Covariant analysis of electromagnetic current on the light cone

exposition with scalar Yukawa theory

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LFQCD Seminar

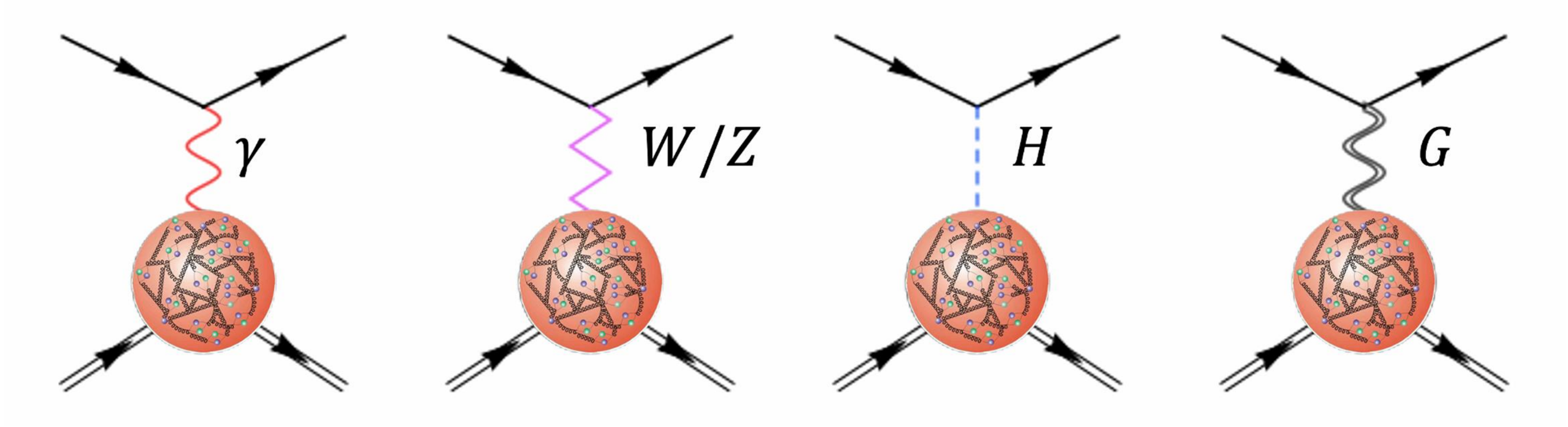
January 8, 2025

Big puzzles of the strong force within hadrons

- Origin of confinement
- Origin of $>99\%$ nucleon mass
- Origin of the nucleon spin

Hadronic electromagnetic form factor

Fundamental interactions:



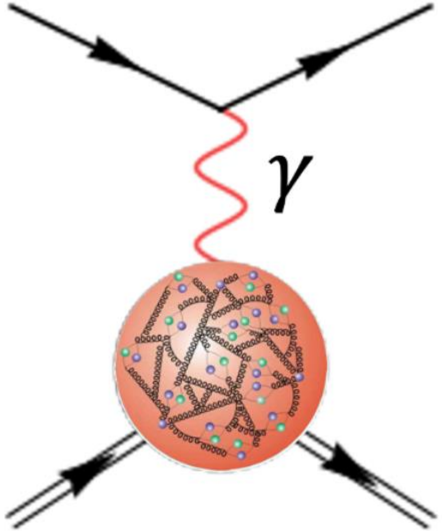
Hadronic electromagnetic(E.M.) form factor encodes the charge densities inside hadrons:

$$\rho(r) = \frac{1}{(2\pi)^3} \int d^3\vec{q}\, F(q^2) e^{-i\vec{q}\cdot\vec{r}}$$

Hadronic electric current & E.M. form factor

Electric current matrix elements and E.M. form factor:

$$j^{\mu} = \langle \psi_{ph}(p') | J^{\mu} | \psi_{ph}(p) \rangle = (p' + p)^{\mu} F(Q^2) \ , Q^2 = -q^2 = -(p' - p)^2$$



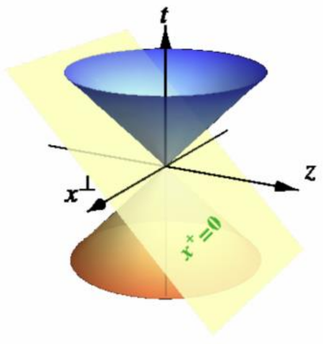
- One of the best measured observables in experiments

  [Hofstadter: 1956 RMP]

- The small Q^2 behavior of $F(Q^2)$, which determines the shape of the hadron, falls into the realm of non-perturbative QCD

The Drell-Yan-West formula

The Drell-Yan-West formula equates the pion form factor $F_\pi(Q^2)$ to the overlap of wave functions on the light front $x^+ = 0$:



$$F_\pi(Q^2) = \int_0^1 \frac{dx}{2x(1-x)} \int \frac{d^2k_\perp}{(2\pi)^3} \psi_\pi^*\left(x, \vec{k}_\perp\right) \psi_\pi\left(x, \vec{k}_\perp - x\vec{q}_\perp\right) + \cdots\cdots$$

higher Fock sector contributions

[S. D. Drell and T. M. Yan: 1970 PRL; G. B. West: 1970 PRL]

- Drell-Yan frame (transverse frame): $q^+ = 0$
- $\psi_\pi\left(x, \vec{k}_\perp\right)$: the pion valence light front wave function (LFWF)
- $x = x_1 = \frac{p_1^+}{p^+}$: the longitudinal momentum fraction of the quark
- $\vec{k}_\perp = \vec{p}_{1\perp} - x_1\, \vec{p}_\perp$: the relative transverse momentum of the quark

Discrepancies in E.M. form factors

- Isgur & Llewellyn-Smith, form factor calculated in longitudinal frame($q^+ \neq 0, q^\perp = 0$) exhibits large discrepancies with the form factor given by Drell-Yan West formula

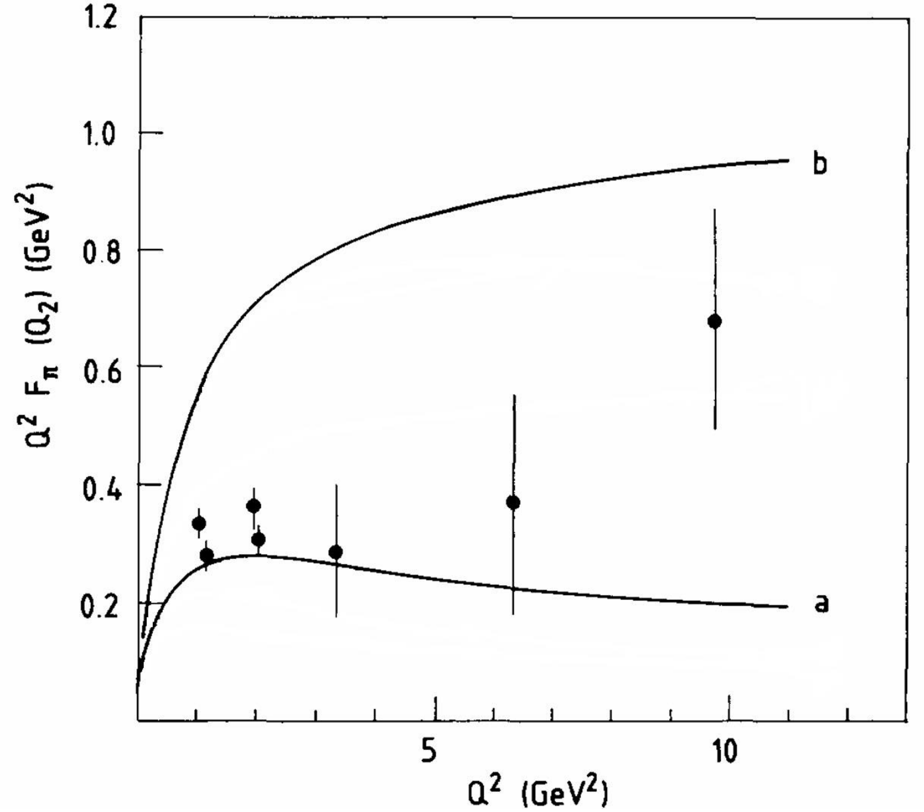


FIG. Pion form as a function of Q^2 . Curve (a) represents the result in longitudinal frame. Curve (b) represents the result given by the Drell-Yan West formula.

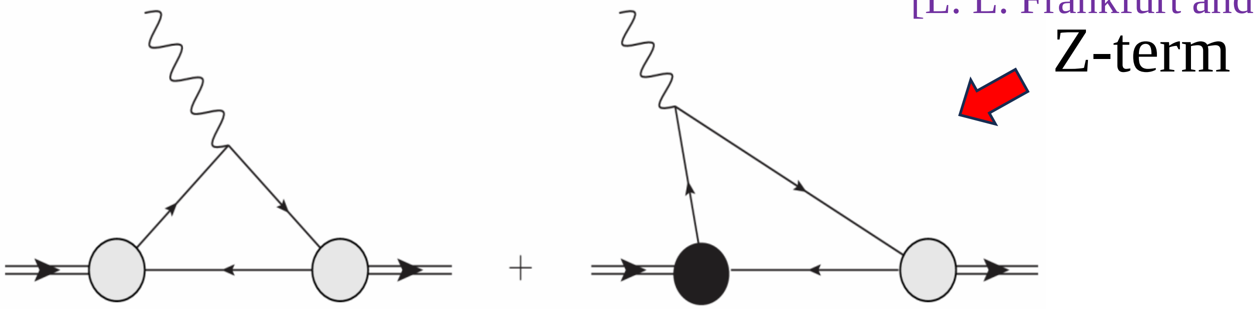
[N. Isgur and C. H. Llewellyn Smith: 1989 NPB]

- This apparent loss of covariance raised doubts to the Drell-Yan-West formula in non-perturbative regime

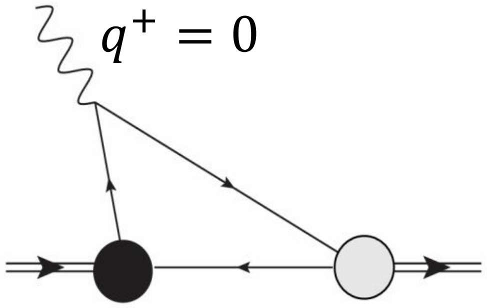
Z-term and zero-mode contributions

- From the Bethe-Salpeter approach, there exists a Z-term (pair creation/ annihilation term), which cannot be represented as the overlap of LFWFs!

[L. L. Frankfurt and M. I. Strikman: 1989 NPB]



- In transverse frame ($q^+ = 0$), Z-term turns into the zero-mode contribution



Partons: $p_i^+ = \frac{p_{i\perp}^2 + m_i^2}{p_i^-} \geq 0$

- We should calculate electric current and E.M. form factor in general frame first, and then take the Drell-Yan limit $q^+ \to 0$ to avoid missing the zero modes

Examples of zero modes in E.M. current

- Using the light cone perturbation theory, Brodsky and Hwang showed that zero modes do not contribute to the " + " component of the E.M. current in a ϕ^3 scalar model

[S. J. Brodsky and D. S. Hwang: 1999 NPB]

- Starting from Bethe-Salpeter approach, de Melo found that zero modes do not contribute to the pion E.M. form factor extracted from the j^+ current in a pseudoscalar field theoretical model

[J. P. B. C. de Melo et. al.: 1999 PRC]

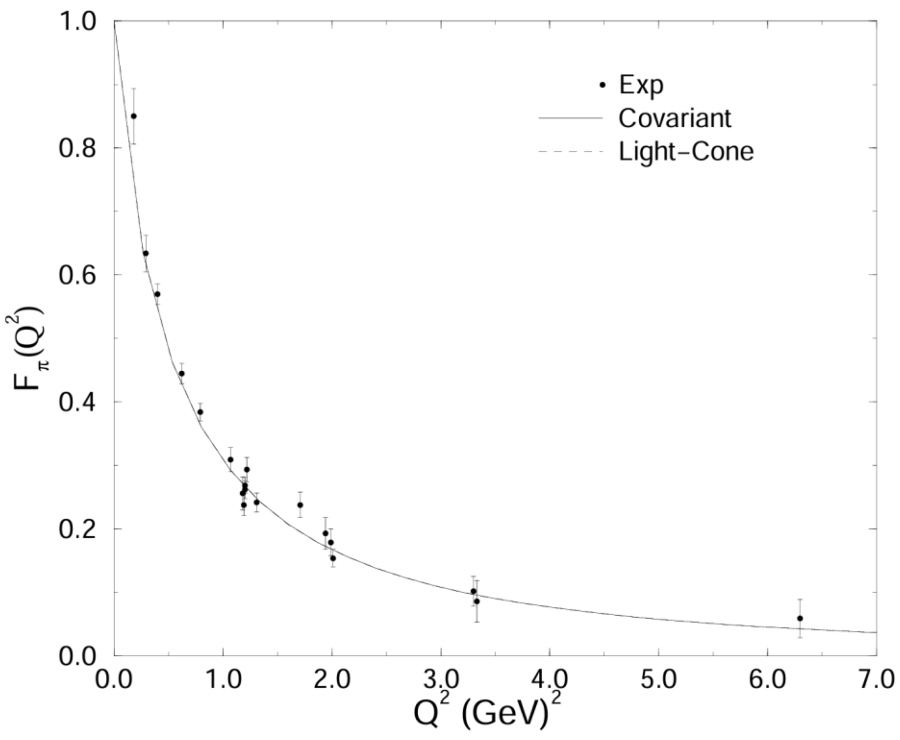


FIG. Pion form factor $F_\pi(Q^2)$ as a function of Q^2

Examples of zero modes in E.M. current

- However, de Melo derived the non-vanishing zero-modes to the E.M. current j^+ of a composite spin-one two-fermion system in Drell-Yan frame

[J. P. B. C. de Melo et. al.: 1999 NPA]

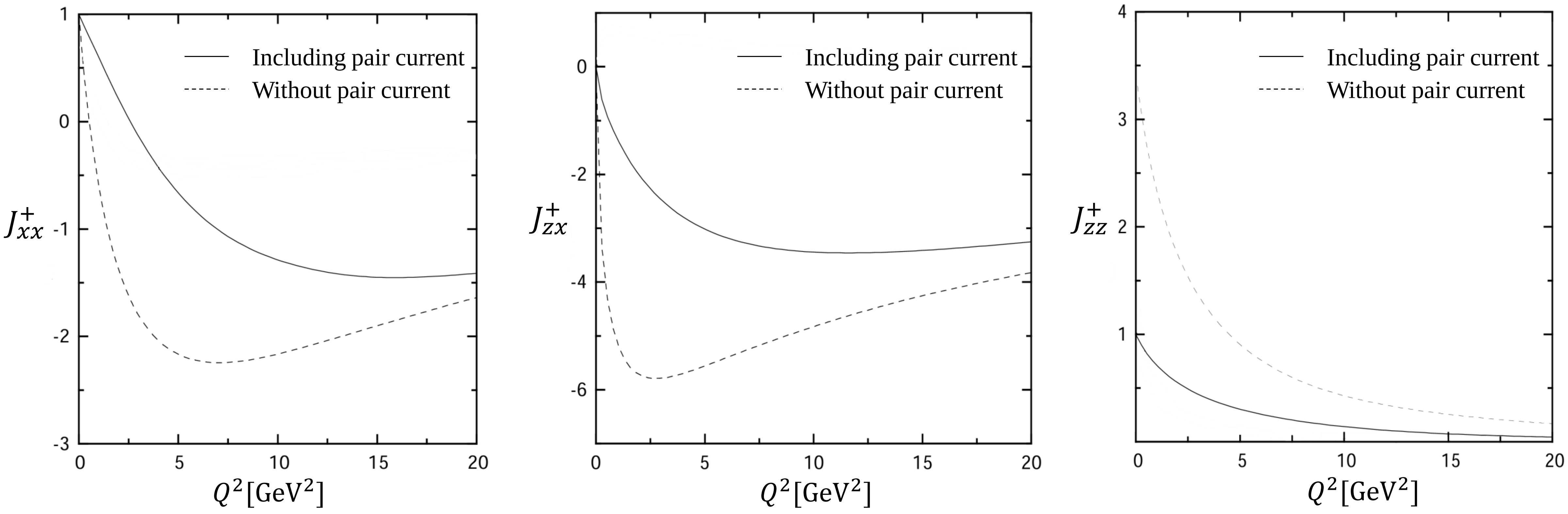


FIG. Current component J^+ as a function of Q^2 in transverse frame

Examples of zero modes in E.M. current

- In the exactly solvable model of (1+1)-dimensional scalar field theory, Choi calculated the Z-term E.M. form factor, and found that it will not vanish in Drell-Yan limit $q^+ \to 0$

[Ho-Meoyng Choi et. al.: 1998 PRD]

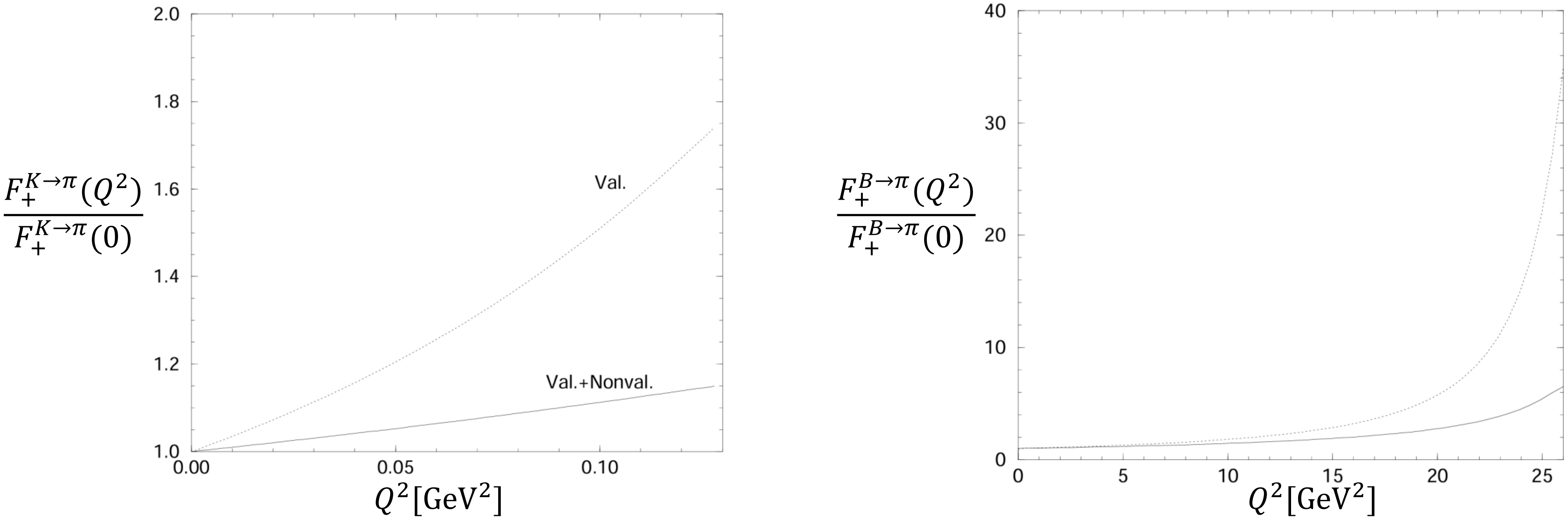


FIG. Normalized form factor of $F_+(Q^2)$ as a function of Q^2 in $(1+1)$ dimension

Examples of zero modes in E.M. current

- Although there are many researches about zero mods, this problem has not been completely resolved. By using different approach, the results of zero mode contributions are different, making the zero-mode problem more mysterious.

- Zero modes had been studied from perturbation theory, Bethe-Salpeter equation etc. But it has not yet been studied from a systematic non-perturbative theory so far.

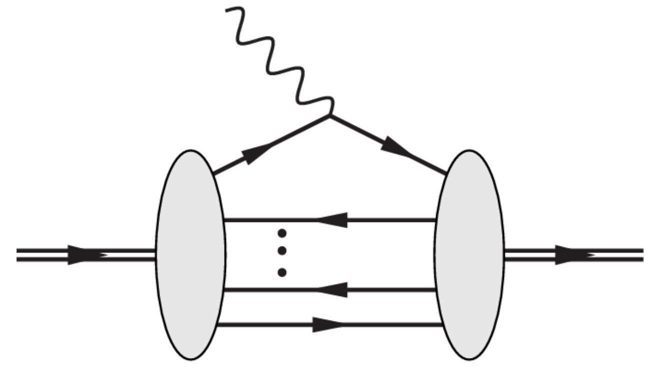
We need a systematic non-perturbative investigation of zero modes

Systematic non-perturbative approach

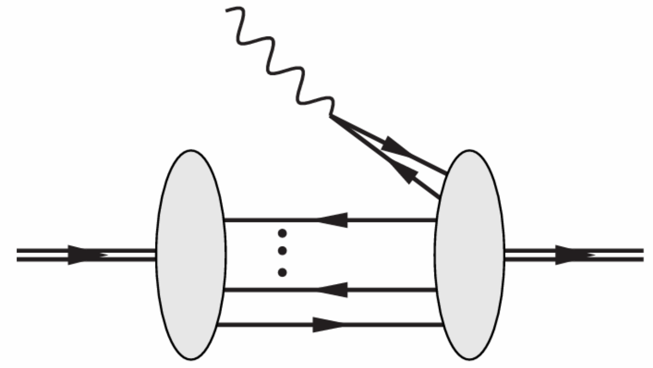
- Fock sector expansion:

$$\left|\psi_{ph}\right\rangle = |1\rangle + |2\rangle + |3\rangle + \cdots\cdots$$

$$j^{\mu} = \langle\psi_{ph}(p')|J^{\mu}\left|\psi_{ph}(p)\right\rangle = j_1^{\mu} + j_2^{\mu} + j_3^{\mu} + \cdots\cdots$$



Diagonal contributions



Non-diagonal contributions
(Z-term contributions)

- Wave functions in full Fock space contain the complete information, but involving all sectors in practical calculations is impossible, truncation is inevitable

Scalar Yukawa theory

- Strongly coupled scalar Yukawa theory:

$$\mathcal{L} = \partial_\mu N^\dagger \partial^\mu N - m^2 N^\dagger N + \frac{1}{2}\partial_\mu \pi \partial^\mu \pi - \frac{1}{2}\mu^2 \pi^2 + g N^\dagger N \pi$$

$$\left|\psi_{ph}\right\rangle = |N\rangle + |\pi N\rangle + |\pi\pi N\rangle + \left|N N \bar{N}\right\rangle$$

N: (Mock) scalar nucleon
$\bar{N}$: (Mock) scalar anti-nucleon
π: (Mock) pion

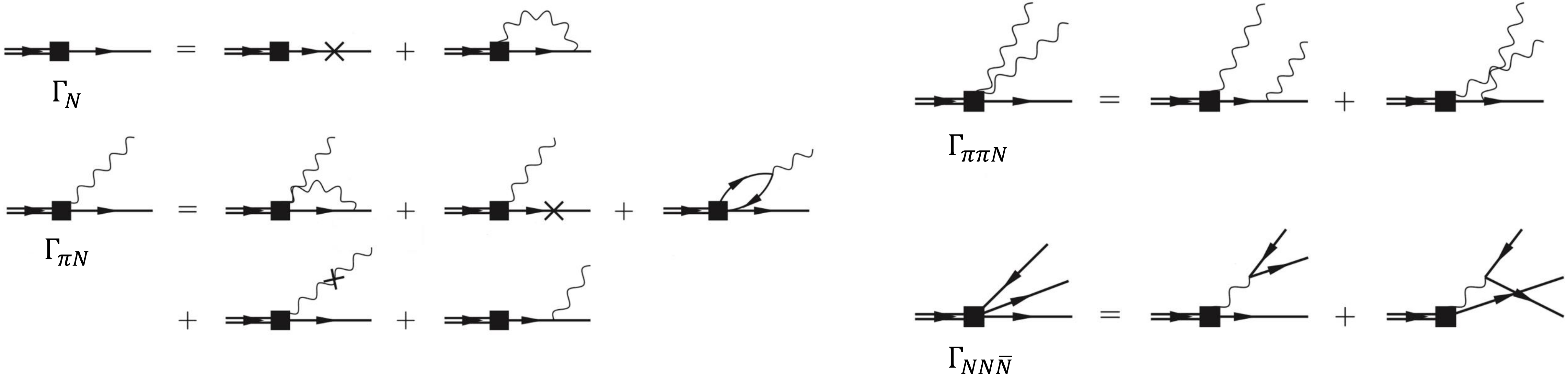
m: The mass of nucleon
μ: The mass of Pion

- Previous work for the same theory with a four-body truncation showed that the three-body truncation is numerically converged for a number of observables, so the three-body truncation suffices for our purpose

[Y. Li et. al.: 2015 PLB]

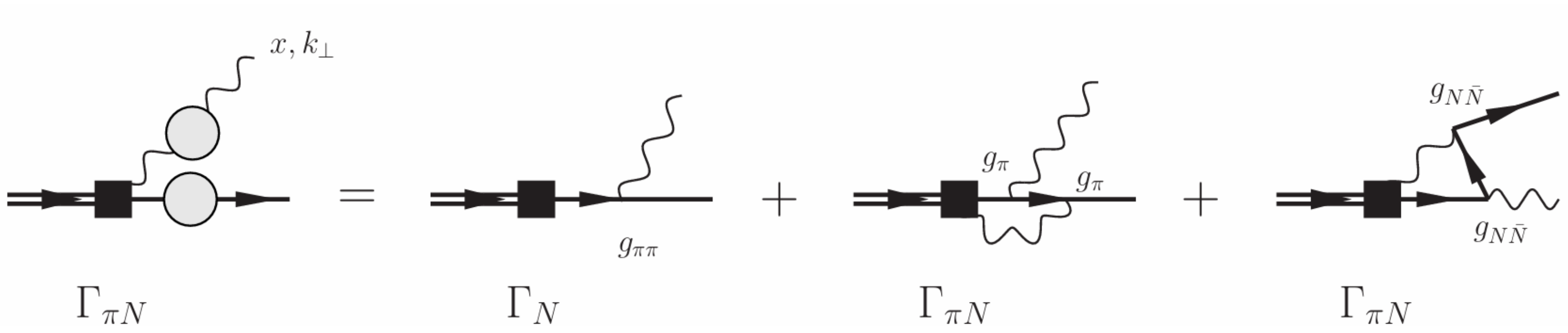
Light-front Hamiltonian approach

- Hamiltonian: $\hat{p}^- = 2\int d^3x\left(|\nabla_\perp N|^2 + m^2 N^\dagger N + \frac{1}{2}|\nabla_\perp \pi|^2 + \frac{1}{2}\mu^2\pi^2 - gN^\dagger N\pi\right)$
- Eigenvalue equation: $\hat{p}^- \left|\psi_{ph}\right\rangle = \frac{p_\perp^2 + M_{ph}^2}{p^+}\left|\psi_{ph}\right\rangle$
- Diagrammatic representation of eigenvalue equation: [V. A. Karmanov et. al.:2008 PRD]



Eigenvalue equation of scalar Yukawa theory

$$\left[1-\frac{\Sigma^R(t_a)}{t_a-m^2}-\frac{\Pi^R(t_b)}{t_b-\mu^2}\right]\Gamma_{\pi N}(x,k_\perp)=\frac{g_\pi g_{N\bar{N}}}{4m\sqrt{\pi\alpha}}$$
$$+g_\pi^2\int_0^{1-x}\frac{dx'}{2x'(1-x')(1-x-x')}\int\frac{d^2k'_\perp}{(2\pi)^3}\frac{\Gamma_{\pi N}(x',k'_\perp)}{s'_2-m^2}\left(\frac{1}{s_{3a}-m^2}-\frac{1}{s^*_{3a}-m^2}\right)$$
$$+g_{N\bar{N}}^2\int_{1-x}^{1}\frac{dx'}{2x'(1-x')(x+x'-1)}\int\frac{d^2k'_\perp}{(2\pi)^3}\frac{\Gamma_{\pi N}(x',k'_\perp)}{s'_2-M^2}\left(\frac{1}{s_{3c}-m^2}-\frac{1}{s^*_{3c}-m^2}\right)$$



Numerical results of the eigenvalue equation

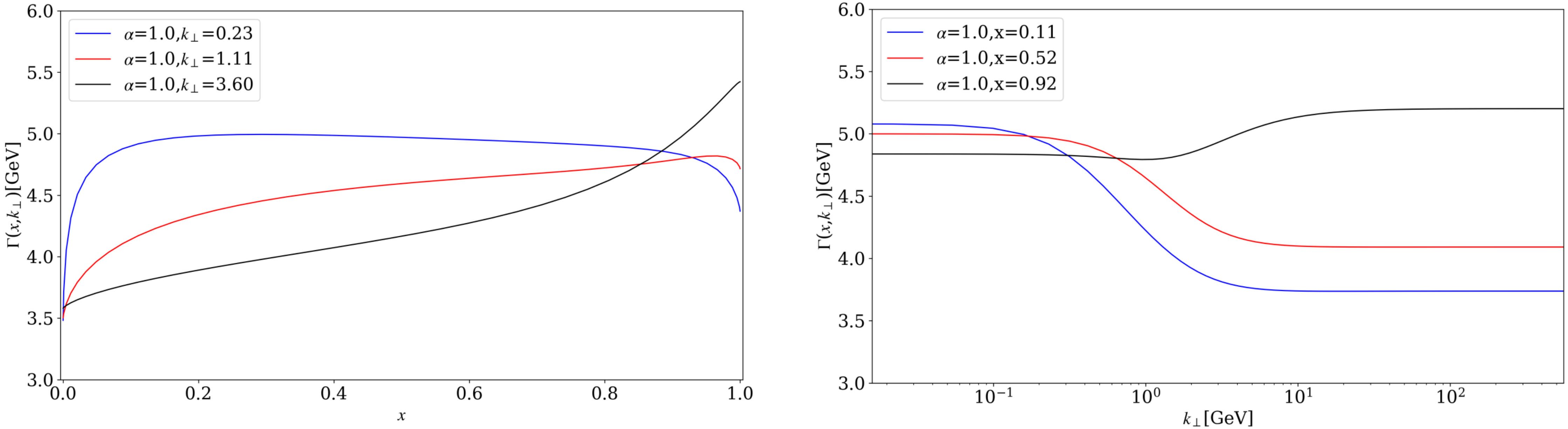


FIG. The two-body vertex function $\Gamma_{\pi N}(x, k_\perp)$ solved numerically from the system of equations with different coupling

- Here we adopt $m = 0.94GeV, \mu = 0.14GeV$
- $\alpha = g^2/(16\pi m^2)$ is the dimensionless coupling of the tree-level Yukawa potential:

$$V(r) = -\frac{\alpha}{r}e^{-\mu r}$$

Numerical result of the eigenvalue equation

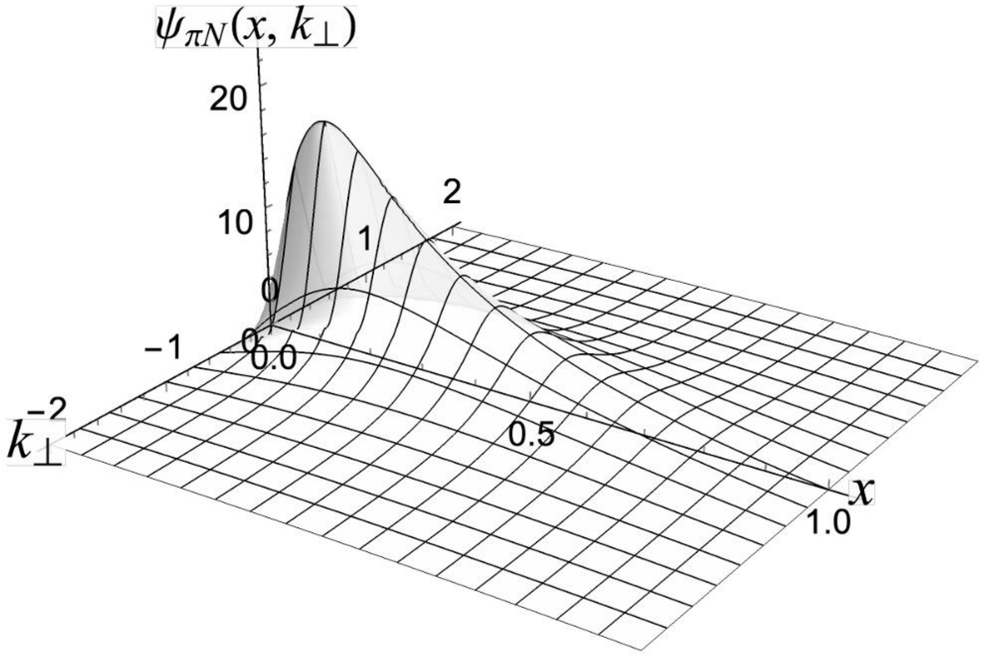


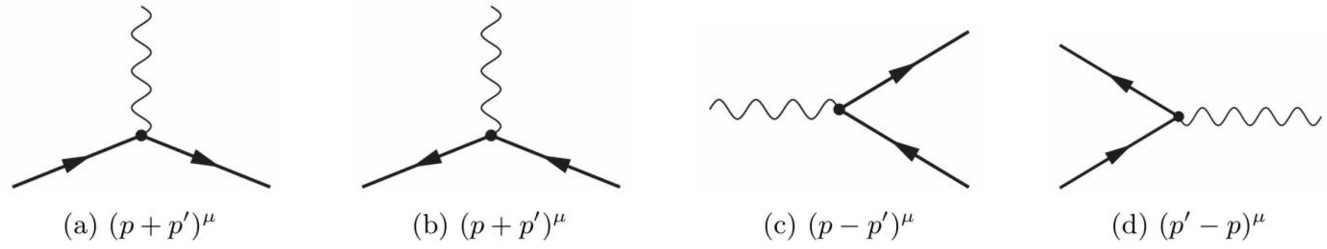
FIG. The two-body wave function $\psi_{\pi N}(x, k_\perp)$ with coupling $\alpha = 1.0$

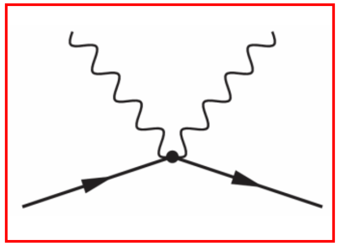
- $\psi_{\pi N}(x, k_\perp) = \Gamma_{\pi N}(x, k_\perp) / (\frac{k_\perp^2 + \mu^2}{x} + \frac{k_\perp^2 + m^2}{1-x} - m^2)$ is the two-body Light Front Wave Function.

E.M. current

- Nucleon field N can couple to the photon by minimal coupling: $\partial_\mu \to D_\mu = \partial_\mu + ieA_\mu$. The corresponding electric current operator is:

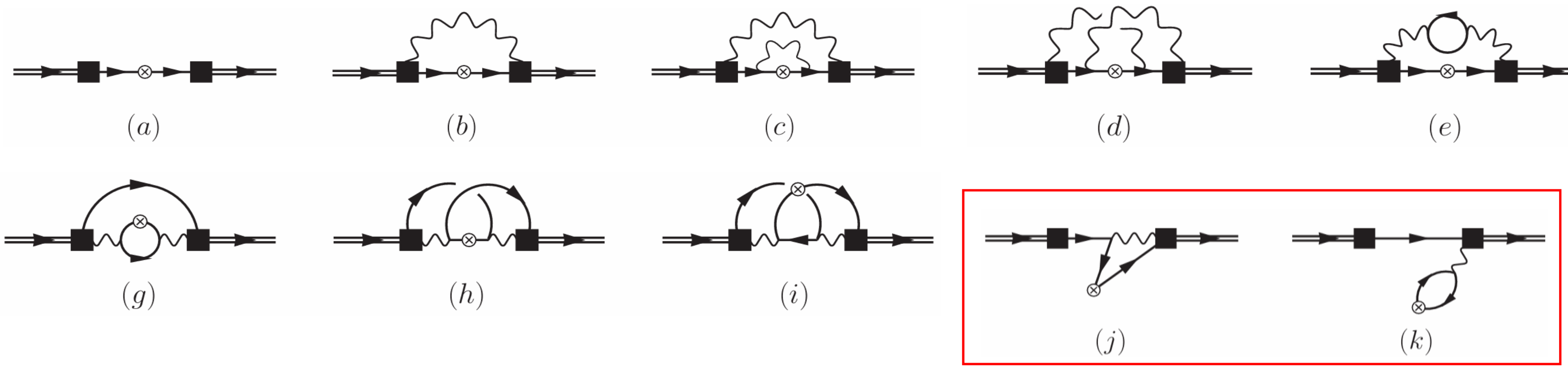
$$J^\mu = iN^\dagger(D^\mu N) - i(D^\mu N)^\dagger = iN^\dagger(\partial^\mu N) - i(\partial^\mu N)^\dagger N + \underline{2eA^\mu N^\dagger N}$$





Seagull term with $\alpha_{em} = e^2/(4\pi)$

- Diagrammatic representation of E.M. current matrix elements:



Z-term current

E.M. form factor in covariant light-front dynamics

$$j^\mu = (p' + p)^\mu F\left(Q^2\right)$$

$$\Downarrow$$

$$j^\mu = (p' + p)^\mu F\left(\zeta, Q^2\right) + \frac{\omega^\mu M^2}{\omega \cdot P} S_1\left(\zeta, Q^2\right) + \frac{q^{[\mu}\omega^{\upsilon]} q_\nu}{\omega \cdot P} S_2\left(\zeta, Q^2\right)$$

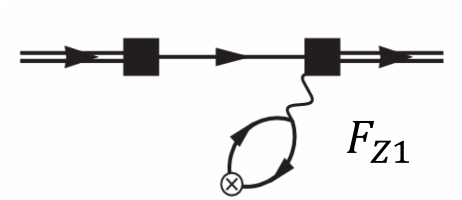
[V. A. Karmanov et. al.:1998 PR]

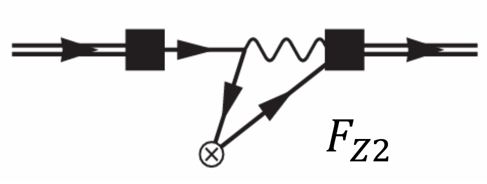
- $\omega^\mu = (1,0,0,-1) \Rightarrow \omega^+ = \omega_\perp = 0\,, \omega^- = 2$
- $P = (p' + p)/2\,,\ a^{[\mu} b^{\upsilon]} = a^\mu b^\upsilon - a^\upsilon b^\mu\,, \zeta = \frac{q^+}{P^+} = (p'^+ - p^+)/P$
- F is the E.M. form factor of the nucleon, and $S_{1,2}$ are spurious form factors due to the possible violation of the Poincare symmetry
- E.M. form factor extracted from the new covariant decomposition in the transverse Breit frame $\vec{P}_\perp = 0$:

$$F\left(\zeta, Q^2\right) = \frac{j^+}{2P^+} - \frac{\zeta}{2} \frac{\vec{j}_\perp}{\vec{q}_\perp}$$

Zero-mode contributions

- Z-term E.M. form factors:





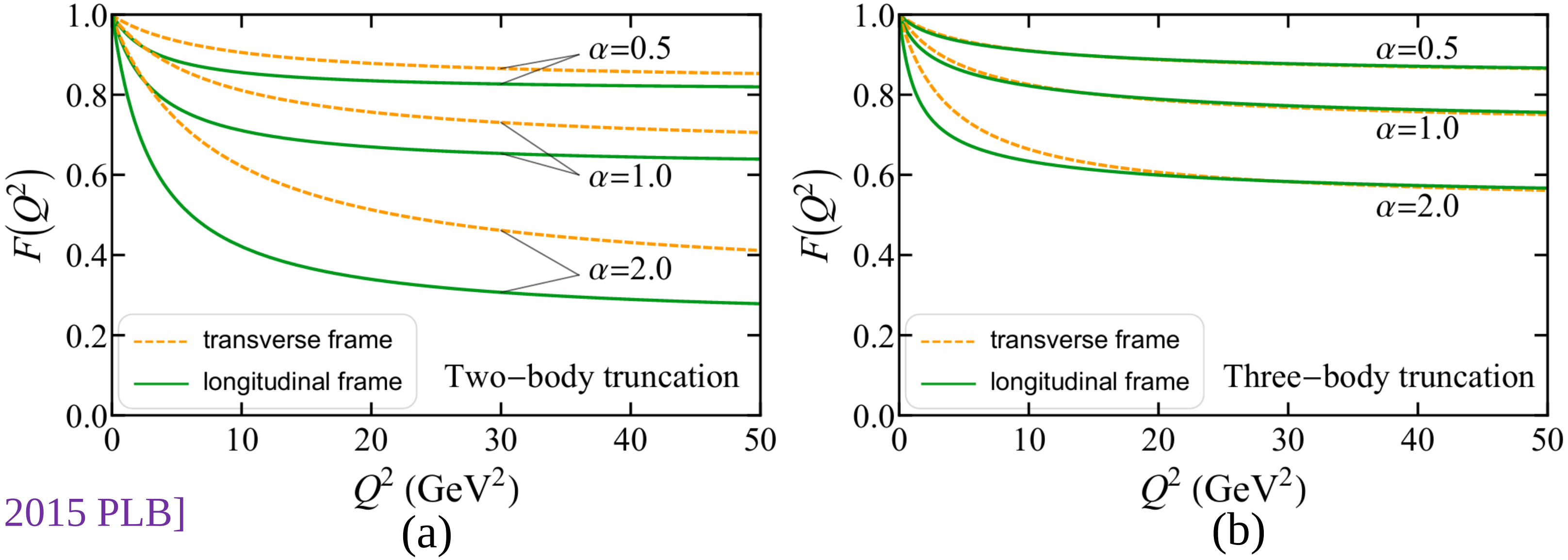
$$F_{Z1}(\zeta,Q^2)=\frac{(1-a)g_{N\bar{N}}\psi_N}{a-2}\psi_{\pi N}\left(a,-\left(1-\frac{a}{2}\right)\vec{q}_\perp\right)\int_0^1 dx\int\frac{d^2\vec{k}_\perp}{(2\pi)^3}\frac{\vec{k}_\perp}{\vec{q}_\perp}\frac{1}{(1-a)\left(\vec{k}_\perp^2+m^2\right)+x(1-x)\left[\left(1-\frac{a}{2}\right)^2\vec{q}_\perp^2+a^2m^2\right]}$$

$$F_{Z2}(\zeta,Q^2)$$

$$=\frac{a^2(1-a)g_{N\bar{N}}\psi_N}{a-2}\int_0^1 dx\int\frac{d^2\vec{k}_\perp}{(2\pi)^3}\frac{x\vec{k}_\perp}{\vec{q}_\perp}\frac{\Gamma_{\pi N}\left(1-ax,\vec{k}_\perp+\left(1-\frac{a}{2}\right)x\vec{q}_\perp\right)}{\left(\vec{k}_\perp+\left(1-\frac{a}{2}\right)x\vec{q}_\perp\right)^2+ax\mu^2+(1-ax)^2m^2}\times\frac{1}{(1-a)\left(\vec{k}_\perp^2+m^2\right)+x(1-x)\left[\left(1-\frac{a}{2}\right)^2\vec{q}_\perp^2+a^2m^2\right]}$$

- Where $a\equiv 2\zeta/(2+\zeta)$, thus $q^+\to 0 \Leftrightarrow \zeta\to 0 \Leftrightarrow a\to 0$.
- In F_{Z1}, since the transverse integral involves an odd integrand caused by $\vec{k}_\perp$, $F_{Z1}(\zeta,Q^2)\equiv 0$
- In F_{Z2}, the integral is finite, thus $\lim_{\zeta\to 0}F_{Z2}(\zeta,Q^2)=0$
- Z-terms don't contain zero-modes in transverse Frame

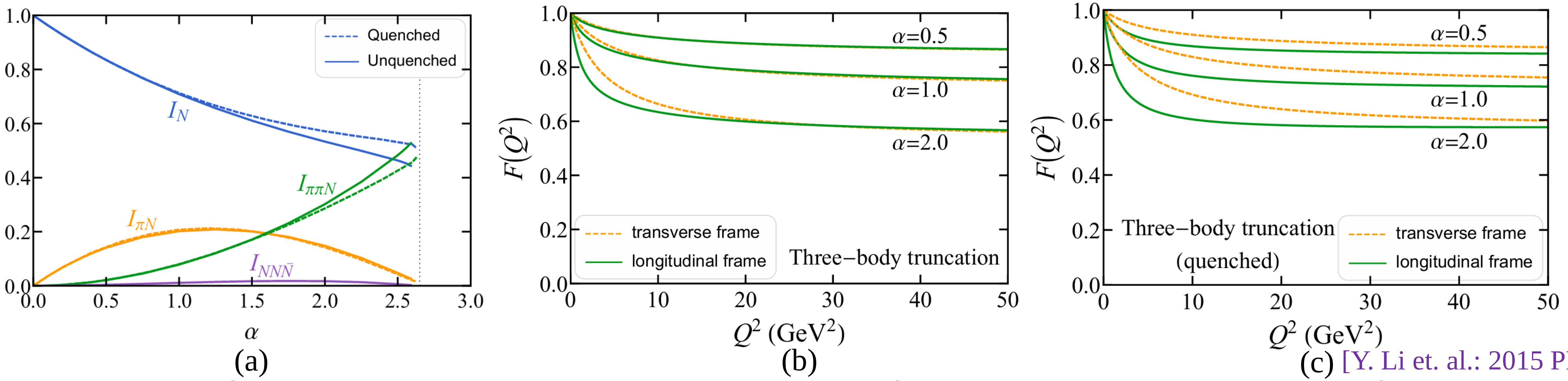
Results of the E.M. form factor



[Y. Li et. al.: 2015 PLB]

- Transverse frame (Drell-Yan frame): $\zeta = 0$; Longitudinal frame: $\Delta_\perp = q_\perp - \zeta P_\perp = 0$
- Two-body: $|\psi_{ph}\rangle = |N\rangle + |\pi N\rangle$; Three-body: $|\psi_{ph}\rangle = |N\rangle + |\pi N\rangle + |\pi\pi N\rangle + |NN\bar{N}\rangle$
- Form factors with two-body truncation which is equivalent to the leading-order of the light-cone perturbation theory, exhibit considerable frame dependence. In three-body truncation, the frame dependence is dramatically reduced as the three-body Fock sectors are included.
- The reduction of the frame dependence in the three-body truncation is also a sign of convergence of the Fock sector expansion.

Results of the E.M. form factor



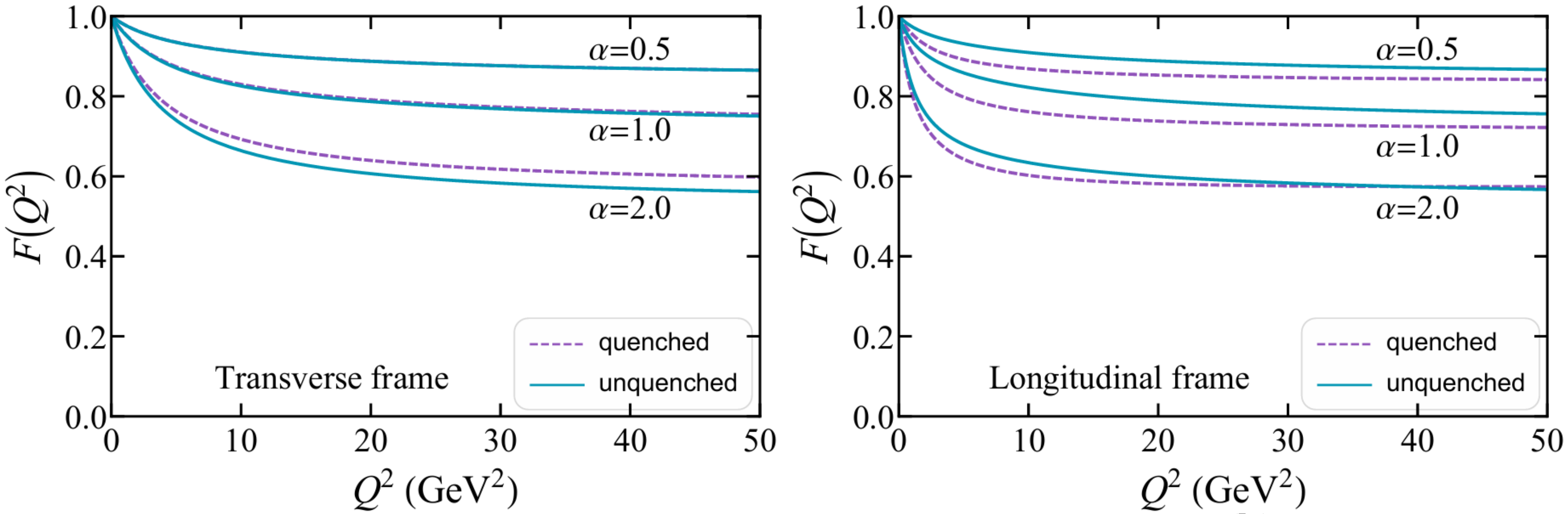
[Y. Li et. al.: 2015 PLB]

- Quenched: $|\psi_{ph}\rangle = |N\rangle + |\pi N\rangle + |\pi\pi N\rangle$; Unquenched: $|\psi_{ph}\rangle = |N\rangle + |\pi N\rangle + |\pi\pi N\rangle + |NN\bar{N}\rangle$
- Figure (a) is the normalization of the Fock sectors as a function of the coupling α, here:

$$I = \frac{1}{S_n}\prod_i \int \frac{dx_i}{2x_i}\int \frac{d^2k_{i\perp}}{(2\pi)^3} 2\delta(\sum_i x_i - 1)(2\pi)^3\delta^2(\vec{k}_{i\perp})|\psi(\{x_i, \vec{k}_{i\perp}\})|^2$$

- Quenched E.M. form factors have a considerable frame dependence, while unquenched E.M. form factors are quite close to each other.
- Although the sector $|NN\bar{N}\rangle$ contributes only a very small portion ($\lesssim 2\%$) of the total physical state, the anti-nucleon degree of freedom plays an important role in the frame dependence of the E.M. form factor.

Results of the E.M. form factor



(a) (b)

- In the transverse frame, the form factors are quite close to each other, suggesting that the anti-nucleon degrees of freedom is not significant for form factors in this frame.
- The anti-nucleon degrees of freedom mainly impact form factors within the longitudinal frame, which corroborates with the analysis based on Bethe-Salpeter equations.
- Drell-Yan frame is a preferred frame within which the Fock sector expansion converges fast, in alignment with the conventional wisdom in light front physics.

Summary and outlook

- In this work, we investigate the zero modes of E.M. form factor of a strongly coupled scalar theory in (3+1)-dimensions by using a systematic non-perturbative Hamiltonian approach.
- With the help of the covariant light front dynamics, we show that a combination of j^+ and $\vec{j}_\perp$ can be used to extract the form factor in general frame without the contamination of spurious contributions.
- From our results, Z-terms in the scalar Yukawa theory do not contain zero-modes in Drell-Yan frame, and the Lorentz covariance of E.M. form factors as gauged by the frame dependence systematically improves as more Fock sectors are incorporated.
- This work may serve as a baseline for developing more sophisticated methods for more complicated quantum field theories in the strong coupling regime.

Thank you!