative Δg violates significantly PDF positivity constraint
- PDF positivity is not a strict requirement in QCD





$$A_{LL}^{\text{jet}}(p_T, y) \propto a_{gg}[\Delta g \otimes \Delta g] + \sum_q a_{qq}[\Delta q \otimes \Delta g] + \sum_{q, q'} a_{qq'}[\Delta q \otimes \Delta q'] + \mathcal{O}(\alpha_s),$$

$$|A_{LL}| < 1 \quad \checkmark$$

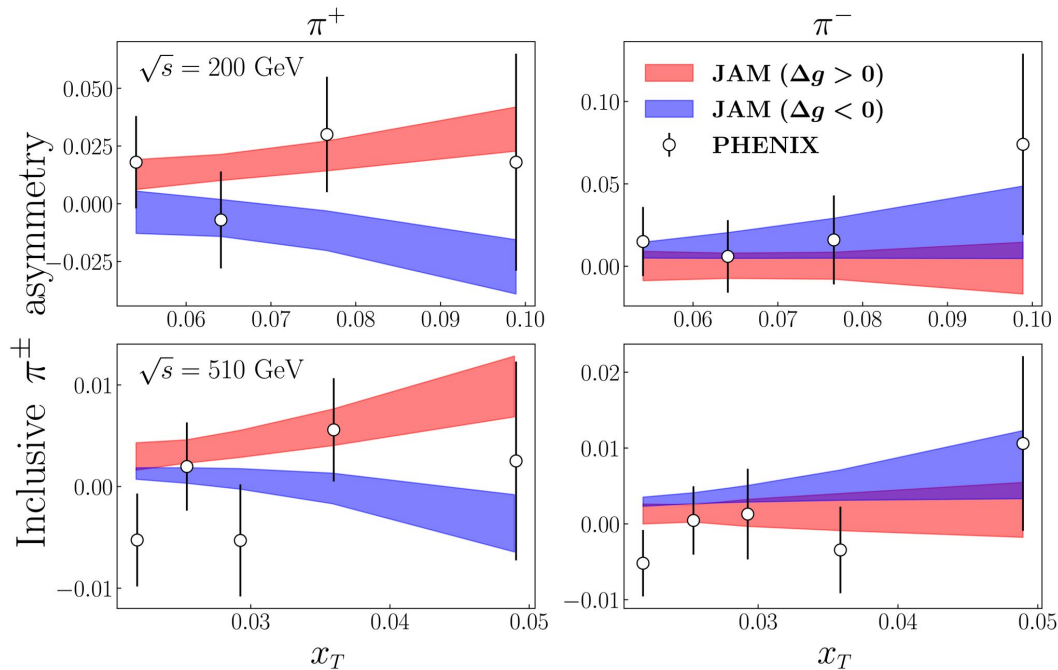
Δg enters quadratically, and different channels contribute with different signs and magnitudes

Charged-pion cross sections and double-helicity asymmetries in polarized $p + p$ collisions at $\sqrt{s} = 200$ GeV

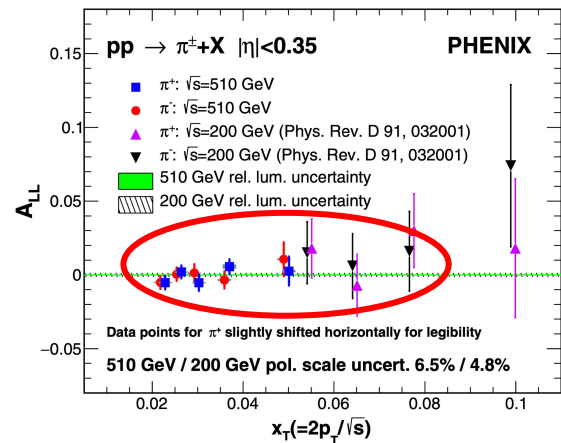
A. Adare *et al.* (PHENIX Collaboration)
Phys. Rev. D **91**, 032001 – Published 2 February 2015

Measurement of charged pion double spin asymmetries at midrapidity in longitudinally polarized $p + p$ collisions at $\sqrt{s} = 510$ GeV

U. Acharya *et al.* (PHENIX Collaboration)
Phys. Rev. D **102**, 032001 – Published 5 August 2020



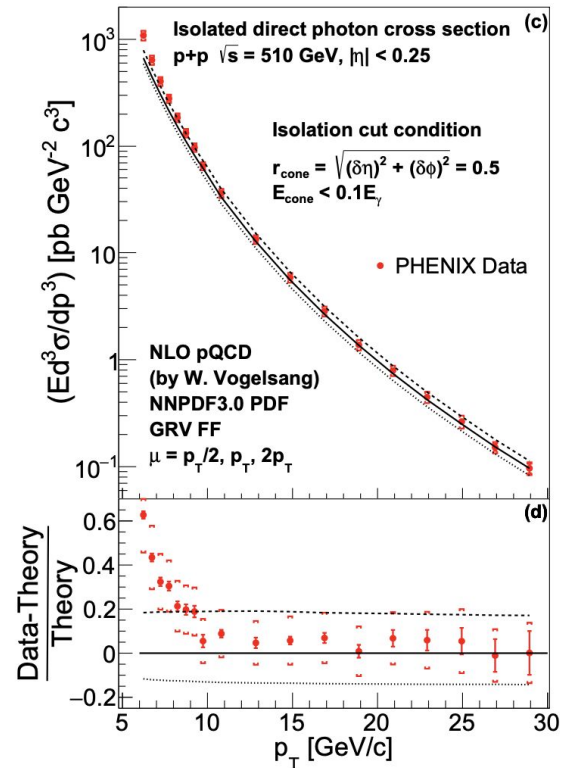
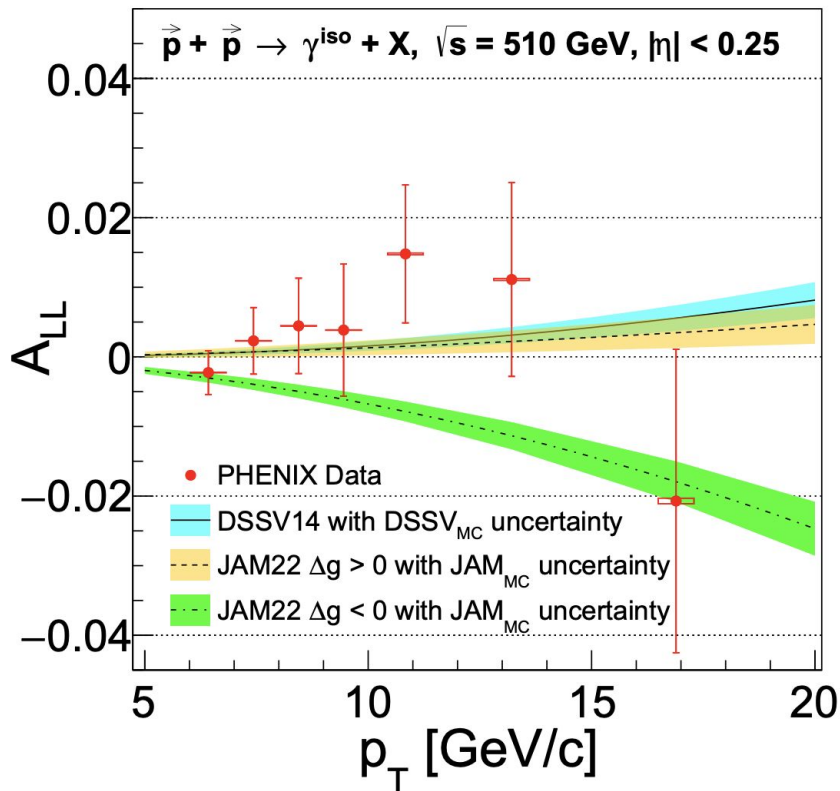
- PHENIX collaboration stated that the ordering of π^+ , π^0 and π^- asymmetries can help discriminate Δg solutions
- The two solutions for Δg found by JAM describe the data equally well



Measurement of Direct-Photon Cross Section and Double-Helicity Asymmetry at $\sqrt{s} = 510$ GeV in $\vec{p} + \vec{p}$ Collisions

N. J. Abdulameer *et al.* (PHENIX Collaboration)
 Phys. Rev. Lett. **130**, 251901 – Published 21 June 2023

$$A_{LL} = \frac{\sigma^{++} - \sigma^{+-}}{\sigma^{++} + \sigma^{+-}}$$

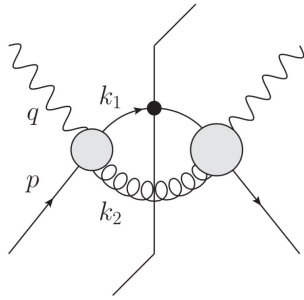


- PHENIX collaboration stated that negative Δg is disfavored by more than 2.8σ
- However, only last 3 high- $p_T A_{LL}$ points are well described in pQCD (see denominator of A_{LL})

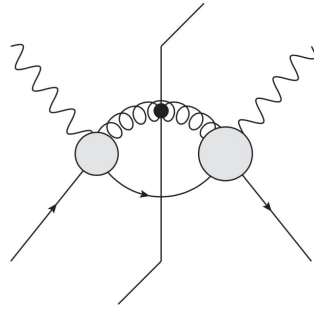
Accessing gluon polarization with high- P_T hadrons in SIDIS

R. M. Whitehill, Yiyu Zhou, N. Sato, and W. Melnitchouk (Jefferson Lab Angular Momentum (JAM) Collaboration)
Phys. Rev. D **107**, 034033 – Published 27 February 2023

SIDIS with large $p_{h,T}$: $e(\ell) + N(P) \rightarrow e(\ell') + h(P_h) + X$

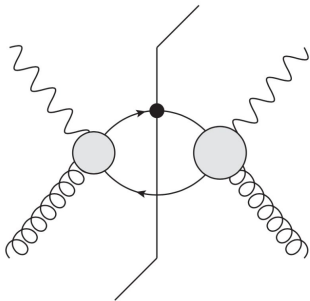


(a)

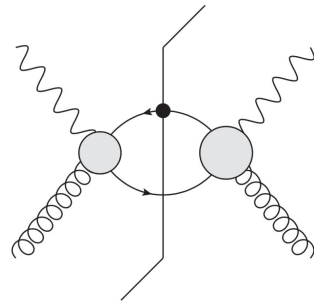


(b)

- q_T is required to be comparable to photon virtuality Q
- Δg starts to contribute at LO
- The cross section depends on Δg linearly



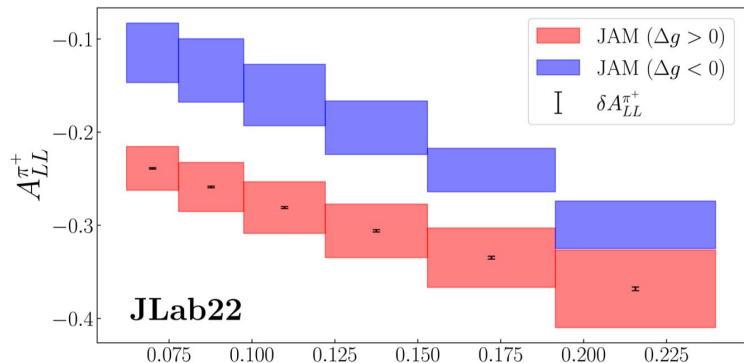
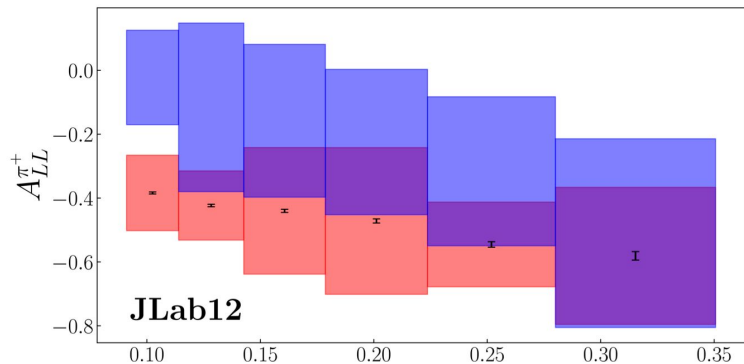
(c)



(d)

$$q_T \equiv \frac{p_{h,T}}{z} \gtrsim Q$$

SIDIS with large $p_{h,T}$: $e(\ell) + N(P) \rightarrow e(\ell') + h(P_h) + X$



$$A_{LL}^{\text{jet}}(p_T, y) \propto a_{gg}[\Delta g \otimes \Delta g] + \sum_q a_{qg}[\Delta q \otimes \Delta g] + \sum_{q,q'} a_{qq'}[\Delta q \otimes \Delta q'] + \mathcal{O}(\alpha_s),$$



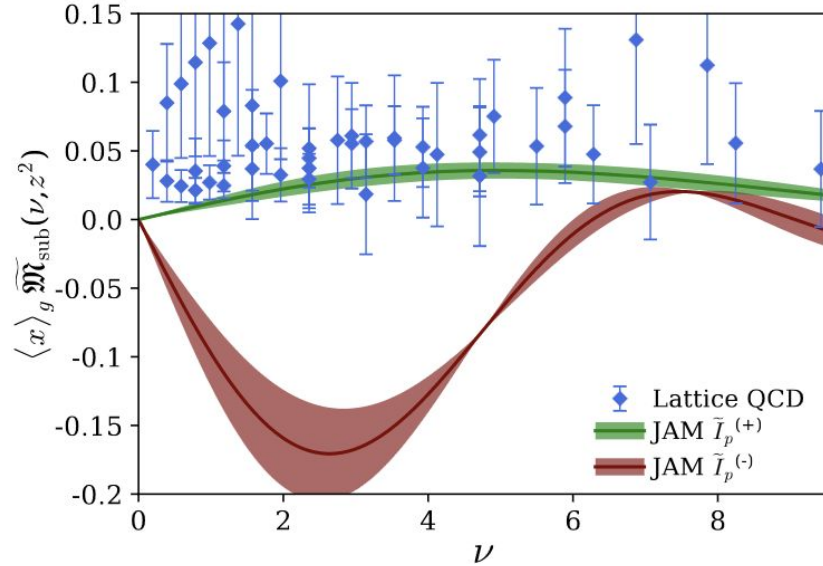
$$A_{LL}^{\text{SIDIS}} \sim \Delta g + \Delta q + \dots$$

Gluon helicity from global analysis of experimental data and lattice QCD Ioffe time distributions

J. Karpie, R. M. Whitehill, W. Melnitchouk, C. Monahan, K. Orginos, J.-W. Qiu, D. G. Richards, N. Sato, and S. Zafeiropoulos (Jefferson Lab Angular Momentum and HadStruc Collaborations)
Phys. Rev. D **109**, 036031 – Published 27 February 2024

Toward the determination of the gluon helicity distribution in the nucleon from lattice quantum chromodynamics

Colin Egerer, Bálint Joó, Joseph Karpie, Nikhil Karthik, Tanjib Khan, Christopher J. Monahan, Wayne Morris, Kostas Orginos, Anatoly Radyushkin, David G. Richards, Eloy Romero, Raza Sabbir Sufian, and Savvas Zafeiropoulos (HadStruc Collaboration)
 Phys. Rev. D **106**, 094511 – Published 28 November 2022

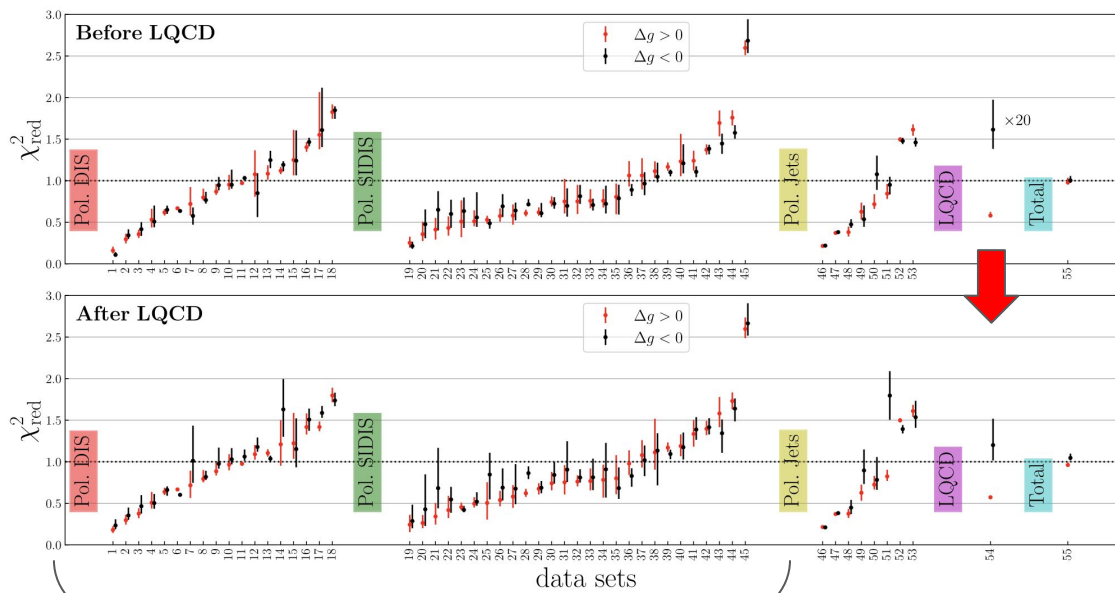


$$\widetilde{M}^{\mu\nu;\alpha\beta}(p, z) = \langle p | F^{\mu\nu}(0) W(0; z) \widetilde{F}^{\alpha\beta}(z) | p \rangle$$

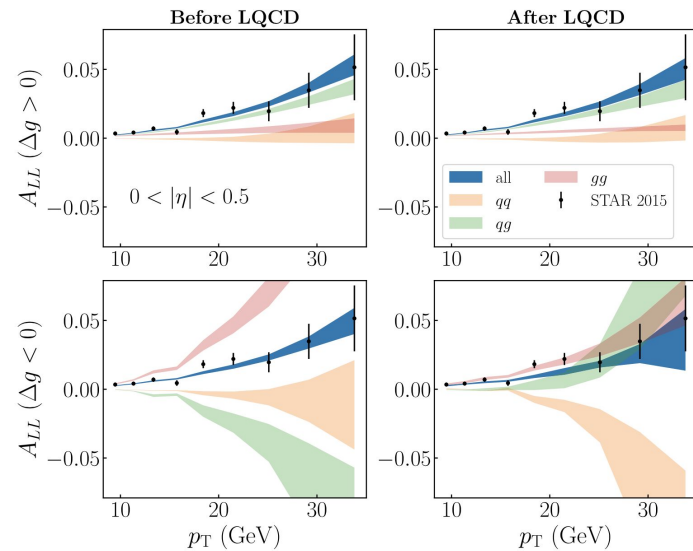
$$\widetilde{\mathfrak{M}}(\nu, z^2) = \frac{\widetilde{M}_{00}(p, z)/p_0 p_3 Z_L(z_3/a)}{M_{00}(p=0, z)/m^2}$$

$$\begin{aligned} \widetilde{\mathfrak{M}}(\nu, z^2) \langle x_g \rangle_{\mu^2} = & \boxed{\tilde{\mathcal{I}}_p(\nu, \mu^2)} - \frac{\alpha_s N_c}{2\pi} \int_0^1 du \tilde{\mathcal{I}}_p(u\nu, \mu^2) \left\{ \ln \left(z^2 \mu^2 \frac{e^{2\gamma_E}}{4} \right) \right. \\ & \left(\left[\frac{2u^2}{\bar{u}} + 4u\bar{u} \right]_+ - \left(\frac{1}{2} + \frac{4 \langle x_S \rangle_{\mu^2}}{3 \langle x_g \rangle_{\mu^2}} \right) \delta(\bar{u}) \right) \\ & + 4 \left[\frac{u + \ln(1-u)}{\bar{u}} \right]_+ - \left(\frac{1}{\bar{u}} - \bar{u} \right)_+ - \frac{1}{2} \delta(\bar{u}) + 2\bar{u}u \left. \right\} \\ & - \frac{\alpha_s C_F}{2\pi} \int_0^1 du \tilde{\mathcal{I}}_S(u\nu, \mu^2) \left\{ \ln \left(z^2 \mu^2 \frac{e^{2\gamma_E}}{4} \right) \tilde{\mathcal{B}}_{gq}(u) + 2\bar{u}u \right\} + \mathcal{O}(\Lambda_{\text{QCD}}^2 z^2), \end{aligned}$$

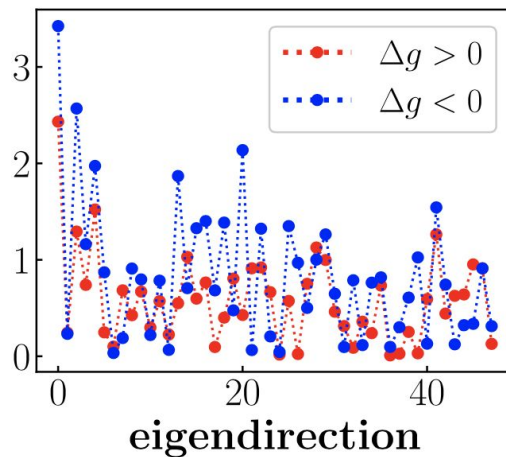
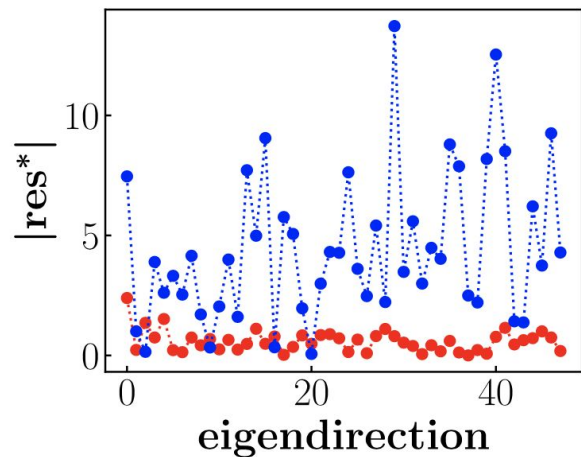
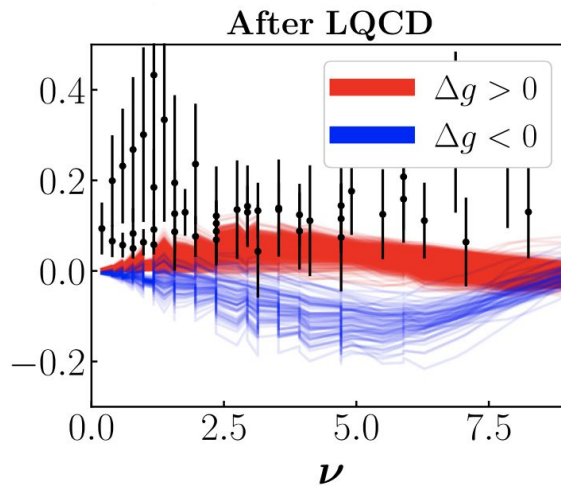
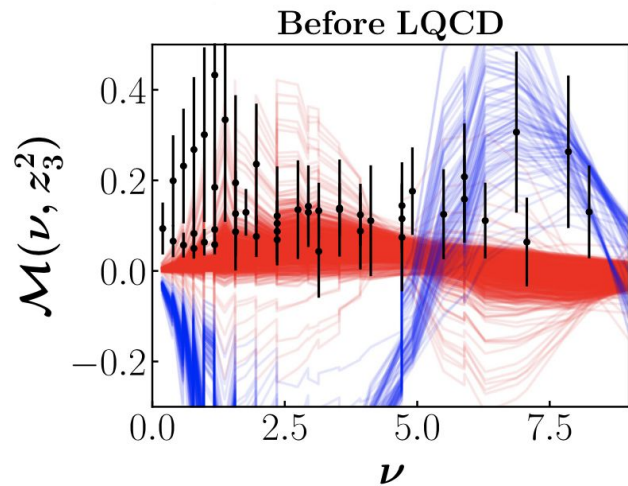
$$\boxed{\mathcal{I}_{\Delta g}(\nu, \mu^2) = \int_0^1 dx x \sin(x\nu) \Delta g(x, \mu^2)}$$



DIS $W^2 > 10 \text{ GeV}^2$



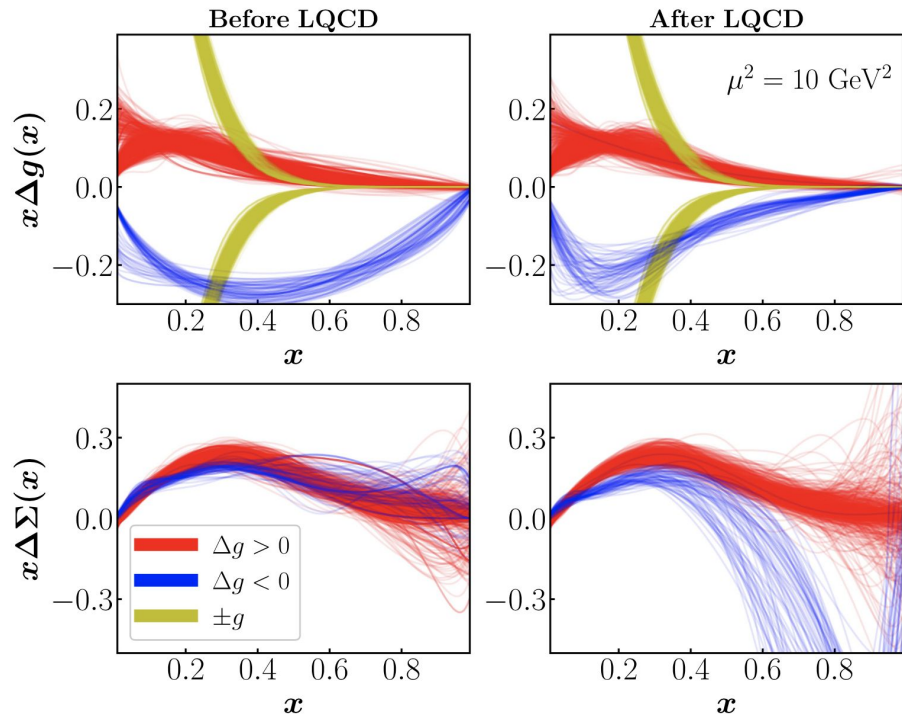
- Good description of global data after inclusion of LQCD for both solutions for Δg
- On the basis of χ^2 , LQCD cannot discriminate fully the sign of Δg



$$\begin{aligned}\chi^2 &= (\mathbf{d} - \mathbf{t})^T \boldsymbol{\Sigma}^{-1} (\mathbf{d} - \mathbf{t}) \\ &= (\mathbf{d} - \mathbf{t})^T \mathbf{U} \mathbf{D}^{-1} \mathbf{U}^T (\mathbf{d} - \mathbf{t}) \\ &= \sum_i \text{res}_i^{*2}.\end{aligned}$$



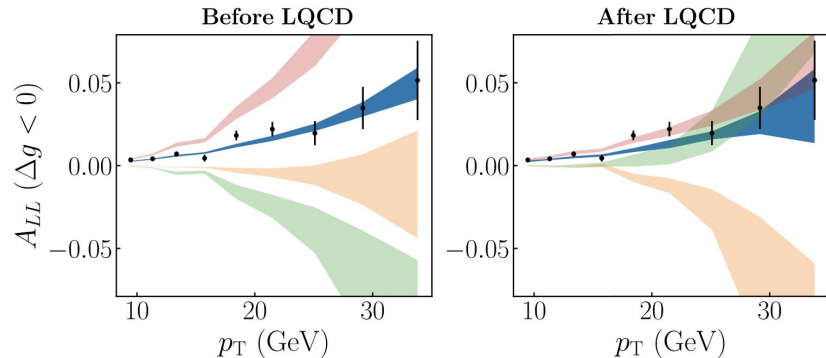
- Projections of residuals reveal strong correlations between LQCD data points
- The correlations prevent determination of sign of Δg



$$\Delta\Sigma = \sum_q (\Delta q + \Delta\bar{q})$$

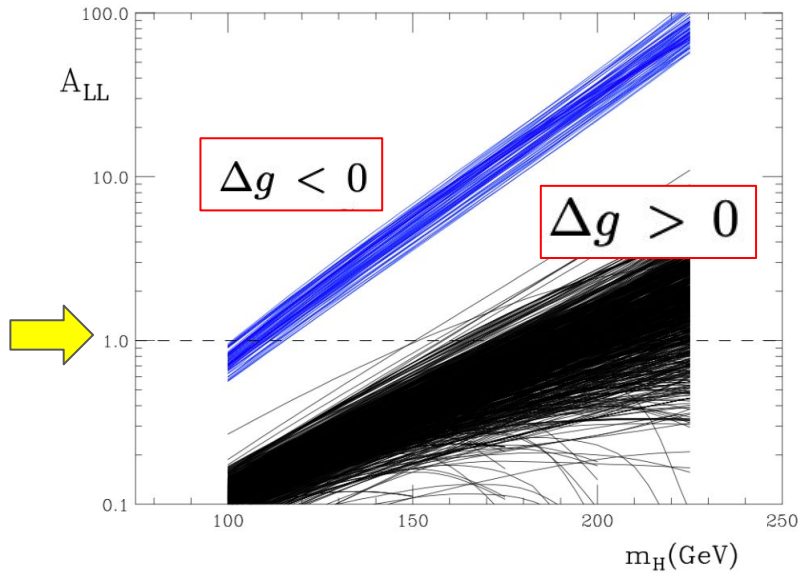


- LQCD distorts significantly the negative Δg at $x > 0.3$
- Note that both solutions violate PDF positivity bounds in $x > 0.3$
- Before inclusion of LQCD data, $\Delta\Sigma$ were stable for both solutions
- Inclusion of LQCD data forces the $\Delta\Sigma$ to become negative at $x > 0.4$ for the negative gluon solution



Higgs production at RHIC and the positivity of the gluon helicity distribution

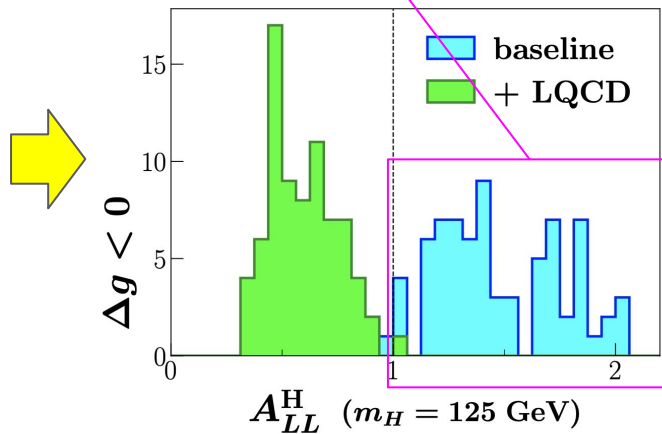
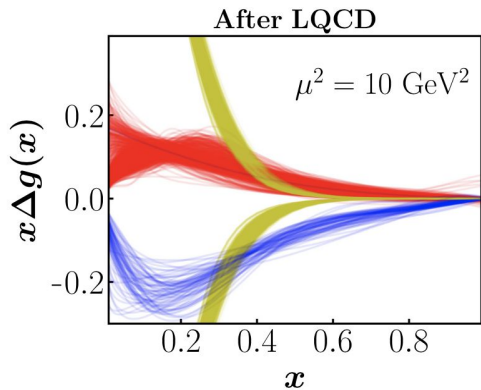
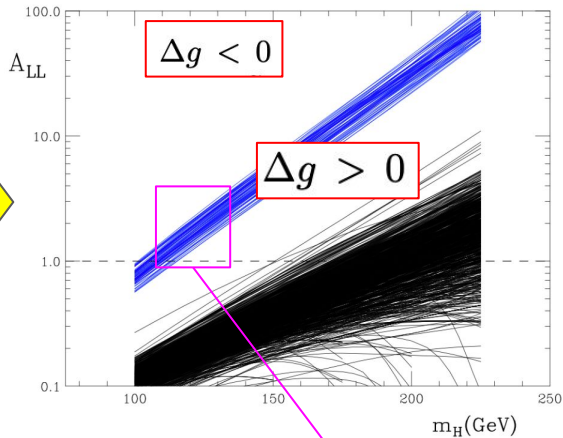
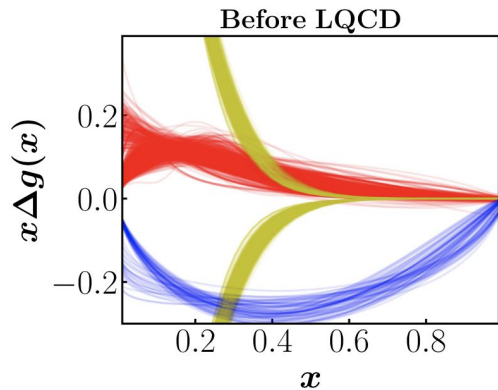
Daniel de Florian, Stefano Forte, and Werner Vogelsang
Phys. Rev. D **109**, 074007 – Published 10 April 2024



- Higgs A_{LL} is directly sensitive to Δg squared at LO
- Calculations of A_{LL} (Higgs) with negative Δg can lead to unphysical results (**using non-LQCD based analysis**)

$$A_{LL}^H(\tau) = \frac{[\Delta g \otimes \Delta g]}{[g \otimes g]} + \mathcal{O}(\alpha_s),$$

Can Higgs A_{LL} fully discriminate negative Δg ?

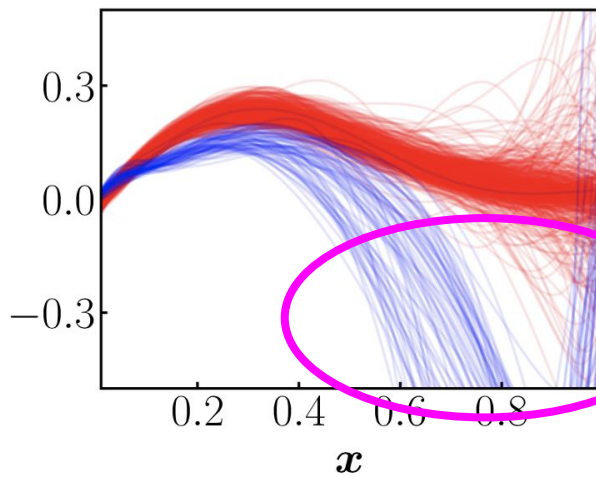
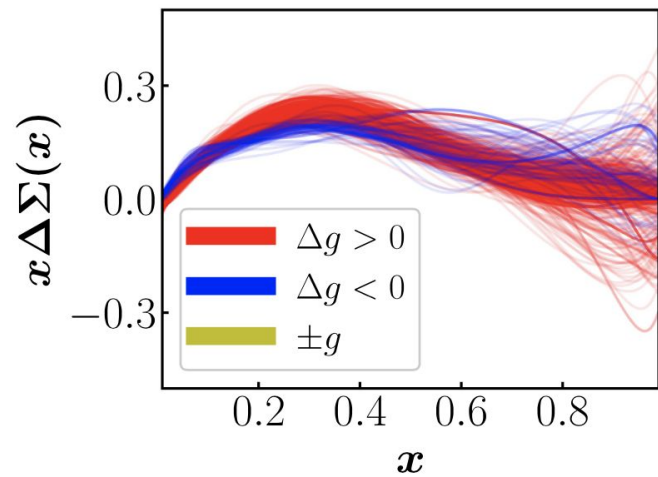
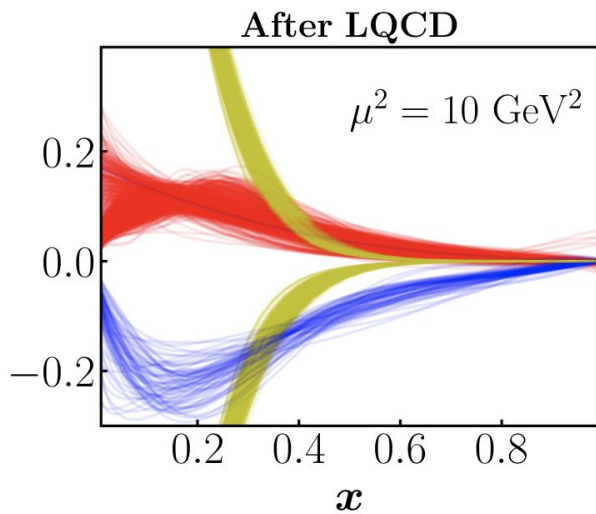
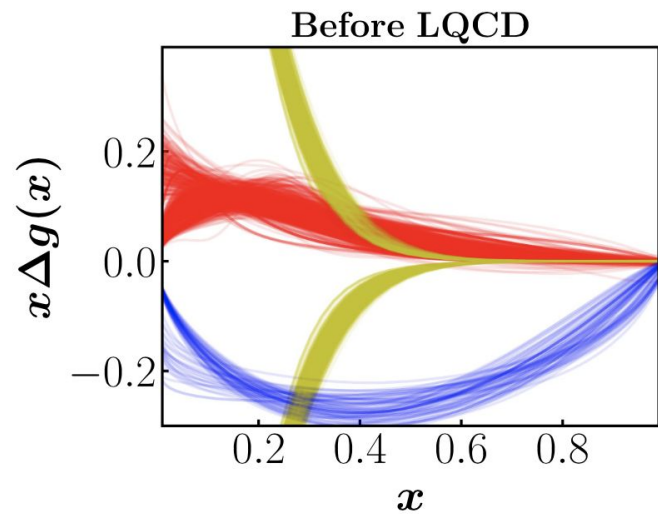


Negative Δg with LQCD constraints still admits a physical Higgs A_{LL}

New Data-Driven Constraints on the Sign of Gluon Polarization in the Proton

N. T. Hunt-Smith, C. Cocuzza, W. Melnitchouk, N. Sato, A. W. Thomas, and M. J. White (JAM Collaboration-Spin PDF Analysis Group)

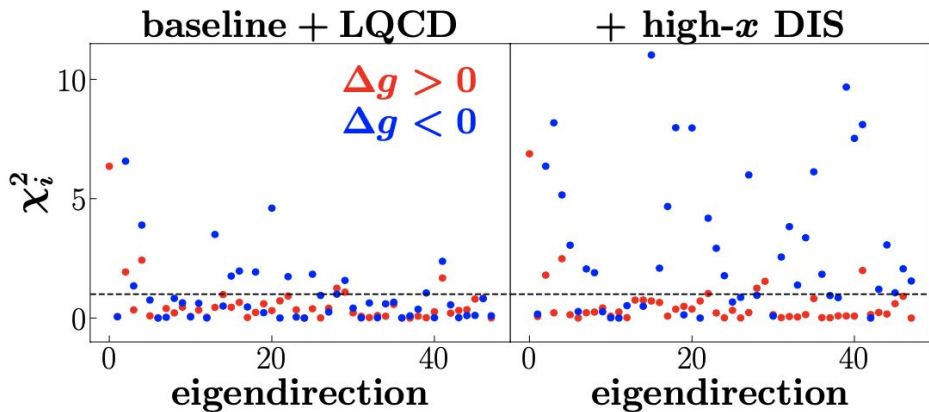
Phys. Rev. Lett. **133**, 161901 – Published 16 October 2024



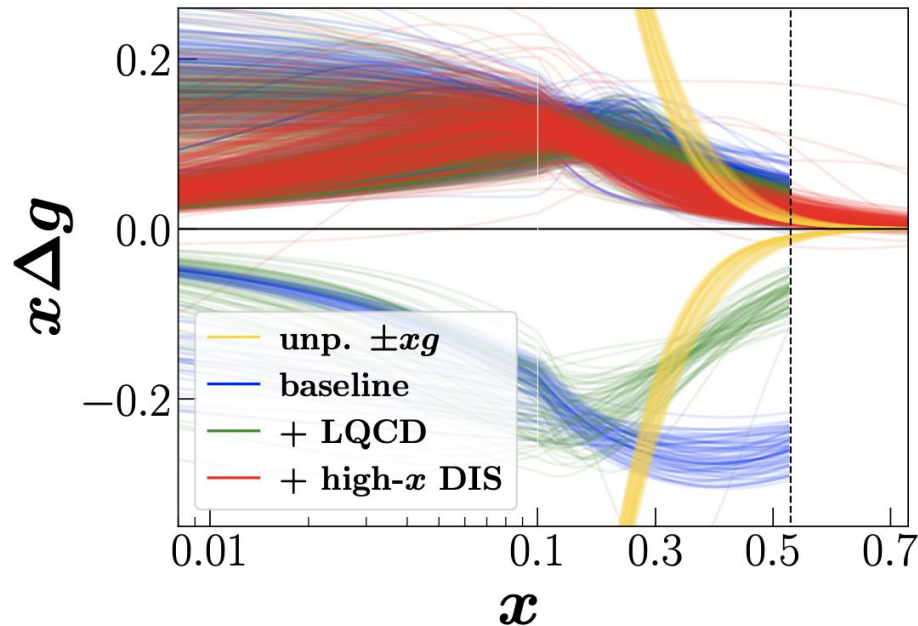
$$\Delta\Sigma = \sum_q (\Delta\mathbf{q} + \Delta\bar{\mathbf{q}})$$

Reaction	$\chi_{\text{red}}^2(\Delta g > 0)$			$\chi_{\text{red}}^2(\Delta g < 0)$			N
	baseline	+ LQCD	+ high- x DIS	baseline	+ LQCD	+ high- x DIS	
<i>Polarized</i>							
Inclusive DIS	0.95	0.96	1.21	0.98	1.12	1.25	1735*
SIDIS	0.85	0.84	1.08	0.84	0.96	1.11	231
Inclusive jets	0.84	0.89	0.90	0.88	1.10	1.44	83
Inclusive W^\pm/Z	0.60	0.60	0.99	0.83	0.84	1.32	18
<i>Total</i>	0.89	0.90	1.18	0.92	1.06	1.24	2067
<i>Unpolarized</i>							
Inclusive DIS	1.17	1.17	1.17	1.18	1.18	1.19	3908
SIDIS	0.99	0.99	1.04	0.99	0.99	1.02	1490
Inclusive jets	1.28	1.28	1.30	1.29	1.29	1.30	198
Drell-Yan	1.21	1.21	1.21	1.24	1.24	1.24	205
Inclusive W^\pm/Z	1.01	1.01	1.01	1.03	1.03	1.04	153
<i>Total</i>	1.14	1.14	1.14	1.15	1.15	1.15	5954
SIA	0.86	0.86	0.89	0.90	0.90	0.92	564
LQCD	—	0.57	0.58	—	1.18	3.92	48
<i>Total</i>	1.08	1.10	1.13	1.10	1.12	1.17	8633

1370 additional data points for pol DIS (+ high- x DIS)

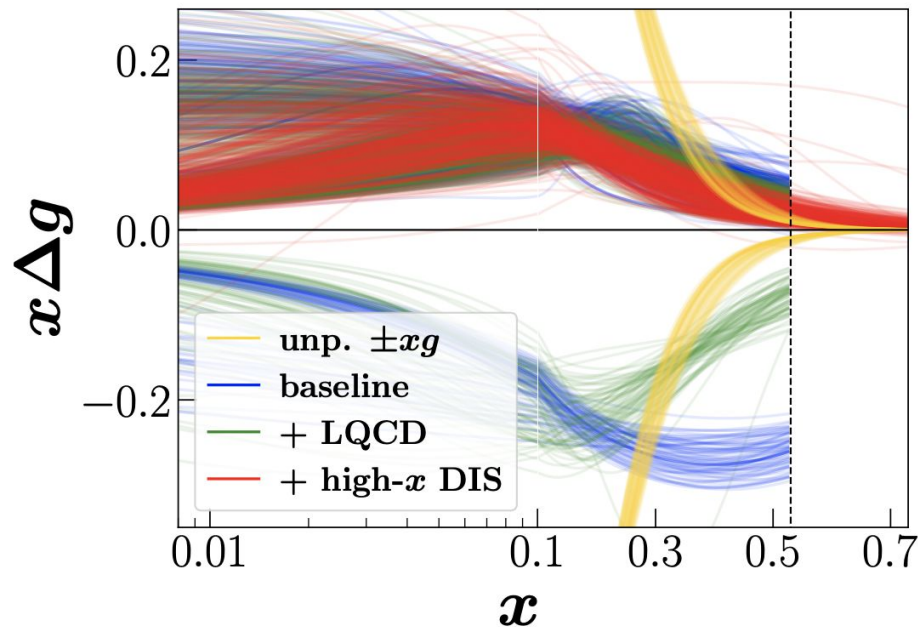


- With inclusion of high- x DIS DSAs, LQCD data strongly disfavor negative Δg solution
- Combined DSA from jet and high- x DIS with LQCD allows us to discriminate the sign of Δg for the first time!



Summary of collinear section

- For the first time, we were able to discriminate the sign of Δg using data-driven approach
- Constraints from LQCD along with DSAs from jets and DIS at large- x were crucial to achieve the resolution of Δg sign
- Inclusion of LQCD is becoming increasingly important in global analysis
- Experimental constraints at large x on Δg are still scarce, and more data are needed to reach precision similar to unpolarized gluon density (RHIC: **dijet**, EIC: small x , JLab-12/22: high x)

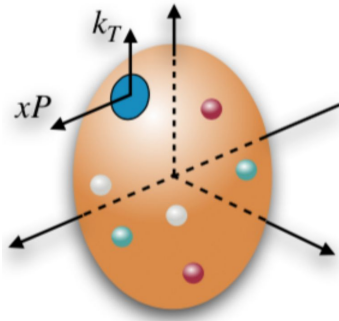


Proton structure in 3D

3D structure in momentum space

TMD (transverse momentum dependent) distributions:

- longitudinal momentum fraction
- transverse momentum k_T



TMD Handbook

A modern introduction to the physics of
Transverse Momentum Dependent distributions

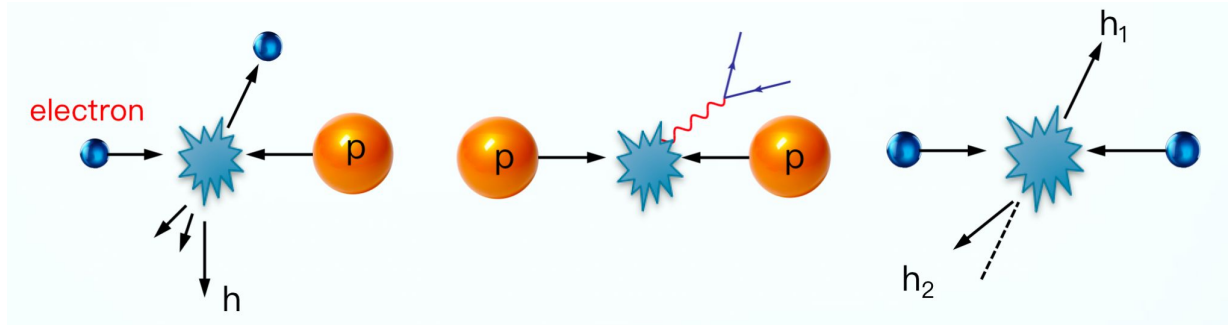


Renaud Boussarie
Matthias Burkardt
Martha Constantinou
William Detmold
Markus Ebert
Michael Engelhardt
Sean Fleming
Leonard Gamberg
Xiangdong Ji
Zhong-Bo Kang
Christopher Lee
Keh-Fei Liu
Simonetta Liuti
Thomas Mehen *
Andreas Metz
John Negele
Daniel Pitonyak
Alexei Prokudin
Jian-Wei Qiu
Abha Rajan
Marc Schlegel
Phiala Shanahan
Peter Schweitzer
Iain W. Stewart *
Andrey Tarasov
Raju Venugopalan
Ivan Vitev
Feng Yuan
Yong Zhao

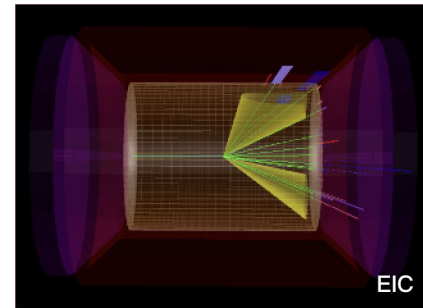
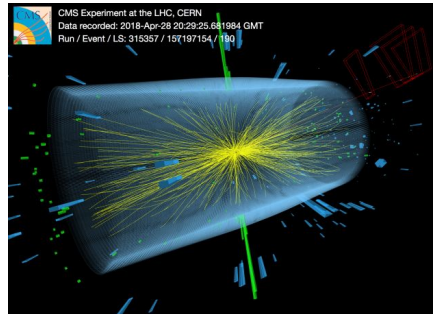
* - Editors

Processes to extract TMDs

- Standard processes: SIDIS, Drell-Yan, e^+e^-

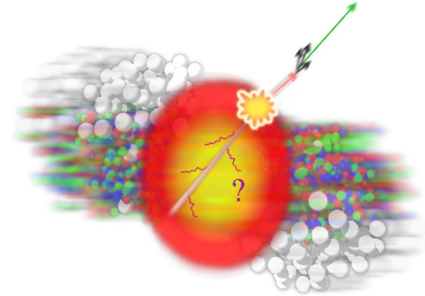


- Focus: using jets for 3D imaging



Why jets?

- Precision probe of QCD
- Explore beyond standard model (BSM) parameters
- Probe quark gluon plasma (QGP)
- ...

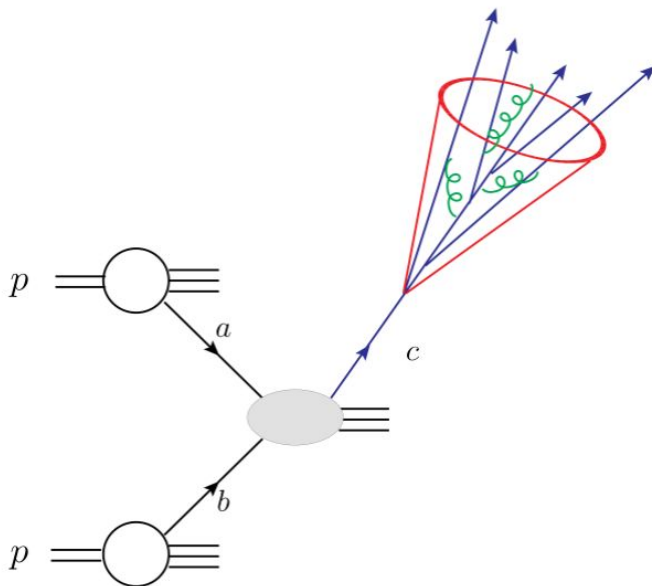


Advantage of jet substructure:

- Clean laboratory for TMD physics: only one TMD function is involved
- Tomography: “scan” the longitudinal momentum fraction z_h distribution

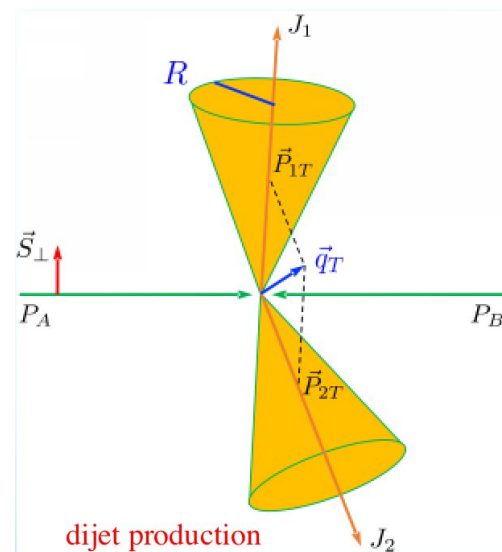
Two types of jet production

Single inclusive: only care about a single jet



Exclusive: a fixed number of final state jets

(back-to-back dijet/Z+jet)



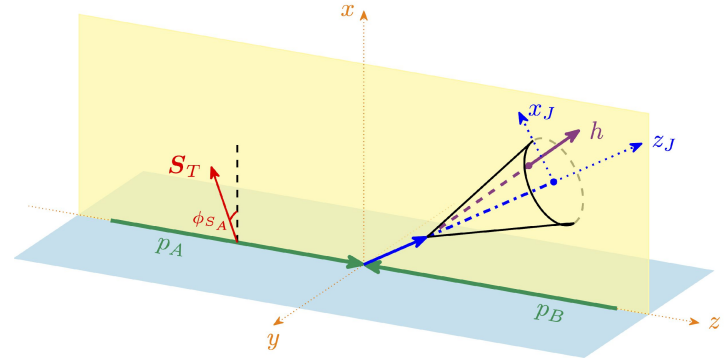
Single inclusive jet production

- Collinear PDFs: only one scale p_T is measured.
- TMD FFs: when hadron transverse momentum distribution is measured.

$$\frac{d\sigma^{pp \rightarrow \text{jet}(h)+X}}{dp_T d\eta dz_h} \propto f_a \otimes f_b \otimes H_{ab \rightarrow c} \otimes \mathcal{D}_1^{h/c}(z, z_h, p_T R, \mu),$$

$$\frac{d\sigma^{pp \rightarrow \text{jet}(h)+X}}{dp_T d\eta dz_h d^2\mathbf{j}_\perp} \propto f_a \otimes f_b \otimes H_{ab \rightarrow c} \otimes \mathcal{G}_1^{h/c}(z, z_h, \mathbf{j}_\perp, p_T R, \mu, \zeta_J),$$

Kang, Ringer & Vitev: 16; Dai, Kim & Leibovich: 16; Kaufmann, Mukherjee & Vogelsang: 15



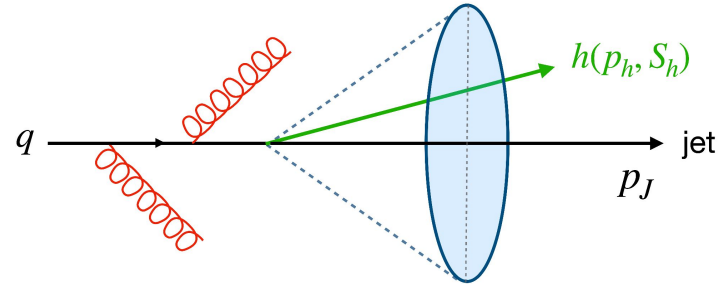
- z_h : large momentum fraction of hadron v.s. jet
- \mathbf{j}_\perp : hadron transverse momentum w.r.t jet axis

Relation between fragmenting **jet** functions (FJFs) and **standard** fragmentation functions

If you measure only collinear z_h distribution

$$\Delta_{(T)} \mathcal{G}_i^h(z, z_h, p_T R, \mu) = \sum_j \int_{z_h}^1 \frac{dz'_h}{z'_h} \Delta_{(T)} \mathcal{J}_{ij}(z, z'_h, p_T R, \mu) \Delta_{(T)} D_{h/j}\left(\frac{z_h}{z'_h}, \mu\right)$$

$$\begin{aligned} \Delta \mathcal{J}_{qq}(z, z_h, p_T R, \mu) = & \delta(1-z)\delta(1-z_h) + \frac{\alpha_s}{2\pi} \left\{ L \left[\Delta P_{qq}(z)\delta(1-z_h) - \Delta P_{qq}(z_h)\delta(1-z) \right] \right. \\ & + \delta(1-z) \left[2C_F(1+z_h^2) \left(\frac{\ln(1-z_h)}{1-z_h} \right)_+ + C_F(1-z_h) + \Delta \mathcal{T}_{qq}^{\text{anti-}k_T}(z_h) \right] \\ & \left. - \delta(1-z_h) \left[2C_F(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ + C_F(1-z) \right] \right\}, \quad (2.43) \end{aligned}$$



Relation between FJFs and **standard** fragmentation function

$$\begin{aligned} \Delta \mathcal{J}_{qq}(z, z_h, p_T R, \mu) = & \delta(1-z)\delta(1-z_h) + \frac{\alpha_s}{2\pi} \left\{ L \left[\Delta P_{qq}(z)\delta(1-z_h) - \Delta P_{qq}(z_h)\delta(1-z) \right] \right. \\ & + \delta(1-z) \left[2C_F(1+z_h^2) \left(\frac{\ln(1-z_h)}{1-z_h} \right)_+ + C_F(1-z_h) + \Delta \mathcal{T}_{qq}^{\text{anti-}k_T}(z_h) \right] \\ & \left. - \delta(1-z_h) \left[2C_F(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ + C_F(1-z) \right] \right\}, \quad (2.43) \end{aligned}$$

$$\begin{aligned} \Delta \mathcal{J}_{gq}(z, z_h, p_T R, \mu) = & \frac{\alpha_s}{2\pi} \left\{ L \left[\Delta P_{gq}(z)\delta(1-z_h) - \Delta P_{gq}(z_h)\delta(1-z) \right] \right. \\ & + \delta(1-z) \left[2\Delta P_{gq}(z_h) \ln(1-z_h) - 2C_F(1-z_h) + \Delta \mathcal{T}_{gq}^{\text{anti-}k_T}(z_h) \right] \\ & \left. - \delta(1-z_h) \left[2\Delta P_{gq}(z) \ln(1-z) - 2C_F(1-z) \right] \right\}, \quad (2.44) \end{aligned}$$

$$\begin{aligned} \Delta \mathcal{J}_{gg}(z, z_h, p_T R, \mu) = & \frac{\alpha_s}{2\pi} \left\{ L \left[\Delta P_{gg}(z)\delta(1-z_h) - \Delta P_{gg}(z_h)\delta(1-z) \right] \right. \\ & + \delta(1-z) \left[2\Delta P_{gg}(z_h) \ln(1-z_h) + 2T_F(1-z_h) + \Delta \mathcal{T}_{gg}^{\text{anti-}k_T}(z_h) \right] \\ & \left. - \delta(1-z_h) \left[2\Delta P_{gg}(z) \ln(1-z) + 2T_F(1-z) \right] \right\}, \quad (2.45) \end{aligned}$$

$$\begin{aligned} \Delta \mathcal{J}_{gq}(z, z_h, p_T R, \mu) = & \delta(1-z)\delta(1-z_h) + \frac{\alpha_s}{2\pi} \left\{ L \left[\Delta P_{gq}(z)\delta(1-z_h) - \Delta P_{gq}(z_h)\delta(1-z) \right] \right. \\ & + \delta(1-z) \left[4C_A(2(1-z_h)^2 + z_h^2) \left(\frac{\ln(1-z_h)}{1-z_h} \right)_+ \right. \\ & \quad \left. - 4C_A(1-z_h) + \Delta \mathcal{T}_{gq}^{\text{anti-}k_T}(z_h) \right] \\ & \left. - \delta(1-z_h) \left[4C_A(2(1-z)^2 + z) \left(\frac{\ln(1-z)}{1-z} \right)_+ - 4C_A(1-z) \right] \right\}, \quad (2.46) \end{aligned}$$

$$\begin{aligned} \Delta_T \mathcal{J}_{qq}(z, z_h, p_T R, \mu) = & \delta(1-z)\delta(1-z_h) + \frac{\alpha_s}{2\pi} \left\{ L \left[\Delta_T P_{qq}(z)\delta(1-z_h) - \Delta_T P_{qq}(z_h)\delta(1-z) \right] \right. \\ & + \delta(1-z) \left[4C_F z_h \left(\frac{\ln(1-z_h)}{1-z_h} \right)_+ + \Delta_T \mathcal{T}_{qq}^{\text{anti-}k_T}(z_h) \right] \\ & \left. - \delta(1-z_h) \left[4C_F z \left(\frac{\ln(1-z)}{1-z} \right)_+ \right] \right\}, \quad (2.47) \end{aligned}$$

$$\begin{aligned} \Delta \mathcal{J}_{qq}(z_h, p_T R, \mu) = & \delta(1-z_h) + \frac{\alpha_s C_F}{2\pi} \left[\delta(1-z_h) \left(\frac{L^2}{2} - \frac{\pi^2}{12} \right) - \Delta P_{qq}(z_h)L \right. \\ & \left. + 1 - z_h + \Delta \hat{\mathcal{J}}_{qq}^{\text{anti-}k_T}(z_h) \right], \end{aligned}$$

$$\Delta \mathcal{J}_{gq}(z_h, p_T R, \mu) = \frac{\alpha_s C_F}{2\pi} \left[-\Delta P_{gq}(z_h)L - 2(1-z_h) + \Delta \hat{\mathcal{J}}_{gq}^{\text{anti-}k_T}(z_h) \right],$$



$$\Delta \mathcal{J}_{gg}(z_h, p_T R, \mu) = \frac{\alpha_s T_F}{2\pi} \left[-\Delta P_{gg}(z_h)L + 2(1-z_h) + \Delta \hat{\mathcal{J}}_{gg}^{\text{anti-}k_T}(z_h) \right],$$









$$\begin{aligned} \Delta \mathcal{J}_{gq}(z_h, p_T R, \mu) = & \delta(1-z_h) + \frac{\alpha_s C_A}{2\pi} \left[\delta(1-z_h) \left(\frac{L^2}{2} - \frac{\pi^2}{12} \right) - \Delta P_{gq}(z_h)L \right. \\ & \left. - 4(1-z_h) + \Delta \hat{\mathcal{J}}_{gq}^{\text{anti-}k_T}(z_h) \right], \end{aligned}$$

$$\begin{aligned} \Delta_T \mathcal{J}_{qq}(z_h, p_T R, \mu) = & \delta(1-z_h) + \frac{\alpha_s C_F}{2\pi} \left[\delta(1-z_h) \left(\frac{L^2}{2} - \frac{\pi^2}{12} \right) \right. \\ & \left. - \Delta_T P_{qq}(z_h)L + \Delta_T \hat{\mathcal{J}}_q^{\text{anti-}k_T}(z_h) \right], \end{aligned}$$

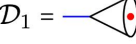
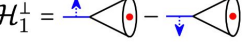
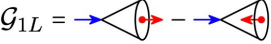

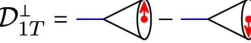



Relation between TMD FFs and TMD FJFs

If you measure both z_h and \mathbf{j}_\perp

Leading Quark TMDFFs  Hadron Spin  Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons		$D_1 = \text{Unpolarized}$ 		$H_1^\perp = \text{Collins}$ 
	L		$G_1 = \text{Helicity}$ 	H_{1L}^\perp 
Polarized Hadrons	T	$D_{1T}^\perp = \text{Polarizing FF}$ 	G_{1T}^\perp 	$H_1 = \text{Transversity}$  H_{1T}^\perp 

[TMD handbook, 2304.03302](#)

		Quark polarization		
		U	L	T
Hadron polarization	U	$\mathcal{D}_1 =$ 		$\mathcal{H}_1^\perp =$ 
	L		$\mathcal{G}_{1L} =$ 	$\mathcal{H}_{1L}^\perp =$ 
	T	$\mathcal{D}_{1T}^\perp =$ 	$\mathcal{G}_{1T} =$ 	$\mathcal{H}_1 =$  $\mathcal{H}_{1T}^\perp =$ 

[Kang, Xing, Zhao and Zhou, 2311.00672](#)

How do we connect them?

TMD FJFs

If you measure both z_h and \mathbf{j}_\perp

$$\mathcal{D}_1(z, z_h, \mathbf{j}_\perp) = \hat{H}_{c \rightarrow i}^U(z) \int \frac{b \, db}{(2\pi)^2} J_0\left(\frac{j_\perp b}{z_h}\right) \tilde{D}_1^{h/i}(z_h, \mathbf{b})$$

The diagram illustrates the decomposition of the TMD FJF into two main components. The first component, $\hat{H}_{c \rightarrow i}^U(z)$, is circled in red and labeled "Hard splitting & jet formation". The second component, $\tilde{D}_1^{h/i}(z_h, \mathbf{b})$, is circled in magenta and labeled "Standard TMD FFs". A magenta arrow points from the second component to the text "Hadron fragmentation & soft correction.".

Kang, Xing, Zhao and **Zhou**, 2311.00672

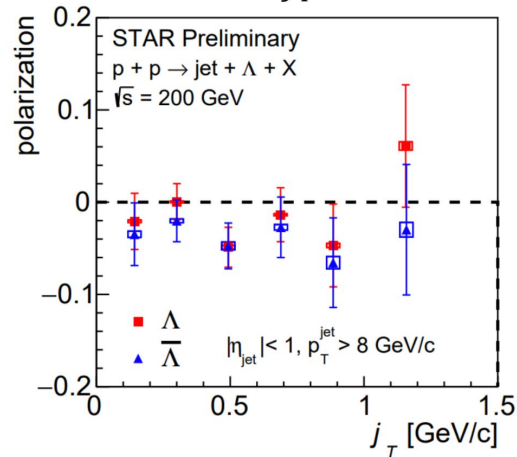
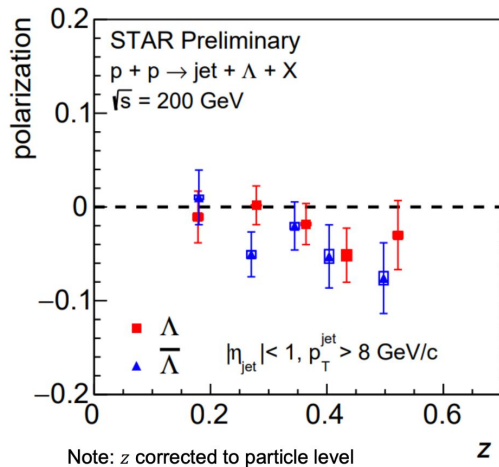
ALL coefficients have been computed in our work!

Λ -baryon polarization

STAR, 2023

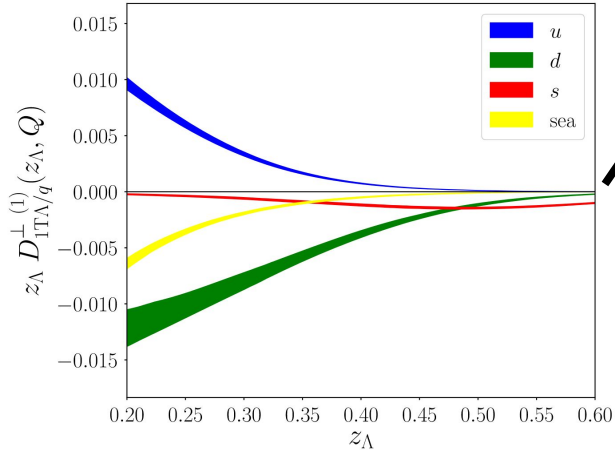
- Large transverse polarization found for Λ produced in unpolarized hadron scattering
- STAR recent measurement: test of universality of Λ polarized FFs

Polarization as a function of z and j_T



Λ -baryon polarization

$$\mathcal{D}_{1T}^\perp(z, z_h, \mathbf{j}_\perp) = \hat{H}_{c \rightarrow i}^U(z) \int \frac{b^2 db}{(2\pi)^2} \frac{z_\Lambda^2 m_\Lambda^2}{j_\perp} J_1\left(\frac{j_\perp b}{z_\Lambda}\right) \tilde{D}_1^{\perp, (1)}(z_h, \mathbf{b})$$



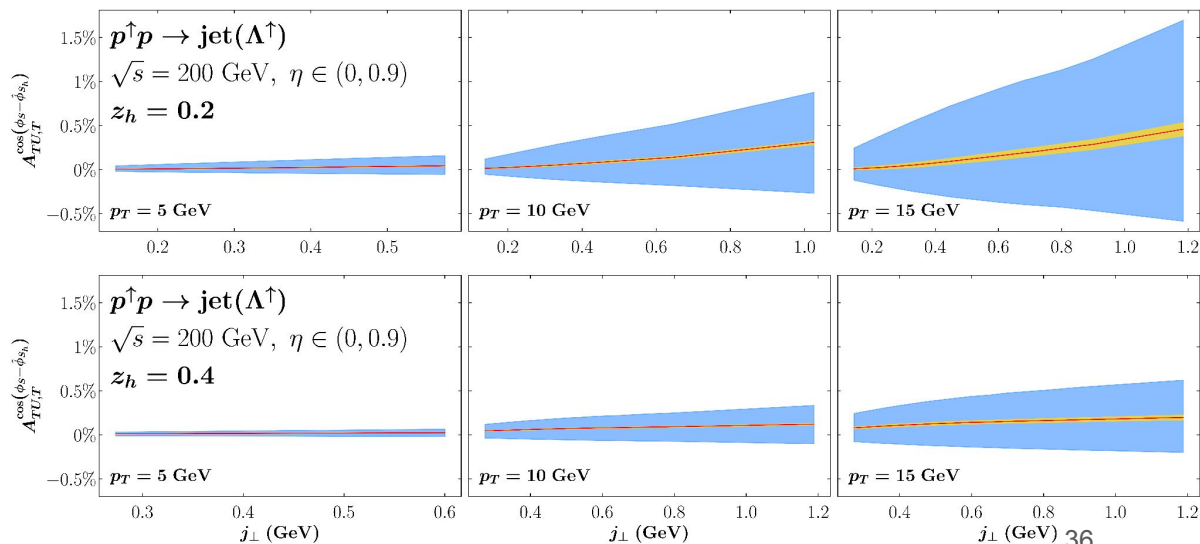
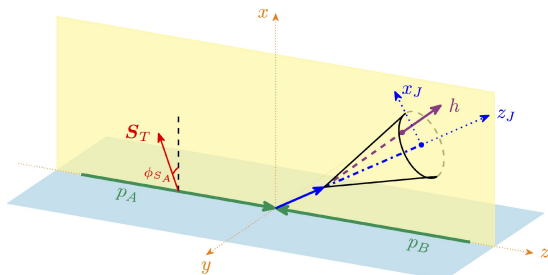
		Quark polarization		
		U	L	T
Hadron polarization	U	$\mathcal{D}_1 = \langle \rightarrow \circ \rangle$		$\mathcal{H}_1^\perp = \langle \uparrow \circ \rangle - \langle \downarrow \circ \rangle$
	L		$\mathcal{G}_{1L} = \langle \rightarrow \circ \rangle - \langle \leftarrow \circ \rangle$	$\mathcal{H}_{1L}^\perp = \langle \uparrow \circ \rangle - \langle \downarrow \circ \rangle$
	T	$\mathcal{D}_{1T}^\perp = \langle \rightarrow \circ \rangle - \langle \leftarrow \circ \rangle$	$\mathcal{G}_{1T} = \langle \rightarrow \circ \rangle + \langle \leftarrow \circ \rangle$	$\mathcal{H}_1 = \langle \uparrow \circ \rangle + \langle \downarrow \circ \rangle$ $\mathcal{H}_{1T}^\perp = \langle \uparrow \circ \rangle - \langle \downarrow \circ \rangle$

currently in preparation

Single inclusive jet production: $pp \rightarrow \text{jet}(h)$

- collinear transversity PDFs
- TMD transversely polarized FFs

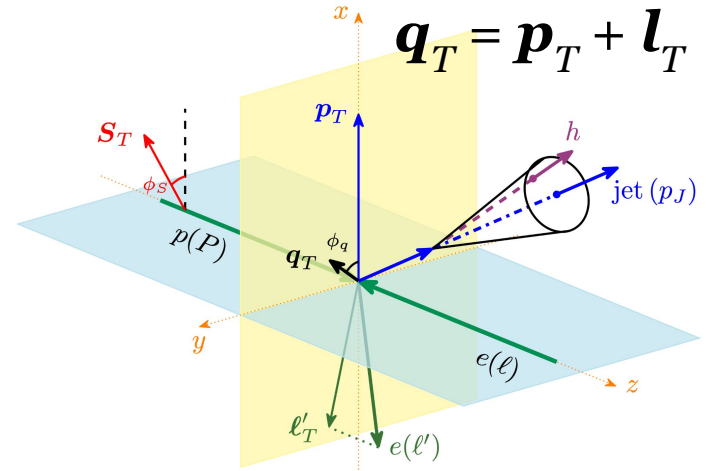
$$A_{TU,T}(z_h, j_\perp) \equiv \frac{h_1 \otimes f \otimes \mathcal{H}_1 \otimes \Delta_T \hat{\sigma}}{f \otimes f \otimes \mathcal{D}_1 \otimes \hat{\sigma}}$$



Exclusive jet production

- Momentum imbalance \mathbf{q}_T : sensitive to initial-state TMD distributions
- Hadron \mathbf{j}_\perp : sensitive to TMD FFs

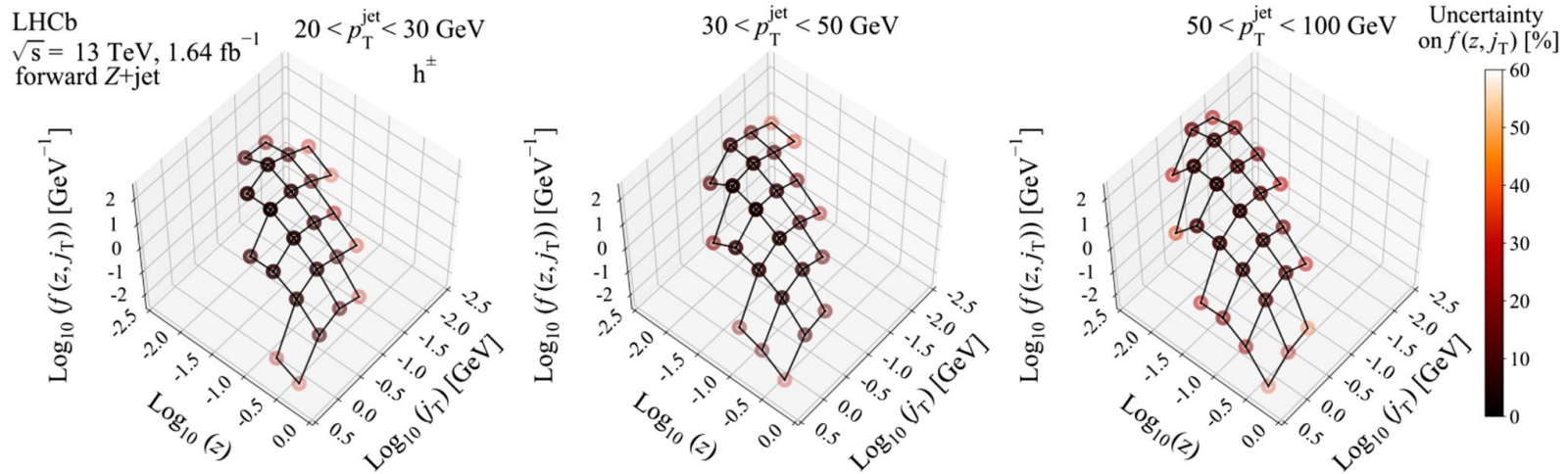
$$\frac{d\sigma_{pp}}{d\mathcal{PS}} = \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{-i\mathbf{q}_T \cdot \mathbf{b}} \tilde{f}_a^{q/p}(x_a, b) \tilde{f}_b^{q/p}(x_b, b) \\ \times \tilde{S}_{n\bar{n}n_J}(\mathbf{b}) \tilde{S}_{n_J}^{cs}(\mathbf{b}, R) H_{ab \rightarrow cZ}(p_T, m_Z) J_c(p_{JT} R)$$



- Kang, Lee, Shao & Zhao: [2106.15624](#)
- Kang, Lee, Xing, Zhao & Zhou: 2505.XXXX

Exclusive jet production: $pp \rightarrow Z + \text{jet}(h)$

- Recent measurement by LHCb ([2208.11691](#))
- First time differential in both z_h and \mathbf{j}_\perp (proposed in [1906.07187](#))



Exclusive jet production: $pp \rightarrow Z + \text{jet}(h)$

- JAM fitted FFs ([2101.04664](#), [2202.03372](#), **Zhou**: in preparation)
- Data included: e^+e^- , SIDIS, polarized SIDIS

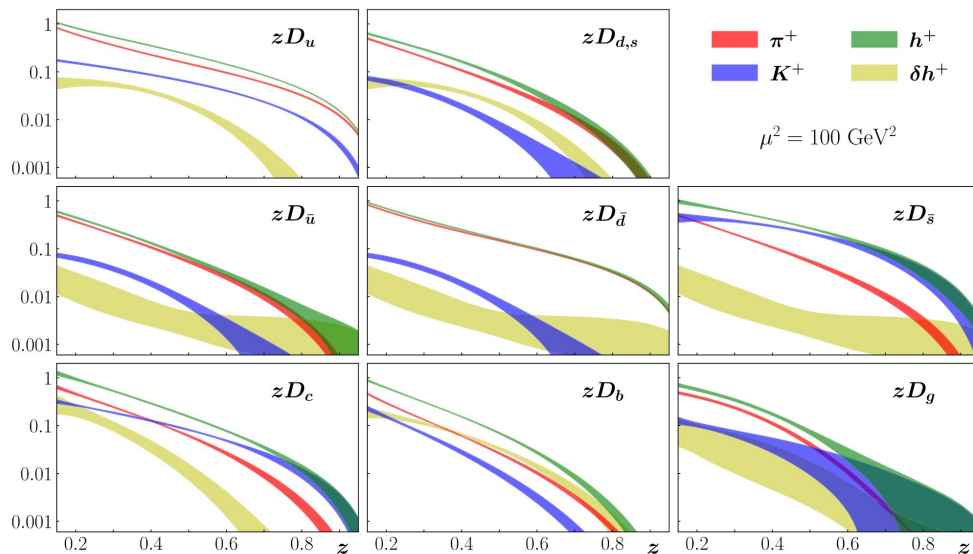
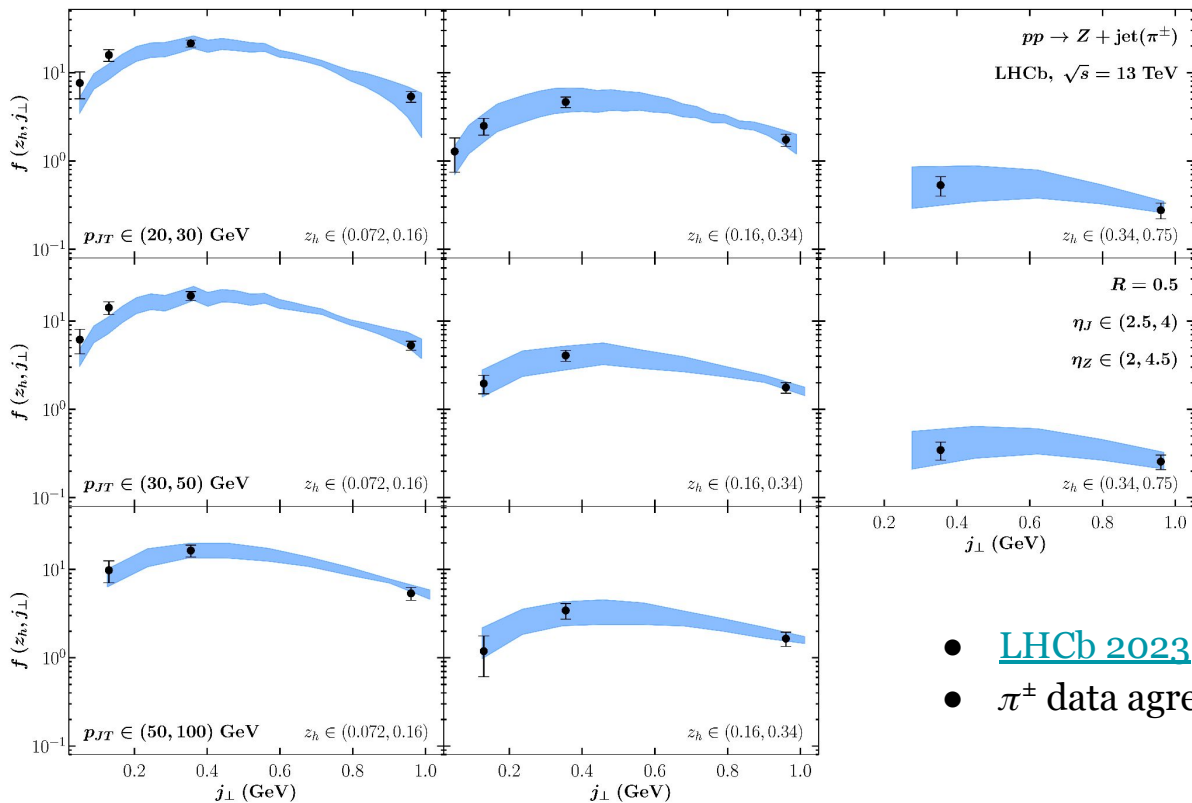


TABLE I. Summary of χ^2 values per number of points N_{dat} for the various datasets used in this analysis.

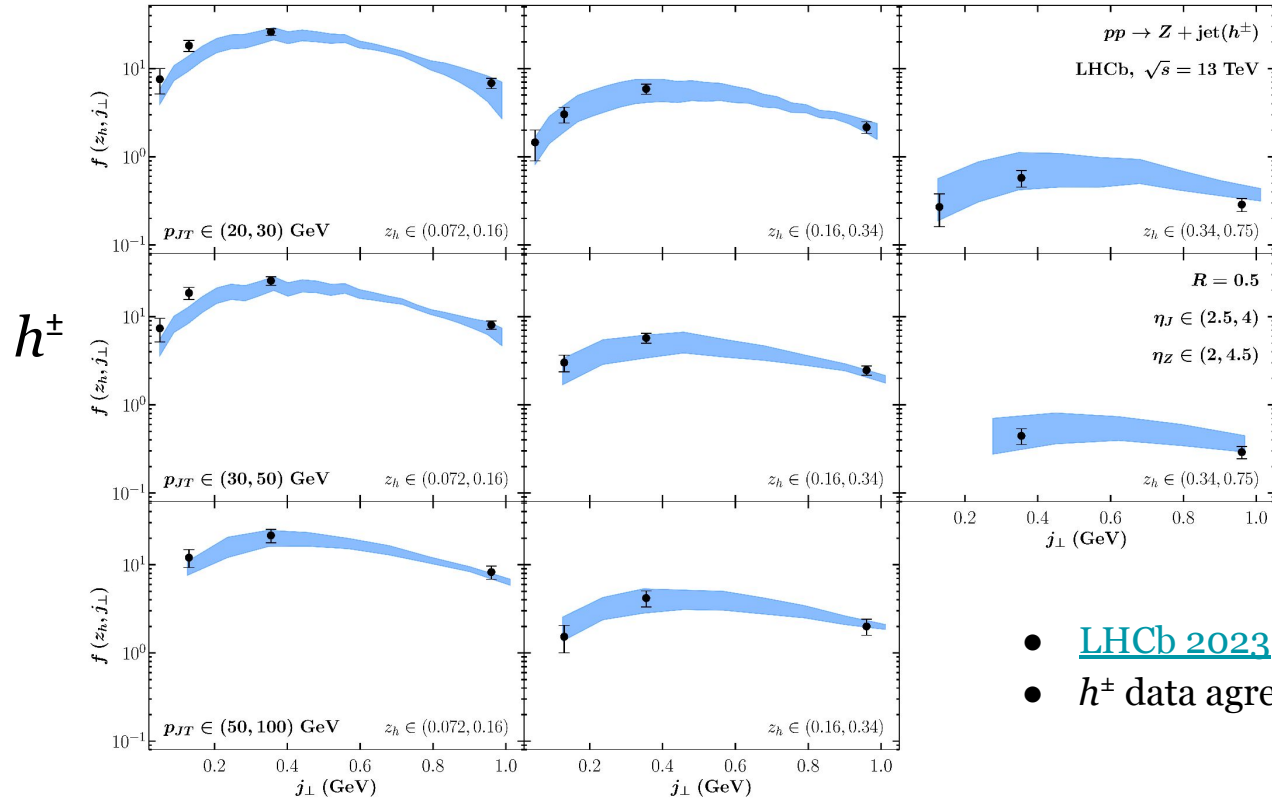
Process	N_{dat}	χ^2/N_{dat}
<i>Polarized</i>		
Inclusive DIS	365	0.95
SIDIS (π^+, π^-)	64	1.05
SIDIS (K^+, K^-)	57	0.42
SIDIS (h^+, h^-)	110	0.95
Inclusive jets	83	0.84
STAR W^\pm	12	0.65
PHENIX W^\pm/Z	6	0.50
Total	697	0.89
<i>Unpolarized</i>		
Inclusive DIS	3908	1.17
SIDIS (π^+, π^-)	498	0.94
SIDIS (K^+, K^-)	494	1.31
SIDIS (h^+, h^-)	498	0.71
Inclusive jets	198	1.28
Drell-Yan	205	1.21
W/Z production	153	1.01
Total	5954	1.12
SIA (π^\pm)	231	0.91
SIA (K^\pm)	213	0.70
SIA (h^\pm)	120	1.07
Total	7215	1.08

Exclusive jet production: $pp \rightarrow Z + \text{jet}(\pi^\pm)$

π^\pm



Exclusive jet production: $pp \rightarrow Z + \text{jet}(h^\pm)$



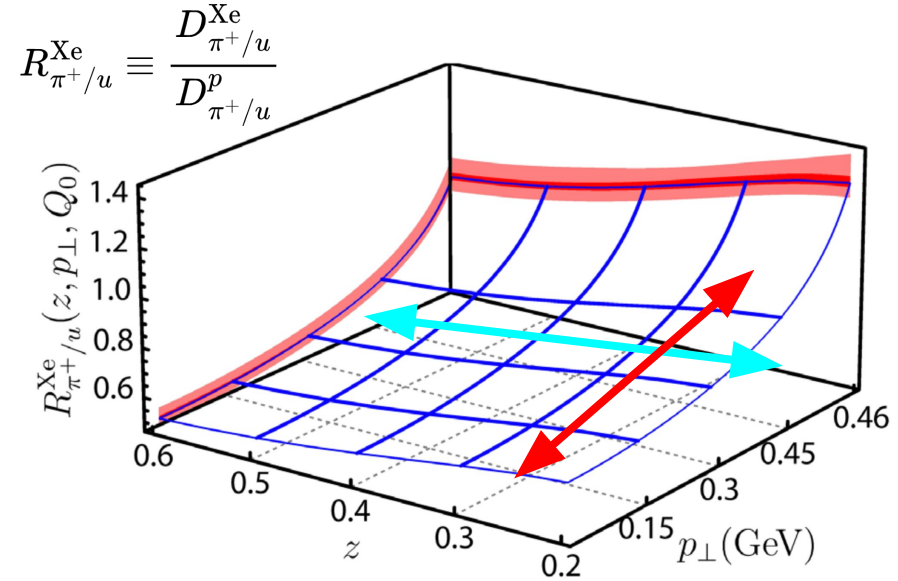
- [LHCb 2023](#)
- h^\pm data agree very well ✓

Exclusive jet production: $p\text{Pb} \rightarrow Z + \text{jet}(h)$

- Nuclear TMD modification extracted in [Alrashed, Anderle, Kang, Terry and Xing, 2107.12401](#)
- Fitted for TMD PDFs & TMD FFs

$$S_{\text{NP}}^{q/A,f}(b, Q_0, \sqrt{\zeta_a}) = \frac{g_2}{2} \ln\left(\frac{b}{b_*}\right) \ln\left(\frac{\sqrt{\zeta_a}}{Q_0}\right) + g_1^{q/A} b^2,$$

$$g_1^{q/A} = g_1 + a_N L, \quad L = A^{1/3} - 1$$

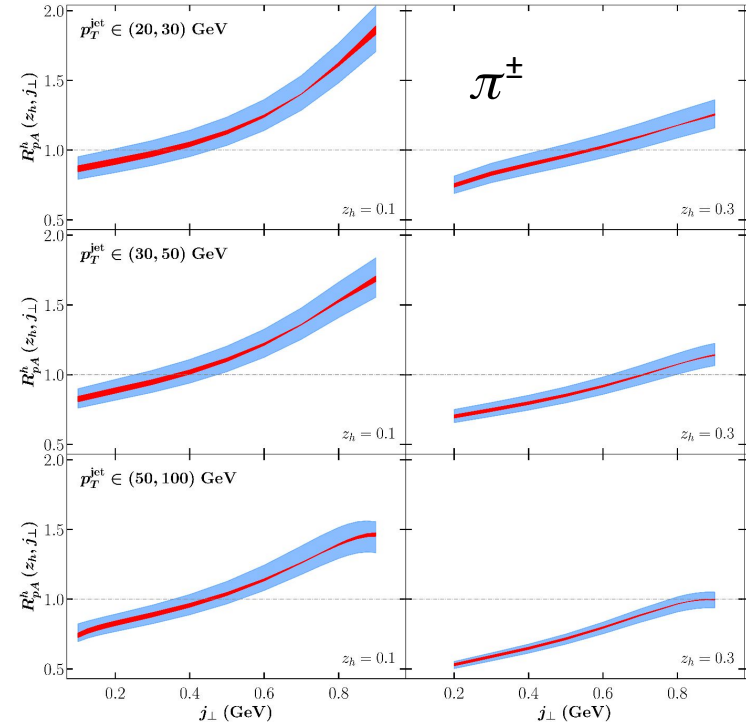


- **Broadening of transverse momentum distribution**
- **Behaviour driven by collinear FFs**

Exclusive jet production: $p\text{Pb} \rightarrow Z + \text{jet}(h)$

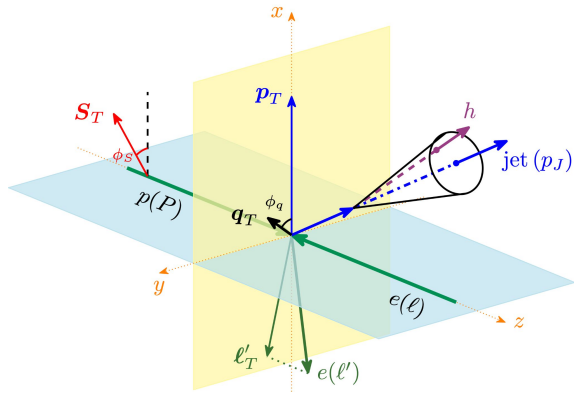
- Nuclear TMD modification extracted in [Alrashed, Anderle, Kang, Terry and Xing, 2107.12401](#)
- The reaction is $p\text{Pb} \rightarrow Z + \text{jet}(\pi^\pm)$
- LHC is interested in the observable and is planning to measure it

$$S_{\text{NP}}^{q/A,f}(b, Q_0, \sqrt{\zeta_a}) = \frac{g_2}{2} \ln\left(\frac{b}{b_*}\right) \ln\left(\frac{\sqrt{\zeta_a}}{Q_0}\right) + g_1^{q/A} b^2,$$

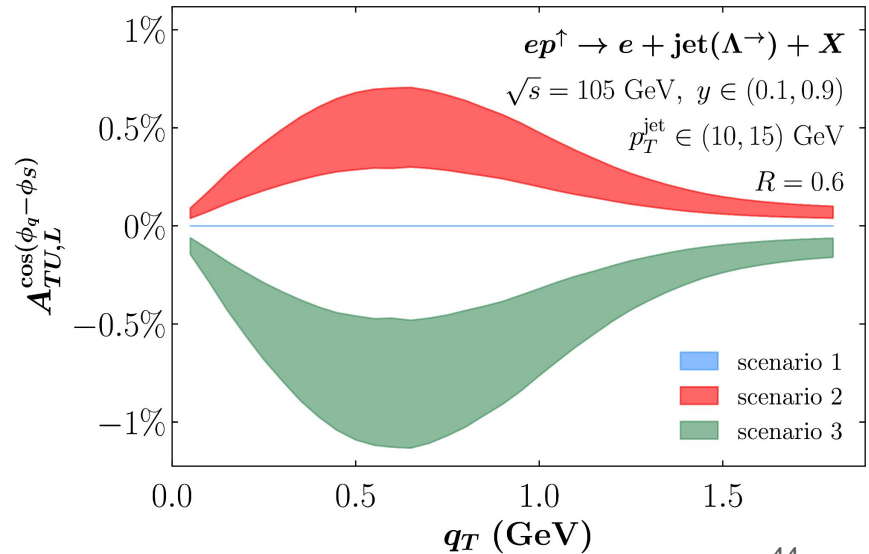


Exclusive jet production: $ep \rightarrow e + \text{jet}(h)$

- worm-gear function
- longitudinally polarized FFs



$$A_{TU,L}(z_h, j_\perp) \equiv \frac{\hat{\sigma}_0 H \mathcal{G}_{1L} \tilde{g}_{1T} \bar{S}_{\text{global}} \bar{S}_{\text{cs}}}{\hat{\sigma}_0 H \mathcal{D}_1 \tilde{f}_1 \bar{S}_{\text{global}} \bar{S}_{\text{cs}}}$$



Summary of TMD section

- We established relations between **jet** fragmentation functions and **standard** fragmentation functions in all possible polarizations
- We use them to describe experimental data at RHIC and LHC
 - Kaon FFs can be further constrained from LHCb data
 - Λ polarization in jet from STAR can be described by our formalism
- Nuclear TMD corrections can also be studied with our formalism