Light-hadron structure and dynamics in Minkowski space 1

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$t = t$ from the one responsible for the e 2 tive mass of the constitution of the constitution of the constituents. In the different second terms of the di ference between the two symmetric PDFs, i.e. *u*_n and two symmetric Patrick Barry. **Pion - Interesting?**

Pions

- Pion is the Goldstone boson associated with spontaneous symmetry breaking of chiral
- Lightest hadron
- Made up of q and \bar{q} constituents

Credits to Patrick Barry $\frac{1}{\sqrt{1-\frac{1$

Light-front hypersurface

$$
\ket{\pi} = \ket{q\bar{q}} + \ket{q\bar{q}g} + \ket{q\bar{q}\,2g} + \cdots
$$

How to look?

FIG. 1. Sullivan process: $ep \rightarrow e' \pi^+ n$ scattering. The black blob represents the half-on-mass shell photo absorption amplitude. Diagrammatic representation of the pion pole amplitude for $p(e, e')\pi^+ n$ process.

off-shell pion EM FF: Choi, TF, Ji, de Melo, PRD 100, 116020 (2019)

How to look in Detail?

Observables associated with the hadron structure

Lorcé, Pasquini, Vanderhaeghen JHEP05(2011)041 $t_{\text{cscy}}, t_{\text{asym,}}$ vandernaction stress of (2011) ⁰

. Pion: SL form factor, PDF, TMD & 3D image \overline{a} and dinal momentum of \overline{a} explaned in Pioner SL t

BSE quark-antiquark & pion model **Fig. 1** The quark-gluon vertex $\mathbf{r} = \mathbf{r} \cdot \mathbf{r}$ *d*−0 *p*¹ + *p*² + *p*³ = 0*.* (1) The one-particle intervals function \mathcal{P} the vertex reads \mathcal{L} How we model: **p**2 −*p p*3 enhancement of ∿ *D* Furthermore, the vertex is enhanced when all momenta κ ing the vertex \blacksquare tends to be pairs, so in this way the compromise that the momenta are restricted **p**₂ + *p*₂ + *p*₂ + *p*₃ + *p*₂ + *p*₃ + *p*₂ \blacksquare antiqual \blacksquare \blacksquare phys. Model the vertex reads *^µ (p*1*, ^p*2*, ^p*3*)* ⁼ *g t^a* ^Γ*µ(p*1*, ^p*2*, ^p*3*),* (2) where \mathbf{y} is the strong constant and \mathbf{y} matrices in the fundamental representation. T , and its spin-diagonal in color and its spin-diagonal in color and its spin-= *p p*3 enhancement occurs preferably at momenta of ∼ Λ*QCD*. Furthermore, the vertex is enhanced when all momenta entering the vertex (see Fig. 1) tends to be parallel in pairs, solving The one-particle irreducible irreducible interaction associated to \mathcal{L}_{max} the vertex reads *^µ (p*1*, ^p*2*, ^p*3*)* ⁼ *g t^a* ^Γ*µ(p*1*, ^p*2*, ^p*3*),* (2) where q is the strong constant and q matrices in the fundamental representation. Eur. Phys. J. C (2019) 79 :116 Page 3 of 33 **116** *p* −**p** *p*¹ + *p*² + *p*³ = 0*.* (1) The one-particle irreducible intervals function associated to \mathcal{L}_{max} the vertex reads of the vertex reads **Fig. 1** The quark-gluon vertex *p*² −*p*¹ *p*3 enhancement occurs preferably at momenta of ∼ Λ*QCD*. Furthermore, the vertex is enhanced when all momenta entering the vertex \mathcal{L} tends to be pairs, solving to be parallel in pairs, so \mathcal{L} in this way the momenta are restricted that the momenta are restricted to a region around *around a region* for the quarkgluon vertex, the dominant form factors are associated with *p*¹ + *p*² + *p*³ = 0*.* (1) The one-particle irreducible is function associated to \mathcal{L}_max *^µ (p*1*, ^p*2*, ^p*3*)* ⁼ *g t^a* ^Γ*µ(p*1*, ^p*2*, ^p*3*),* (2) where the strong constant and **t** matrices in the fundamental representation. T and in color and its spin--diagonal in color and its spin-Lorentz structure is given by

Ladder approximation (L): suppression of XL for Nc=3 in a bosonic system [A. Nogueira, CR Ji, Ydrefors, TF, PLB777(2017) 207] nation (L): suppression of XL for Nc=3 in a bosonic system \mathcal{F} i, turcjors, tr \mathcal{F} the momenta are restricted as matrices in the fundamental representation. $T_{\rm eff}$ propagator is diagonal in color and its spin- \mathbf{L} and \mathbf{L} structure is given by gluon σ dominant form σf χ nation (L): suppression of XL for Nc=3 in a bo Ii $Vdvofovs$ TF PI R777(2017) \cdot , \cdot *<i>A(p*₂)*/p*₂*(p*₂) *A*2*(p*2*) p*² − *B*2*(p*2*)* ⁼ *i Z(p*2*) /^p* ⁺ *^M(p*2*)* to a region around Λ*QCD*. Within our solution for the quark- U is suppression of X (L): suppression of XL for Nc=3 in a bos \mathcal{C} and \mathcal{D} the sub-leading contributions of the contributions of the contributions to the contribution of the contributions of 'IOI In the current work, the vertex is written using the vertex is written using the \mathcal{N} *A*2*(p*2*) p*² − *B*2*(p*2*) i IF. PLB777(2017)* 2071 Furthermore, the vertex is \mathcal{L} VI \mathcal{L} \mathcal{S} uddi ession of AL to in this way the momenta are restricted way that the momenta are restricted way to \mathcal{L} g, II, I DD $\frac{1}{2}$ (2017) 20 $Nc=3$ in a hose $\sum_{i=1}^n$ in a coloring spin- \mathbf{L} rank tensor structures give sub-leading contributions to the \mathbf{v}_1 I' $ETLDIPIZ(2017) 20$, IF, $FLD///201/120$ are associated to the transverse form factors (see definitions \mathcal{S} *x <i>x <i>x <i>d*</sup> *m a bosonic sy* where *^Z(p*2*)* ⁼ ¹*/A(p*2*)* stands for the quark wave function and *^M(p*2*)* ⁼ *^B(p*2*)/A(p*2*)* is the renormalisation group

> **In the current work** with works with the vertex is written ⁼ *i Z(p*2*) /^p* ⁺ *^M(p*2*)* below). These singularities can be avoided by considering are associated to the transverse form factors (see definitions (see definitions (see definitions (see definitions) α different tensor basis for the full vertex as described, for the full vertex as α $\frac{1}{2}$ α example, in [37]. However, the singularities are not associated to the longitudinal form for the local form α takes into account this class of form factors. *^S*−1*(p)* ⁼ [−]*i Z*2*(/^p* [−] *^m*bm*)* ⁺ ^Σ*(p*2*),* (4) are associated to the transverse form factors (see definitions bbcu uudik Diodde a different tensor basis for the full vertex as \mathbf{a} invariant running quark mass. Ω r equation for the quark propagator, see the quark propagator, see the quark propagator, see the quark propagator, see the Ω also named the quark gap equation, is represented in Fig. 2 The paper is organised as follows. In Sect. 2 we introssed duark propag equations and the quark-gluon vertex. Moreover, we use a $\mathbf{r}_{\mathbf{Q}}$ \triangleright dressed quark propagator

- → dressed gluon propagator below). The singularities can be avoid by considering the single singular considering the constant of the singular singular considering the constant of the co \triangleright dressed gluon propagator also named the quark gap equation, is represented in Fig. 2. \sim are bodd grupp pro- $\frac{1}{2}$ where *Z*² is the quark renormalisation constant, *m*bm the bare quark propagators functions and the quark-ghost kernel. The parametrisation of the quark-ghost kernel is also discussed. \overline{a} \overline{b} \overline{a} \overline{b} \overline{a} \overline{b} \overline{a} \overline{b} \overline{a} \overline{b} \overline{a} \overline{b} \overline{b} \overline{a} \overline{b} \overline{a} \overline{b} \overline{b} \overline{b} \overline{a} \overline{b} \overline{b} \overline{b} \overline{c} $\overline{$ *(*2π*)*⁴ [×]*Dab ^µ*^ν *(q) (igt^b*γν *) ^S(^p* [−] *^q)* ^Γ *^a* takes into account this class of form factors. η α α degree as α duce the notation for the propagators, the Dyson–Schwinger where *Z*² is the quark renormalisation constant, *m*bm the bare equations and the quark-gluon vertex. Moreover, we use a In Sect. 3 the scalar and vector components of the DSE in esseg given broda kernels. In Sect. 4 the DSE are rewritten in Euclidean space, and the DSE are rewritten in Euclidean space, and where *Z*¹ is a combination of several renormalisation con-stants, *Dab*
- ▶ dressed quark-gluon vertex \sim are been quark grave. \triangleright dressed quark-gluon vertex *►* dressed quark-gluon vertex parametrisation of the quark-ghost kernel is also discussed. ! *d*4*q* the integral equations. In Sect. 5 we give the details of the lattice data used in the current work for the various propa- \mathbf{u} v \mathbf{v} \mathbf{u} \mathbf{u} \mathbf{v} \mathbf{v} In Sect. 3 the scalar and vector components of the DSE in Mindia Space and Given are given the corresponding to the correction of the co kernels. In Sect. 4 the DSE are rewritten in Euclidean space, [×]*Dab ^µ*^ν *(q) (igt^b*γν *) ^S(^p* [−] *^q)* ^Γ *^a* where *Z*¹ is a combination of several renormalisation con-stants, *Dab* gators and on the functions that parametrise the lattice data. The Standard Netzels ho Sect. 6, together with the solutions for the vertex of the gap below both *Dab ^µ*^ν *(q)* and *^D(q*2*)* will be referred to as the gluon

 $M_{\rm{max}} = 1$ space are given, together with the corresponding to \sim ers. quark and giuon ma introduce the vertex *ansatz* and perform a scaling analysis of where *Z*¹ is a combination of several renormalisation con**es & quark-gluon vertex** gauge, is given by ers: auark and oluon mar T are discussed in the Euclidean space T $\frac{1}{2}$ and gluon masses $\frac{1}{2}$ quark-gluon vertey In this section in defined. In this first part of this work, the equations discussed *x* $\frac{1}{2}$ or, equivalently Γ a_{nd} concludes. α and gluon masses & quark gluon w are written in Minkowski space with the diagonal metric *g* = *(*1*,* −1*,* −1*,* −1*)*. Let us follow the notation of [38] for the [*^p*] [−]¹ ⁼ *^p* [] [−]¹ ⁺ *^p k* Ø Model parameters: quark and gluon masses & quark-gluon vertex

OM LQCD in Landau gauge: SL momenta work, we also the lattice of the current work, we are the gap of the gap o **INPUTS FROM LQCD in Landau gauge: SL momenta**

Gluon propagator with the gluon anomalous dimension being γ = $12/22$

$$
D_{\mu\nu}^{ab}(q) = -i \delta^{ab} \left(g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2} \right) D(q^2)
$$

Dudal, Oliveira, Silva, Ann. Phys. 397, 351 (2018)

Ghost propagator

$$
D_{gh}(p^2) = \frac{F(p^2)}{p^2}
$$

Quark propagator Oliveira, Silva, Skullerud an

Silva, Skullerud and Sternbeck, PRD 094506 **magazine expression and all of above expressions** $\frac{1}{2}$.

 $\frac{1}{2}$ are singular buildings Parametrizations summarized in Oliveira, de Paula, Frederico, de Melo, EPJ C 79 (2019) 116 Parametrizations summai harized in Oliveira, de Paula, Parametrizations summarized in Oliveira, de Paula, Frederico, de Melo, EPJ C 79 (2019) 116 functions from full QCD simulations with *N ^f* = 2 Parametrizations summarized in Oliveira, de Paula, Frederico, de Melo, EPJ C 79 (2019) 116

The Quark-Gap Equation and the Quark-Gluon Vertex 1 and the waard-Uldon vertex

Spontaneous Chiral symmetry breaking & pion as a Goldstone boson It quarks, Roberts, Maris, Tandy, Oloet, Maris...*)*
Γ (origin of the nucleon mass – "constituent quarks", Roberts, Maris, Tandy, Cloet, Maris...)

Schwinger-Dyson eq. Quark propagator Quark-gluon vertex *p*² −*p*¹ *p*3 Furthermore, the vertex is enhanced when all momenta entering the vertex (see Fig. 1) tends to be parallel in pairs, solving in this way the compromise that the momenta are restricted rank tensor structures give sub-leading contributions to the vertex. In the current work, the vertex is written using the Ball-The one-particle irreducible Green's function associated to the vertex reads where *g* is the strong coupling constant and *t^a* are the color matrices in the fundamental representation. The quark propagator is diagonal in color and its spin-Lorentz structure is given by *^A(p*2*)/^p* [−] *^B(p*2*)* ⁼ *ⁱ* ⁼ *i Z(p*2*) /^p* ⁺ *^M(p*2*) p*² − *M*2*(p*2*) ,* (3) where *^Z(p*2*)* ⁼ ¹*/A(p*2*)* stands for the quark wave function and *^M(p*2*)* ⁼ *^B(p*2*)/A(p*2*)* is the renormalisation group invariant running quark mass. The Dyson–Schwinger equation for the quark propagator, *p*² −*p*¹ enhancement occurs preferably at momenta of ∼ Λ*QCD*. Furthermore, the vertex is enhanced when all momenta entering the vertex (see Fig. 1) tends to be parallel in pairs, solving in this way the compromise that the momenta are restricted to a region around Λ*QCD*. Within our solution for the quark*p*¹ + *p*² + *p*³ = 0*.* (1) The one-particle irreducible Green's function associated to Γ *a ^µ (p*1*, ^p*2*, ^p*3*)* ⁼ *g t^a* ^Γ*µ(p*1*, ^p*2*, ^p*3*),* (2) where *g* is the strong coupling constant and *t^a* are the color matrices in the fundamental representation. The quark propagator is diagonal in color and its spin-Lorentz structure is given by *S(p)* = *A(p*2*)/p* − *B(p*2*)* = *i ^A(p*2*)/^p* ⁺ *^B(p*2*) A*2*(p*2*) p*² − *B*2*(p*2*)* tudinal Γ *(L)* and transverse Γ *(^T)* components relative to the gluon momenta, i.e. one writes ^Γ*µ(p*1*, ^p*2*, ^p*3*)* ⁼ ^Γ *(L) ^µ (p*1*, ^p*2*, ^p*3*)* ⁺ ^Γ *(^T)* where, by definition, *pµ* ³ ^Γ *(^T) ^µ (p*1*, p*2*, p*3*)* = 0*.* (8) By choosing a suitable tensor basis in the spinor-Lorentz Longitudinal component

Rojas, de Melo, El-Bennich, Oliveira, Frederico, JHEP 1310 (2013) 193; Oliveira, Paula, Frederico, de Melo EPJC 78(7), 553 (2018) & EPJC 79 (2019) 116 & Oliveira, Frederico, de Paula, EPJC 80 (2020) 484 $\overline{D}_{\text{total}} = \overline{D}_{\text{total}}$ Paula, Frederico, de Melo EPJC 78(7), 553 (2018) & EPJC 79 (2019) 116 & Oliveira, Frederico, de Paula, EPJC 80 (2020) 484 $\frac{2}{\pi}$ is the so $\left(2020\right)$ for *p*2 − *M*₂ $16 \&$

Ladder approximation (*L*): suppression of *XL* for Nc=3 [Alvarenga Nogueira, CR Ji, Ydrefors, TF, PLB777(2017) 207] *enformation (L): suppression of* α proximation (L): suppression of XL for $Nc=$ **5** \sim the compromise that the momenta are restricted as \sim $777(2017)$ 207] to a region around Λ*QCD*. Within our solution for the quark*sroximation (L): suppression of XL for N* $t_{\text{reduction}}$ CD *k* V_{def} CT ^{*n*} rank tensor structures and the sub-leading contributions of the theorem is the theorem in the theorem in the t *B*^{*(*}*p***₁)/***p* $\frac{1}{2}$ *(n*₁)/*p*₂*017* ⁼ *i Z(p*2*) /^p* ⁺ *^M(p*2*) ,* (3) u iuon (L) . suppression *ira CR Ii Vdrefors TH* $\ddot{}$ $\overline{\mathbf{a}}$ structure is given by *f*(*b*)*/py nc*=*5 ^A(p*2*)/^p* [−] *^B(p*2*)* ⁼ *ⁱ A*2*(p*2*) p*² − *B*2*(p*2*)* ⁼ *i Z(p*2*) /^p* ⁺ *^M(p*2*) p*² − *M*2*(p*2*) ,* (3) *i* (L) suppression of ing the vertex (see Fig. 1) tends to be parallel in pairs, solving $CDI: VIC$ are $TITDI$ CR JI, *Turejors*, *IF*, *FL.* $\ddot{}$ \overline{f} \overline{f} is \overline{f} and \overline{f} are the color L for the -5 \mathbf{C} $\mathbb{E}[\mathbf{E} \mid \mathbf{E}[\mathbf{E}] \mathbf{E}[\mathbf{E}[\mathbf{E}] \mathbf{E}[\mathbf{E}] \mathbf{E}[\mathbf{E}] \mathbf{E}[\mathbf{E}] \mathbf{E}[\mathbf{E}[\mathbf{E}] \mathbf{E}] \mathbf{E}[\mathbf{E}[\mathbf{E}] \mathbf{E}[\mathbf{E}] \mathbf{E}[\mathbf{E}[\mathbf{E}] \mathbf{E}[\mathbf{E}] \mathbf{E}[\mathbf{E}[\mathbf{E}] \mathbf{E}[\mathbf{E}] \mathbf{E}[\mathbf{E}[\mathbf{E}] \mathbf{E}[\math$ *^S(p)* ⁼ *ⁱ ^A(p*2*)/^p* [−] *^B(p*2*)* ⁼ *ⁱ A*2*(p*2*) p*² − *B*2*(p*2*) ^A(p*2*)/^p* ⁺ *^B(p*2*)* $\mathcal{F}(I)$ and tensor structures give sub- ι (1 CR li Ydretors TH PL \mathcal{L} is kinematical singularities for the Landau gauge \mathcal{L} *^A(p*2*)/^p* [−] *^B(p*2*)* ⁼ *ⁱ* ⁼ *i Z(p*2*) /^p* ⁺ *^M(p*2*) p*² − *M*2*(p*2*) ,* (3) where *^Z(p*2*)* ⁼ ¹*/A(p*2*)* stands for the quark wave function and *^M(p*2*)* ⁼ *^B(p*2*)/A(p*2*)* is the renormalisation group

constituent quark mass ~ 200 - 300 MeV $S(P) = \frac{P}{P - m + i\epsilon}$ $g_{\text{max}} = \text{max}$ form factors are associated with max $111d55 \approx 200 - 300 \text{ NIEV}$ $\frac{1}{200}$ construction. It is $\frac{1}{200}$ $\frac{1000}{300}$ $\frac{200}{300}$ MC has kinematical singularities for the Landau gauge [37] that \sim 200 $-$ 200 MeV below). These singularities can be avoided by considering $S(D) = \frac{1}{2\pi i}$ $J(1)$ – λ $\mathbb{P} - m$ vertex. $\rm I\rm O\,O\,$ the current work, the vertex is written using the Ball-Chiu construction. It is known that the Ball-Chiu vertex a different tensor basis for the full vertex as described, for $\Omega \Omega = 2 \Omega \Omega$. Not associated $\overline{}$ \triangleright constituent quark mass ~ 200 – 300 MeV

► vector exchange
Feynman gauge
$$
iK_V^{(Ld)\mu\nu}(k, k') = -ig^2 \frac{g^{\mu\nu}}{(k - k')^2 - \mu^2 + i\epsilon}
$$

 $S(P) = \frac{1}{p}$

where *^Z(p*2*)* ⁼ ¹*/A(p*2*)* stands for the quark wave function

 $\mathbf{r}-$

 (D) —

*A*2*(p*2*) p*² − *B*2*(p*2*)*

 $\mathbf{r} - \mathbf{m} + \mathbf{i}\epsilon$

 $\zeta(p) = \frac{1}{\sqrt{p^2 + 4p^2}}$ $a(f) = \frac{b}{p}$

 $\frac{l}{p_1 - p_2}$ \sqrt{p} $\frac{m}{2}$ $\frac{m}{2}$ \mathbf{r} = $m + i\epsilon$

 $\mathbf{p} - m + i\epsilon$

The Dyson–Schwinger equation for the quark propagator, also named the quark gap equation, is represented in Fig. 2.2 μ

^S−1*(p)* ⁼ [−]*i Z*2*(/^p* [−] *^m*bm*)* ⁺ ^Σ*(p*2*),* (4)

Ø quark-gluon vertex form-factor ^L *^µ* (*p*1*, p*2*, p*3) = *i* \sim 300 MeV form-factor $\lambda_1 \gamma_\mu$ $F(q) = \frac{\mu^2 - \Lambda^2}{q^2 - \Lambda^2 + i\epsilon}$ x form-factor λ , \sim equations and the quark-gluon vertex. The μ $\mathbf{M_0} \mathbf{V}$ $E(z)$ **i** μ \bar{z} is the \bar{z} $r(q) = \frac{q}{q}$! *d*4*q* $\Delta 1$ γ_{II} \mathbf{M} $F(a) = \frac{P^a - 4P^b}{P^a - P^a}$ $\alpha^2 - \Delta^2 + i\epsilon$ \mathbf{v} **d** \mathbf{v} **c** \mathbf{v} \overline{a} used in the current work for the various propa- c and c that parameters the lattice data. T recent λ 1 γ . \mathcal{S} . The solutions for the solutions for the solutions for the gap \mathcal{S} equation. The quark-gluon vertex form factors are reported vertex for ϵ *g*_{*x*} *d*_{*x*} *d*_{*x}* μ^2 - $P'(q) = \frac{q^2}{q^2}$ \mathcal{A} is the quark-gluon vertex. $k = 1$ the DSE are rewritten in Euclidean space. $CIOI = \lambda - \Delta$ $\sqrt{1 + \frac{1}{11}}$ $\frac{1}{\sqrt{2}}$ in the various propagation of the various propagations $\frac{1}{\sqrt{2}}$ $\mu^2 - A$ q^2 $2 - A^2 +$ $S_{\rm eff}$ is the solutions for the solutions for the solutions for the gap \sim $\mathsf{cfor} \quad \mathsf{C}$ \mathcal{L} ion \mathcal{L} 1 \mathcal{L} \mathbb{R} we summarize and conclude the summarise and conclude \mathbb{R} $\mu^2 - A^2$ A key ingredient in gap Eq. (4) is the quark-gluon vertex. Indeed, it is only after knowing Γ *^a ^µ* or, equivalently Γ*µ*, that $Q^2 - A^2 +$

> SOLUTION IN MINKOWSKI SPACE introduce the vertex *ansatz* and perform a scaling analysis of the integral equations. In Sect. 5 we give the details of the $[pion mass \rightarrow g]$ gators and on the functions that parametrise the lattice data. $\overline{}$ SOLUTION IN MINKOWSKI SPACI Sect. 6, together with the solutions for the vertex of the gap below both *Dab* \mathbf{r} in Sect. 7 for several kinematical configurations. Finally, one of \mathbf{r} *^µ*^ν *(q)* and *^D(q*2*)* will be referred to as the gluon \sim and \sim for \sim for \sim mass \rightarrow q. λ and λ is the quark-gluon vertex. In the notation the notation used through \sim defined. In this first part of this work, the equations discussed are written in Minkowski space with the diagonal metric *g* = [*^p*] [−]¹ ⁼ *^p* [] [−]¹ ⁺ *^p* **2 The quark gap equation and the quark-gluon vertex** momenta are incoming and, therefore, verify denote dressed propagators and vertices α [pion mass \rightarrow g] [−]¹ ⁼ *^p* [] $ON IN MINKOWSK$ SOLUTION IN MINKOWSKI SPACE

Pion BS amplitude

$$
\Phi(k, p) = S_1 \phi_1 + S_2 \phi_2 + S_3 \phi_3 + S_4 \phi_4
$$

$$
S_1 = \gamma_5 \quad S_2 = \frac{1}{M} p \gamma_5 \quad S_3 = \frac{k \cdot p}{M^3} p \gamma_5 - \frac{1}{M} k \gamma_5 \quad S_4 = \frac{i}{M^2} \sigma_{\mu\nu} p^{\mu} k^{\nu} \gamma_5
$$

10

(Nakanishi 1962) **Main Tool: Nakanishi Integral Representation (NIR)**

Each BS amplitude component:

$$
\Phi_{l}(k, p) = \int_{-1}^{1} dz' \int_{0}^{\infty} d\gamma' \frac{g_{l}(\gamma', z')}{(\gamma' + \kappa^{2} - k^{2} - p.kz' - i\epsilon)^{3}} \kappa^{2} = m^{2} - \frac{M^{2}}{4}
$$

Bosons: Kusaka and Williams, PRD 51 (1995) 7026;

Light-front projection: integration in k- Carbonell&Karmanov EPJA27(2006)1;EPJA27(2006)11;

TF, Salme, Viviani PRD89(2014) 016010,… Fermions (0⁻): Carbonell and Karmanov EPJA 46 (2010) 387; de Paula, TF,Salmè, Viviani PRD 94 (2016) 071901; de Paula, TF, Pimentel, Salmè, Viviani, EPJC 77 (2017) 764

Projecting BSE onto the LF hyper-plane $x^+=0$

Carbonell and Karmanov EPJA 46 (2010) 387

Light-Front coordinates: $x^{\mu} = (x^{+}, x^{-}, \mathbf{x}_{\perp})$

Within the LF framework, the valence wf is obtained by integrating the BSA on k-(elimination of the relative LF time)

$$
\textsf{LF amplitudes} \enspace \psi_i(\gamma,\xi) = \int \frac{dk^-}{2\pi} \; \phi_i(k,p) = -\frac{i}{M} \int_0^\infty d\gamma' \frac{g_i(\gamma^-,z)}{[\gamma+\gamma'+m^2z^2+(1-z^2)\kappa^2]^2}
$$

 $= (x^2, x^2)$

The coupled equation system is (NIR+LF projection, Karmanov & Carbonell 2010)

$$
\int_0^\infty d\gamma' \frac{g_i(\gamma',z')}{[\gamma+\gamma'+m^2z^2+(1-z^2)\kappa^2]^2} = iMg^2\sum_j \int_0^\infty d\gamma' \int_{-1}^1 dz' \mathcal{L}_{ij}(\gamma,z;\gamma'z')g_j(\gamma,z')
$$

Generalized Stietjes transform: inversible Carbonell, TF, Karmanov PLB769 (2017) 418

Kernel contains singular contributions:

de Paula, TF,Salmè, Viviani PRD 94 (2016) 071901; de Paula, TF, Pimentel, Salmè, Viviani, EPJC 77 (2017) 764

The coupled equations are formally equivalent to BSE, once NIR is applied, and the validity of NIR is assessed by the existence of unique solutions to the GEVP!

LF-time

 $x^+ = x^0 + x^3$

BS norm, valence wave function, decay constant theory constant *T r* Z *^d*4*^k* norm valence waye function decay consta */*2)¯(*k, p*)*S*1(*k* + *p* 1 where *Q*² = *|*~*q*?*|* , = *| k*?*|* , ⁰ = *|* ?*|* , ⁰⁰ = *|* ?*|* wave functic *^M "*5*, S*3(*k, p*) = ^Ë (*^k · ^p*) *M*₂ *M*₂ *<i>µ <i>n <i>n***₂** *p <i>n***₂** *p <i>n***₂** *p <i>n***₂** *p <i>n***₂** *p <i>n***₂** *p n***₂** *p <i>n***₂** *p n*

Paula, Ydrefors, Alvarenga Nogueira, TF and Salme PRD 103 014002 (2021). Paula, Ydrefors, Alvarenga Nogueira, TF and Salme PRD 103 014002 (2021).

 $m = 1$ **Normalization:** $i N_c$ $\int \frac{d^2 n}{(2\pi)^4} |\phi_1 \phi_1 + \phi_2 \phi_2 + b\phi_3 \phi_3 + b\phi_4 \phi_4 - 4 b\phi_1 \phi_4 - 4 \frac{d^2 n}{M} \phi_2 \phi_1| = -1$ BS amplitude. Alternatively, the valence wave function can be obtained using the quasi-potential expansion method adapted to perform the LF projection $\partial \alpha$ $\partial \alpha$ ₂ $(\gamma', z)/\partial z$ $\frac{1}{\sqrt{5}}\frac{1}{\sqrt{4-x^2+y^2+2}}$ $=\frac{\sqrt{\gamma}}{M}\psi_4(r)$ $4\sqrt{7}$ $\frac{1}{2}$ $\gamma = k_{\perp}^2$ and $z = 2\xi - 1$ $\langle \cdot, \kappa^2 \rangle$ $\frac{1}{1}$ nell, TF, Karmanov PLB769 (2017) 418 $\int d^4k$ $(2\pi)^4$ $\left[\phi_1 \phi_1 + \phi_2 \phi_2 + b \phi_3 \phi_3 + b \phi_4 \phi_4 - 4 b \phi_1 \phi_4 - 4 \frac{m}{M} \phi_2 \phi_1 \right]$ $\overline{}$ $=-1$ $(k;$ $\int d^4k$ **r** IN Eq. (2 π) the antiparallel spin component ($(2\pi)^4$, $(2\pi)^4$, $(2\pi)^4$, $(2\pi)^4$ $\psi_{\uparrow\downarrow}(\gamma,z) = -i$ *M* 4*p*⁺ $\int dk^{-}$ 2π $\text{Tr}[\gamma^+\gamma_5\Phi(k;p)]$ $=\psi_2(\gamma, z) + \frac{z}{2}$ 2 $\psi_3(\gamma, z) + \int^\infty$ 0 $d\gamma'$ *M*³ $\partial g_3(\gamma',z)/\partial z$ $[\gamma + \gamma' + z^2m^2 + (1 - z^2)\kappa^2]$ $\left(\begin{array}{cc} a & y \\ y & z \end{array}\right)$ p*M* $\frac{V}{4}$ \overline{I} \overline{C} \overline{I} $\frac{u}{2}$ x^{-}
 $T_{\rm eff}$ $\pm i$ $\pm (l_{\rm eff})$ $=\frac{\sqrt{\gamma}}{2\pi}\psi_4(\gamma,$ $\frac{1}{\sqrt{2}}$ *, z*) $\gamma = k_{\perp}^2$ and $z = 2\xi - 1$ $\mathcal{U}(\gamma \propto \kappa^2) = \int d\gamma' \frac{g_l(\gamma, z, \kappa)}{1 - \gamma^2}$ \mathcal{U} is the plus component of the axial-current associates of the axial-current associates $\mathcal{U} + \gamma' + m^2 z$ Generalized Sueyes transform
Carbonell, TF, Karmanov PLB769 (2017) 418 *Generalized Stietjes transg k*0 ? *·* [~] *k*00 ? ⁼ (1 *^z*)² ¹⁶ *^Q*² and [~] *k*? *·* ~*q*? = *| k*?*||*~*q*?*|* cos ✓ *.* **on:** $i N_c \int \frac{d^2 n}{(2\pi)^4} \left[\phi_1 \phi_1 + \phi_2 \phi_2 + b \phi_3 \phi_3 + b \phi_4 \phi_4 - 4 b \phi_1 \phi_4 - 4 \frac{m}{M} \phi_2 \phi_1 \right] = -1$ $\Uparrow\downarrow$ (γ, z) $=$ $-i\frac{\pi}{4p}$ *M* $\frac{1}{\sqrt{2}}$ $\frac{dK}{dx}$ T $\left[\gamma^{+}\gamma_{5}\Phi(\kappa;p)\right]$ $\psi_2(\gamma, z) + \frac{z}{2}\psi_3$ $\partial_i(\gamma, z) + \int_0^{\ldots} \frac{d\gamma}{M}$ $\frac{1}{2}$ *M*³ $\frac{3(7, 2)}{2}$ $\frac{2}{1}$ $\frac{1}{2}$ $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 2 & 10 \\ 2 & 0 \end{bmatrix}$ $\psi_{\uparrow\uparrow}(\gamma,z)$ \models $\sqrt{\gamma}M$ 4*ip*⁺ $\int dk^ 2\pi$ $\text{Tr}[\sigma^{+i}\gamma_5\Phi(k;p)] =$ $\sqrt{\gamma}$ $\frac{\mathbf{V}}{M} \psi_4(\gamma, z) \,,$ \int ∞ $g_i(\gamma', z; \kappa^2)$ $\mathcal{P}_i \quad (1, 2, 3, 6)$ decay constant is obtained for $[\gamma + \gamma' + m^2 z^2 + (1 - z^2) \kappa^2]^2$ G ana \overline{G} Carbonell, TF, Karmanov PLB76 "#(*, z*)*,* (19) **Valence wf:** $\left[\phi _{4}-4\frac{\pi }{M}\phi _{2}\phi _{1}\right] =-1.$ $\frac{1}{4p^+}\int\frac{1}{2\pi}\text{Tr}[\gamma^+\gamma_5\Phi(k)]$ $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\gamma'$ $\frac{\partial q_3(\gamma', z)}{\partial z}$ $(v_2(\gamma, z) + \frac{1}{2}\psi_3(\gamma, z) + \int_0^{\infty} \frac{1}{M^3} \frac{1}{[\gamma + \gamma' + z^2 m^2 + (1 - z^2)\kappa^2]}$ extra singularities, not present for the boson-boson or fermion-boson systems. $\frac{dW}{dt} \int \frac{dk^-}{k^-} \text{Tr}[\sigma^{+i} \gamma_r \Phi(k \cdot n)] = \frac{\sqrt{\gamma}}{k} \int \gamma_k(\gamma) \gamma_k(n)$ $4ip$ ⁺ J 2π 2π 5π (equations) set of M $(1, 1, 0)$ $\frac{1}{3}$ and \int^{∞} 0 $d\gamma'$ ^{*gi*}(γ' , *z*; *K*²)
^{*f*} $\psi_i(\gamma, z; \kappa^2) = \int_0^{\infty} d\gamma' \frac{z \kappa^2}{[\gamma + \gamma' + m^2 z^2 + (1 - z^2) \kappa^2]^2}$ *ener dized Stietjes transform*
11 TF Karmanov PLB769 *Generalized Stietjes transform*
Carbonell, TF, Karmanov PLB769 (2017) 418 *Generalized Stietjes transform*

Purely relativistic nature of the aligned spin component! Eq. (17) for the antiparallel spin component. On the other side the decay constant can be also with *k*⁺ = *p*⁺*z/*2 (*p*⁺ = *M*), and where we have used that *d*⁴*k* = ¹ \longrightarrow Pu

Valence probability:
$$
P_{\text{val}} = \frac{N_c}{16 \pi^2} \int_{-1}^{1} dz \int_{0}^{\infty} d\gamma \left[|\psi^{\uparrow\downarrow}(\gamma, z)|^2 + |\psi^{\uparrow\downarrow}(\gamma, z)|^2 \right]
$$

$f_{\pi} = -i$ *N^c p*⁺ $\int \frac{d^4k}{(2\pi)^4} \text{Tr}[\,\gamma^+ \,\gamma^5 \,\Phi(p,k)] = \frac{2\,N_c}{M}$ $\int \frac{d^2k_{\perp}}{k_{\perp}}$ $(2\pi)^2$ dk^+ $\frac{\partial}{\partial z_1} \psi_{\uparrow \downarrow}(\gamma, z)$ N_c $\int d^4k$ m_i , \pm $5 \pi (n-l_i)^2$ $2N_c$ $\int d^2k_{\perp} dk^+$ **Decay constant: Decay constant:** $f_{\pi} = -i \frac{1}{n^+} \int \frac{1}{(2\pi)^4} \text{Tr}[\gamma^+ \gamma^0 \Phi(p, k)] = \frac{1}{M} \int \frac{1}{(2\pi)^2} \frac{1}{2\pi} \psi_{\uparrow\downarrow}(\gamma, z)$ $\sum_{i=1}^{n}$ radius, valence and non-valence charge radii. The experimental pion charge radius is 0*.*657 *±* 0*.*003 fm [34] with

The experimental value of f_{π} is 130.50 \pm 0.017 MeV *P* experimental value of f_{-} is 130.50 + 0.017 MeV The experimental value of f_{π} is 130.50 \pm 0.017 MeV

IX 187 1.25 1 2 0.514 0.71 96 0.913 0.975 0.742

TABLE I. Pion model with $m_{\pi} = 140$ MeV for different parameter sets, m and f_{π} in MeV. Calculated valence probability, total, antiparallel and parallel, and decay constant. The values of the coupling constant α_s and the effective strength, defined in Eq. (46), are also given. defined in Eq. (46), are also given. decay constant. The values of the coupling constant α_s and the effective strength, total, antiparallel and parallel, and decay constant. The values of the coupling constant α_s and the effective strength,
defined in Eq. (46), are also given.

$$
\overline{\alpha}_s = \frac{\alpha_s}{\frac{\mu^2}{m^2} + 0.2} \quad \text{with} \quad \alpha_s = \frac{g^2}{4\pi} (1 - \mu^2/\Lambda^2)^2
$$

$$
\varphi_{\uparrow\downarrow}(\xi) = \frac{\int_0^\infty d\gamma \, \psi_{\uparrow\downarrow}(\gamma, z)}{\int_0^1 d\xi \int_0^\infty d\gamma \psi_{\uparrow\downarrow}(\gamma, z)},
$$

$$
\varphi_{\uparrow\uparrow}(\xi) = \frac{\int_0^\infty d\gamma \psi_{\uparrow\uparrow}(\gamma, z)}{\int_0^1 d\xi \int_0^\infty d\gamma \psi_{\uparrow\uparrow}(\gamma, z)}.
$$

$$
\varphi_{\uparrow\downarrow}^T(\gamma) = \frac{\int_0^1 d\xi \, \psi_{\uparrow\downarrow}(\gamma, z)}{\int_0^1 d\xi \int_0^\infty d\gamma \, \psi_{\uparrow\downarrow}(\gamma, z)},
$$

$$
\varphi_{\uparrow\uparrow}^T(\gamma) = \frac{\int_0^1 d\xi \, \psi_{\uparrow\uparrow}(\gamma, z)}{\int_0^1 d\xi \int_0^\infty d\gamma \, \psi_{\uparrow\uparrow}(\gamma, z)},
$$

$$
P_{\text{val}} = \int_{-1}^{1} dz \int_{0}^{\infty} d\gamma \, \mathcal{P}_{val}(\gamma, z)
$$

|
|-
| 6 ft function (1 $\frac{1}{2}$ **h** $\frac{1}{2}$ the antiparallel, parallel and total valence distributions. or *N* TABLE II. Exponent of the fit function $(1-\xi)^{\eta}$ $(\xi \to 1)$ for

FIG. 3. 3D-valence momentum distribution as a function of ξ and $\gamma = k_{\perp}^2$. Panels from top to bottom represent the results for the parameter sets (II) , (IV) and (VII) , respectively. respectively. The results for the parameter sets (11) , $(1 \nvert$
respectively. (mion
ent
II) of ζ and $\eta = \kappa_+$. Fancis from top to bottom letter to the pion center. The pion center. The pion center. The Table II is controlled to the Table II in the Table II is controlled to the Table II in the Table II is controlled to the Table II in the Table II is control sent between the *f*⇡*/m* ratio and the exponent of the func-

3D Pion image on the null-plane

The probability distribution of the quarks inside the pion, on the light-front, is evaluated in the space given by the Cartesian product of the Ioffe-time and the plane spanned by the transverse coordinates.

Our goal is to use the configuration space in order to have a more detailed information of the space-time structure of the hadrons.

The Ioffe-time is useful for studying the relative importance of short and long light-like distances. It is defined as:

 $\tilde{z} = x \cdot P_{\text{target}} = x^{-} P_{\text{target}}^{+} / 2$ on the hyperplane $x^{+} = 0$

Miller & Brodsky, PRC 102, 022201 (2020)

(2010), arXiv:1009.2481 [hep-ph].

 \mathcal{D}

 $\sqrt{2}$

 \rightarrow $\sum_{i=1}^{n}$

K. Jansen, G. Koutsou, B. Kostrzewa, F. Ste↵ens,

b₂₀

dinal mo-
and dash

 $rac{d}{dt}$

3D Pion image on the null-plane: Spin configurations

Space-time structure of the pion in terms of $\tilde{z} = x^-p^+/2$ and transverse coord. {b_x, b_y}

 \mathcal{F} and \mathcal{F} is valuence contribution by the monopole form \mathbf{G} . Lepage, b. J. Drousky, I hys. Lett. D of (1919) $\partial \mathcal{F}$ $Q^2 F_{\text{asymp}}(Q^2) = 8\pi\alpha_s(Q^2)f_\pi^2$ G. Lepage, S. J. Brodsky, Phys. Lett. B 87 (1979) 359-

$$
F_{\pi}(Q^2) = \sum_{n} F_n(Q^2) = F_{val}(Q^2) + F_{nval}(Q^2)
$$

[56] S. J. Brodsky, H. C. Pauli and S. S. Pinsky, Phys. Rept. 301 (1998) 299.

$$
r_{\pi}^{2} = P_{val} r_{val}^{2} + (1 - P_{val}) r_{nval}^{2}
$$

 0.657 ± 0.003 fm \overline{I} 0.657 ± 0.003 fm B. Ananthanarayan, I. Caprini, D. Das, Phys. Rev. Lett. 119 (2017) 132002 component is *rval* = 0*.*709 fm, showing an extended valence quark charge dis- $\mathcal{I}(\mathcal{I})$

 $\frac{1}{2}$ i and are distribution from the standard way $\frac{1}{2}$ parton distribution function in a p *m* 5*,* PDF. The starting point is the unpolarized transverseu pion with ivinikowskian dynamics *<u>S*^{*x*} *R*₁</sub> *Phe parton distribution function in a pion with <i>Physon*</u> *^S*4(*k*; *^P*) = *ⁱ f*₁(*k*) <u>die</u> *n*^d**d**</sup> *n*¹ **d**₂ *d*₂ *d*₂ *d*₂ *d*₂ *d*₂ *d*₂ The parton distribution function in a pion with Minkowskian dynamics

 ∂_{α} Poule V ∂_{α} because ∂_{α} de Paula, Y drefors, Alvarenga N *^M /k*5*,* ra Noqueira, TE Salmè, PRD 105, L071505 (2022 *A*
 de Paula, Y drefors, Alvarenga Nogueira, TF, Salmè, PRD 105, L0 α faula, Furtions, Alvarenga Inoguella, TT, Sallite, FKD 109, E071909 (2022) de Paula,Ydrefors, Alvarenga Nogueira, TF, Salmè, PRD 105, L071505 (2022)

*^d*kˆ?

⇥

dp⁺

of fermionic fields, dressed by QCD interactions, that is a strategic interactions, th

dk⁺(*k*⁺ ⁺ *^P* ⁺*/*² ⇠*^P* ⁺)

^q (*p*⁺

^q ⇠*^P* ⁺)

tool, since one can recover a probabilistic framework,

Ref. [12] for the pion electromagnetic form factor),

 Γ even looding twist (twist 2) TMD (transverse *g* = 0 (d) and the distribution is determined to the 1-cycle in the $\frac{1}{2}$ *S*_{*M*} *S*_{*M*} *C*_{*R*} *C*_{*k*} *n*_{*E*} *d*_{*x*} *= d*_{*z*} $\overline{\text{y}}$ = $\overline{\text{y}}$ \mathfrak{p} dent) functions T_{reco} and $\frac{1}{2}$ and $\frac{1}{2}$ im-commutation rules in ℓ pose that the functions *ⁱ* are even for *i* = 1*,* 2*,* 4, un-2) - TMD (tr *z*)⁺ *^q*(*P*_momen nt der $t_{\rm t}$ and $t_{\rm t}$ and σ and σ dressed σ ading-twist (twist 2) - TMD (transverse-moment depe α fermionic fields, dressed by α ng-twist (twist 2) - TMD (transverse-moment dependent) functions \mathbf{c} and the use a dressed quark-pion \mathbf{c} noment dependent) functions $\overline{}$ fermionic fields, dressed by $\overline{}$ noment dependent) functions T even leading twist (twist²) TMD (transverse moment dependent) functions 1-even reading-twist (twist 2) - TiviD (transverse-moment dependent) functions T-even leading-twist (twist 2) - TMD (transverse-moment dependent) functions

$$
f_1(\gamma,\xi)=\ \frac{N_c}{4}\int d\phi_{\hat{\bf k}_\perp}\int \frac{dz^-d{\bf z}_\perp}{2(2\pi)^3}e^{i[\xi P^+z^-/2-{\bf k}_\perp\cdot{\bf z}_\perp]}\ \langle P|\bar\psi_q(-\frac{1}{2}z)\gamma^+\psi_q(\frac{1}{2}z)|P\rangle\big|_{z^+=0}\ \ {\rm LC\ gauge}
$$

The parton distribution of the pion is obtained for the first time for the solution of α dynamical for a dynamical for α

⁼ *^N^c*

2(2⇡)³ *^eⁱ*[(*p*⁺

⇥ h*P[|]* ¯*q*(¹

z
Z 1910

2

der the change *k* ! *k*, and odd for *i* = 3.

that heuristically amounts to use a dressed quark-pion of use a dressed quark-pion of use a dressed quark-pion

 $\mathcal{L}(\mathcal{L})$ is given by (see Ref. $\mathcal{L}(\mathcal{L})$

z)⁺ *^q*(

2

z)*|P*i

*^f*1(*,* ⇠) = ¹

l *z*+=0

(2⇡)⁴

⇥

1

8

A⁺

$$
f_1(\gamma,\xi) = \frac{1}{(2\pi)^4} \frac{1}{8} \int_{-\infty}^{\infty} dk^+ \delta(k^+ + P^+/2 - \xi P^+) \int_{-\infty}^{\infty} dk^- \int_{0}^{2\pi} d\phi_{\hat{k}_\perp}
$$

\n
$$
\times \left\{ Tr \left[S^{-1}(k - P/2) \bar{\Phi}(k, P) \frac{\gamma^+}{2} \Phi(k, P) \right] - Tr \left[S^{-1}(k + P/2) \Phi(k, P) \frac{\gamma^+}{2} \bar{\Phi}(k, P) \right] \right\}
$$

\n
$$
0.5
$$

\n
$$
\frac{\partial}{\partial t} \left\{ \frac{1}{2} \sum_{n=0}^{\infty} \frac{1}{2} \left(\frac{\gamma}{2} \right) \left[\frac{1}{2} \left(\frac{\gamma}{2} \right) \left(\frac{\gamma}{2} \right) \right] + \frac{\gamma^+}{2} \Phi(k, P) \right] \right\}
$$

\n
$$
0.5
$$
<

4

up the square modulus of each amplitude present in the square present in the square present in the square pres

dk

the Fock expansion, we obtain

Parton distribution function

WP, Ydrefors, Nogueira, Frederico and Salme PRD 105 L071505 (2022).

Low order Mellin moments at scales $Q = 2.0$ GeV and $Q = 5.2$ GeV.

 $\langle x^2 \rangle$ and $\langle x^3 \rangle$ - Alexandrou et al PRD 104, 054504 (2021)

LQCD, $Q = 5.0$ **GeV:** $\langle x \rangle$ - Alexandrou et al PRD 103, 014508 (2021)

Hadronic scale and effective charge for DGLAP

 $Q_0 = 0.330 \pm 0.030$ GeV - Cui et al EPJC 2020 80 1064

Within the error, we choose $Q_0 = 0.360$ GeV to fit the first Mellin moment. We used lowest order DGLAP equations for evolution

Parton distribution function

WP, Ydrefors, Nogueira, Frederico and Salmè PRD 105, L071505 (2022).

Comparison with other theoretical calculations

Solid line: full calculation of the BSE evolved from the initial scale $Q_0 = 0.360$ GeV to $Q = 5.2$ GeV Dashed line: DSE calculation (Cui et al) Dash-dotted line: DSE calculation with dressed quark-photon vertex from Bednar et al PRL 124, 042002 (2020) Dotted line: BLFQ colaboration, PLB 825, 136890 (2022) Gray area: LQCD results from C. Alexandrou et al (2021) It is in agreement with PQCD, exponent greater than 2

Evolved $\xi u(\xi)$, for $\xi \to 1$, the exponent of $(1 - \xi)^{\eta_5}$ is $\eta_5 = 2.94$ LQCD: Alexandrou et al PRD 104, 054504 (2021) obtained 2*.*20 *±* 0*.*64 Cuit et al EPJA 58, 10 (2022) obtained 2*.*81 *±* 0*.*08

 \overline{a} 7, 1780 (1973). [32] J. Conway *et al.*, Experimental Study of Muon Pairs Produced by 252-GeV Pions on Tungsten, Phys. Rev. D 39, 92 (1989).

[33] M. Aicher, A. Schäfer, and W. Vogelsang, Soft-gluon resummation and the valence parton distribution function of the pion, Phys. Rev. Lett. 105 , 252003 (2010) , arXiv:1009.2481 [hep-ph].

FIG. 2. (Color online). The distribution function $\xi u(\xi)$ in a pion. Solid line: full calculation (see Eqs. (7) and (8)), obtained from the BS amplitude solution of the BSE with $m = 255 \text{ MeV}, \mu = 637.5 \text{ MeV}$ and $\Lambda = 306 \text{ MeV},$ and evolved from the initial scale $Q_0 = 0.360$ GeV to $Q =$ 5*.*2 GeV (see text). Dashed line: the evolved LF valence component, Eq. (9). Full dots: experimental data from Ref. [32]. Full squares: reanalyzed data by using the ratio between the fit 3 of Ref. [33], evolved to 5*.*2 GeV, and the experimental data [32], at each data point, so that the resummation effects (see text) are accounted for.

Quark unpolarized transverse-momentum distribution functions of the pion Vuark unporarized transverse-monetation distribution functions of the profit
Ydrefors, de Paula, TF, Salmè, e-Print: 2301.11599 [hep-ph] ordinates are *^a[±]* ⁼ *^a*⁰ *[±] ^a*³), and by adopting both Quark unpolarized transverse-momentum dis component *P*ˆ is interacting in the LF dynamics, see, uuon runcuon $\overline{}$ $\frac{1}{\sqrt{2}}$ ρ pion and ρ and ρ and ρ *a propertion distribut* k unpolarized transverse-moniemum distribut. molarized transverse-momentum produced a and only information <u>ribution</u> functions of the nion **Primidistribution functions of the pion**

Bethe-Salpeter amplitude: beyond the valence states **Light-front projection** au. *p*u**l** that individually declared that has a particular that α \mathbf{p} is the negligible probability of the higher probability of \mathbf{A}^{α}

- \triangleright higher Fock-components $|\pi\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q} \, 2g\rangle + \cdots$
- ^Ø **gluon radiation from initial state interaction (ISI)** Due to the charge symmetry, each Fock-component is
- ^Ø **No gluon radiation in the final state (FSI)** *in the final state (r***oi**)

Sales, TF, Carlson,Sauer, PRC 63, 064003 (2001); Marinho, TF, Pace,Salme,Sauer, PRD 77, 116010 (2008)

$\overline{C_1}$ **that Gluon momentum in the pion** α \overline{a} The beat momentum in the $\overline{3}$ $\frac{1}{2}$ Gluon momentum in the pion

$$
|\pi\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}2g\rangle + \cdots
$$
quark momentum distribution

componentum distribution DUCHARGE SYMMETRY, EACH FOCK-component is a component in the component is a component in the component is a component is a component in the component is a component in the component is a component in the component is a com *<u>quark</u> momentum distribution* is small since the valence component *|qq*¯i has 70% of probability (as q u

$$
u^{q}(\xi) = \sum_{n=2}^{\infty} \left\{ \prod_{i=1}^{n} \int \frac{d^{2}k_{i\perp}}{(2\pi)^{2}} \int_{0}^{1} d\xi_{i} \right\}
$$

\n
$$
\times \delta (\xi - \xi_{1}) \delta \left(1 - \sum_{i=1}^{n} \xi_{i} \right) \delta \left(\sum_{i=1}^{n} \mathbf{k}_{i\perp} \right)
$$

\n
$$
\times \left| \Psi_{n}(\xi_{1}, \mathbf{k}_{1\perp}, \xi_{2}, \mathbf{k}_{2\perp}, \ldots) \right|^{2},
$$

\n
$$
\left| \Psi_{n}(\xi_{2}, \mathbf{k}_{2\perp}, \ldots) \right|^{2},
$$

\n
$$
\left| \Psi_{n}(\xi_{1}, \mathbf{k}_{1\perp}, \xi_{2}, \mathbf{k}_{2\perp}, \ldots) \right|^{2},
$$

\n
$$
\left| \Psi_{n}(\xi_{2}, \mathbf{k}_{2\perp}, \ldots) \right|^{2},
$$

\n
$$
\left| \Psi_{n}(\xi_{1}, \mathbf{k}_{1\perp}, \xi_{2}, \mathbf{k}_{2\perp}, \ldots) \right|^{2},
$$

\n
$$
\left| \Psi_{n}(\xi_{1}, \mathbf{k}_{1\perp}, \xi_{2}, \mathbf{k}_{2\perp}, \ldots) \right|^{2},
$$

\n
$$
\left| \Psi_{n}(\xi_{1}, \mathbf{k}_{1\perp}, \xi_{2}, \mathbf{k}_{2\perp}, \ldots) \right|^{2},
$$

\n
$$
\left| \Psi_{n}(\xi_{1}, \mathbf{k}_{1\perp}, \xi_{2}, \mathbf{k}_{2\perp}, \ldots) \right|^{2},
$$

\n
$$
\left| \Psi_{n}(\xi_{1}, \mathbf{k}_{1\perp}, \xi_{2}, \mathbf{k}_{2\perp}, \ldots) \right|^{2},
$$

\n
$$
\left| \Psi_{n}(\xi_{1}, \mathbf{k}_{1\perp}, \xi_{2}, \mathbf{k}_{2\perp}, \ldots) \right|^{2},
$$

\n
$$
\left| \Psi_{n}(\xi_{1}, \mathbf{k}_{1\perp}, \xi_{2}, \mathbf{k}_{2\
$$

$$
\delta \left(1 - \sum_{i=1}^{n} \xi_i \right) \delta \left(\sum_{i=1}^{n} \mathbf{k}_i \right)
$$
\n
$$
\delta \left(1 - \sum_{i=1}^{n} \xi_i \right) \delta \left(\sum_{i=1}^{n} \mathbf{k}_i \right)
$$
\n
$$
\delta \left(\sum_{i=1}^{n} \mathbf{k}_i \right)
$$
\n
$$
\delta \left(\sum_{i=1}
$$

momentum sum-rule in the HFS
The HFS \sim 1, \sim 1, \sim posed by a *qq*¯ pair and *n* 2 gluons, generated by sum-rule in the H *Pⁿ* h⇠*q*i*ⁿ*

$$
\langle \xi_q \rangle_{HFS} = 1 - \langle \xi_{\bar{q}} \rangle_{HFS} - \langle \xi_g \rangle
$$
0.2

 $P(x)$ are severally $f(x)$ of the la constitution of the result of the pion, the pion, the pion, the electronic section, the electronic sec \mathbf{m} of the pion! where the gluon bosonic nature leads to the factor α is small since the valence component *|qq*¯i has 70% of Gluons carry 6% of the longitudinal momentum of the pi tion), and hence is largely dominant. Gradis carry the longitu **P**_n(*n*) *n n* **Gluons carry 6% of the longitudinal momentum of the pion!**

is small since the valence component ω the pi probability (as generated by our dynamical calcula*n* **2.** *n Pn*(*n* 2) h⇠*g*i*ⁿ .* (41) **@** the p $@$ the pion scale

Transverse Momentum Distributions

Unpolarized transverse-momentum dependent quark distributions

TMD's are important for parametrizing the hadronic quark-quark correlator One can define the T-even subleading quark uTMDs, starting from the decomposition of the pion correlator (Mulders and Tangerman, Nucl. Phys. B 461, 197 (1996)). Ω ne can decompo (*recall a Figure in Follopeium currical the cone the setting from the setting from the setting from the setting the setting from the setting from the setting from the setting of* $\frac{1}{2}$ *A* and the prevent subleading quark univitys, starting from the
A of the pion correlator (Mulders and Tangerman, Nucl. Phys. B 461, 197 (1996)) the profit correlator

 $\tt twist-3$ $\sf uTMD$ (See Lorcé, Pasquini, and Schweitzer, EPJC 76, 415 (2016)): ¹ (*,* ⇠), is defined as follows (for a

$$
\frac{M}{P^+} e^q(\gamma,\xi) = \frac{N_c}{4} \int d\phi_{\hat{\mathbf{k}}_{\perp}} \int_{-\infty}^{\infty} \frac{dy^{-} dy_{\perp}}{2(2\pi)^3} e^{i[\xi P^+ y^{-}/2 - \mathbf{k}_{\perp} \cdot \mathbf{y}_{\perp}]} \langle P | \bar{\psi}_q(-\frac{y}{2}) \hat{1} \psi_q(\frac{y}{2}) | P \rangle \big|_{y^+ = 0}
$$

$$
\frac{M}{P^+} f^{\perp q}(\gamma,\xi) = \frac{N_c M}{4\gamma} \int d\phi_{\hat{\mathbf{k}}_{\perp}} \int_{-\infty}^{\infty} \frac{dy^{-} dy_{\perp}}{2(2\pi)^3} e^{i[\xi P^+ y^-/2 - \mathbf{k}_{\perp} \cdot \mathbf{y}_{\perp}]} \langle P | \bar{\psi}_q(-\frac{y}{2}) \mathbf{k}_{\perp} \cdot \gamma_{\perp} \psi_q(\frac{y}{2}) | P \rangle \big|_{y^+ = 0}
$$

$$
\gamma=|\mathbf{k}_\perp|^2
$$

of the BS amplitude, obtaining an integral equation

Subleading-twist 3 uTMDs \mathbf{t} this case, one resorts to a suitable method (based one resorts to a suitable method (based one) one resorts to a suitable method (based one) one resorts to a suitable method (based one) one resorts to a suitable **range of the singularity at the singular singular single singular single singular single singular singular singular singular singular singular single singular single single single single single single single single single**

PION MODEL FROM LQCD RUNNING QUAK MASS

Fig. 1. The running quark mass, Eq. (3), as a function of the Euclidean momentum $p = \sqrt{-p^{\mu}p_{\mu}}$, with parameters from (4), is given by the solid line and compared to lattice QCD calculations from [37]. The dashed line shows the parametrization used in reference [5] L

C.S. Mello et al. / Physics Letters B 766 (2017) 86-

$$
S_F(k) = i \left[k - M(k^2) + i\epsilon \right]^{-1}
$$

$$
M(k^2) = m_0 - m^3 \left[k^2 - \lambda^2 + i\epsilon \right]^{-1}
$$

$$
m_0 = 0.014 \text{ GeV}, \ m = 0.574 \text{ GeV} \text{ and } \lambda = 0.846 \text{ GeV}
$$

$$
\psi_{\pi}(k; P) = S_F(k + P/2) \Gamma_{\pi}(k; P) S_F(k - P/2)
$$

$$
\Gamma_{\pi}(k; P) = i \mathcal{N} \gamma_5 M(k^2)|_{m_0 = 0}
$$

 $\Phi(k,p) = S_1\phi_1 + S_2\phi_2 + S_3\phi_3 + S_4\phi_4$ $S_1 = \gamma_5 \quad S_2 = \frac{1}{M} p \gamma_5 \quad S_3 = \frac{k \cdot p}{M^3} p \gamma_5 - \frac{1}{M} k \gamma_5 \quad S_4 = \frac{i}{M^2} \sigma_{\mu\nu} p^{\mu} k^{\nu} \gamma_5$ $\phi_i(k,p) = \int_{-1}^1 dz' \int_{-\infty}^{\infty} d\gamma' \frac{g_i(\gamma',z')}{[k^2 + z'k \cdot p - \gamma' + i\epsilon]^3}.$

FIG. 5. Weight function $g_1(\gamma, z)$ as a function of z for $\gamma =$ 0.45 GeV^2 coming from the covariant model with fixed constituent mass (dashed line) and with the QCD pion inspired model (the single constituent mass pole model has $g_3 = 0$). For the plot, we use the arbitrary value of $\mathcal{N} = 100$, drop out the factor of i and all parameters are in units of GeV.

FIG. 6. Weight function $g_2(\gamma, z)$ as a function of z for $\gamma =$ 0.45 GeV^2 coming from the covariant model with fixed constituent mass (dashed line) and with the QCD pion inspired model (continuous line). For the plot, we use the arbitrary value of $\mathcal{N} = 100$, drop out the factor of *i* and all parameters are in units of GeV.

 $0.00-$ 0.002 $(z',\lambda)^{\dagger_S}$ -0.002 -0.004 -0.006 -0.008 -0.5 0.5 $\boldsymbol{0}$ -1 z^{\prime}

FIG. 7. Weight function $g_3(\gamma, z)$ as a function of γ computed with the QCD pion inspired model for $z = 0.5$ (continuous line) and for $z = -0.5$ (dashed line). For the plot, we use the arbitrary value of $\mathcal{N} = 100$, drop out the factor of i and all parameters are in units of GeV.

FIG. 8. Weight function $g_4(\gamma, z)$ as a function of z for $\gamma =$ 0.45 GeV^2 coming from the covariant model with fixed constituent mass (dashed line) and with the QCD pion inspired model (continuous line). For the plot, we use the arbitrary value of $\mathcal{N} = 100$, drop out the factor of i and all parameters are in units of GeV.

 P_{val} $P_{\uparrow\downarrow}$ $P_{\uparrow\uparrow}$ $f_{\pi}(\text{MeV})$ 0.70 0.58 0.12 130.1

FIG. 5. (color online). Charged pion electromagnetic form factor. Experimental points are a compilation adopted from Ref. [56]. Red solid circles and blue diamonds are calculated using the ADFM quark propagator Ansatz and the MMF quark propagator Ansatz, respectively. In the both cases, three different methods of calculation (detailed in the text) yielded the same results. Black dashed line represents the result of Mello *et al.* $\boxed{32}$. Black solid line corresponds to the perturbative QCD result (27) with asymptotic PDA.

FIG. 7. (color online). Blue dots represent π^0 transition form factor calculated using the MMF quark propagator Ansatz, Eqs. \Box and \Box). The red pluses are calculated using the ADFM quark propagator, Eqs. (6). The blue solid line and red dashed line represent the Brodsky-Lepage interpolation formula, Eq. (29), for the MMF quark propagator and ADFM quark propagator models, respectively. Solid circles and diamonds (with error bars) represent the measurements of BaBar $\left| 62 \right|$ and Belle $\left| 63 \right|$ collaboration, respectively.

FIG. 8. (color online). Pion distribution amplitudes $\phi_{\pi}(u)$. Blue solid line and red dashed line correspond to the MMF and ADFM Ansätze, respectively. Black dotted line represents the asymptotic form, $\phi_{\pi}^{as}(u) = 6u(1-u)$.

(MMF) Mello, de Melo, Frederico, PLB766, 86 (2017)

(ADFM) Alkofer, Detmold, Fischer, Maris, PRD70, 014014 (2004)

PION DYNAMICS & QUAK SELF ENERGY

0⁻ Bound State with Running quark mass function

Abigail Castro, WP, Ydrefors, Frederico, Salmè - in preparation

Dressed quark propagator: $S(p) = i Z(p^2) \frac{p + M(p^2)}{p^2 - M^2(p^2)}$ Phenomenological model to reproduce Lattice Data for $\mathcal{M}(p^2)$ (Mello, de Melo, Frederico, PLB 766, 86 (2017)).

$$
\mathcal{M}(\rho^2) = m_0 - \frac{m^3}{\rho^2 - \lambda^2 + i\epsilon}
$$

From Lattice simulations: bare quark mass $m_0 = 8$ MeV and $\mathcal{M}(0) = 0.344$ GeV. Our fit (solid line): $m = 0.648$ GeV and $\lambda = 0.9$ GeV.

Lattice data (dashed line) from: Oliveira, Silva, Skullerud and Sternbec, PRD 99 (2019) 094506

0⁻ Bound State with Running quark mass function

Abigail Castro, WP, Ydrefors, Frederico, Salmè - $S^V(p^2) = \int_0^\infty ds \frac{\rho^V(s)}{\rho^2 - s + i\epsilon}$; $S^S(p^2) = \int_0^\infty ds \frac{\rho^S(s)}{\rho^2 - s + i\epsilon}$ Phenomenological model: $\mathcal{M}(p^2) = m_0 - \frac{m^3}{p^2 - \lambda^2 + i\epsilon}$ $\rho^{S(V)}(s) = \sum_{a}^{S} R_{a}^{S(V)} \delta(s - m_{a}^{2}),$ $a=1$

where $R_a^{S(V)}$ are the residues, that read

$$
R_a^V = \frac{(\lambda^2 - m_a^2)^2}{(m_a^2 - m_b^2)(m_a^2 - m_c^2)},
$$

$$
R_a^S = R_a^V \mathcal{M}(m_a^2),
$$

with the indices $\{a, b, c\}$ following the cyclic permutation $\{1, 2, 3\}$.

0⁻ Bound State with Running quark mass function

Abigail Castro, WP, Ydrefors, Frederico, Salmè - in preparation

Integral Representation:
$$
S^V(p^2) = \int_0^\infty ds \frac{\rho^V(s)}{p^2 - s + i\epsilon}
$$
; $S^S(p^2) = \int_0^\infty ds \frac{\rho^S(s)}{p^2 - s + i\epsilon}$

Using the Nakanishi integral representation for $\phi_i(k, p)$, performing the loop integral and projecting onto the LF, one obtains the BSE as

$$
\int_0^\infty d\gamma' \frac{g_i(\gamma',z)}{\left[\gamma + z^2 M^2/4 + \gamma' + \kappa^2 - i\epsilon\right]^2} = \frac{\alpha}{2\pi}
$$

$$
\times \sum_j \int_{-1}^1 dz' \int_0^\infty d\gamma' \mathcal{L}_{ij}(\gamma,z;\gamma',z') g_j(\gamma',z').
$$

Longitudinal momentum distribution

Abigail Castro, WP, Ydrefors, Frederico, Salmè - in preparation

Thin solid line: running mass model for $M = 0.516$ GeV.

Thin dashed line: fixed quark mass (344 MeV) for $M = 0.516$ GeV.

Dressing the Quark: Schwinger-Dyson equation

The model: Bare vertices, massive vector boson, Pauli-Villars regulator

Credits to Wayne de Paula

The rainbow ladder Schwinger-Dyson equation in Minkowski space is:

$$
S_q^{-1}(k) = k - m_B + ig^2 \int \frac{d^4q}{(2\pi)^4} \Gamma_\mu(q,k) S_q(k-q) \gamma_\nu D^{\mu\nu}(q) ,
$$

where m_B is the quark bare mass and g is the coupling constant. The massive gauge boson is given by

$$
D^{\mu\nu}\left(q\right)=\frac{1}{q^2-m_g^2+\imath\epsilon}\left[g^{\mu\nu}-\frac{(1-\xi)q^\mu q^\nu}{q^2-\xi m_g^2+\imath\epsilon}\right]\,,
$$

where we have introduce an effective gluon mass m_g , as suggested by LQCD calculations (see Dudal, Oliveira and Silva, PRD 89 (2014) 014010). The dressed fermion propagator is

$$
S_q(k) = \left[\sharp A(k^2) - B(k^2) + i\epsilon \right]^{-1}.
$$

Duarte, TF, de Paula, Ydrefors, PRD105, 114055 (2022)

Schwinger-Dyson equation in Rainbow ladder truncation The vector and scalar self-energies are given by the NIR, respectively as:

$$
A(k^{2}) = 1 + \int_{0}^{\infty} ds \frac{\rho_{A}(s)}{k^{2} - s + i\epsilon},
$$

$$
B(k^{2}) = m_{B} + \int_{0}^{\infty} ds \frac{\rho_{B}(s)}{k^{2} - s + i\epsilon}.
$$

The quark propagator can also be written as:

$$
S_q(k)=R\frac{\cancel{k}+\overline{m}_0}{k^2-\overline{m}_0^2+i\epsilon}+\cancel{k}\int_0^\infty ds\frac{\rho_v(s)}{k^2-s+i\epsilon}+\int_0^\infty ds\frac{\rho_s(s)}{k^2-s+i\epsilon},
$$

where \overline{m}_0 is the renormalized mass.

$$
\begin{aligned}\n\mathcal{J}(k^2) - B(k^2) &= ig^2 \int \frac{d^4q}{(2\pi)^4} \frac{\gamma_\mu S_f(k-q)\gamma_\nu}{q^2 - m_g^2 + i\epsilon} \left[g^{\mu\nu} - \frac{(1-\xi)q^\mu q^\nu}{q^2 - \xi m_g^2 + i\epsilon} \right] \\
&\quad - ig^2 \int \frac{d^4q}{(2\pi)^4} \frac{\gamma_\mu S_f(k-q)\gamma_\nu}{q^2 - \Lambda^2 + i\epsilon} \left[g^{\mu\nu} - \frac{(1-\xi)q^\mu q^\nu}{q^2 - \xi \Lambda^2 + i\epsilon} \right] \end{aligned}
$$
\nPauli-Villars

Fermion Schwinger-Dyson equation

• Parameters:
$$
\alpha = \frac{g^2}{4\pi}
$$
, Λ , m_g , \overline{m}_0 .

• Spectral densities are obtained from the IR of the self-energy:

$$
\rho_A(\gamma) = -\frac{1}{\pi} \text{Im} [A(\gamma)]
$$

$$
\rho_B(\gamma) = -\frac{1}{\pi} \text{Im} [B(\gamma)]
$$

• Solutions of DSE obtained writing the trivial relation $S_f^{-1}S_f = 1$ in a suitable form:

$$
\frac{R}{\gamma - \overline{m}_0^2 + i\epsilon} + \int_0^\infty ds \frac{\rho_v(s)}{\gamma - s + i\epsilon} = \frac{A(\gamma)}{\gamma A^2(\gamma) - B^2(\gamma) + i\epsilon}
$$

$$
\frac{R\overline{m}_0}{\gamma - \overline{m}_0^2 + i\epsilon} + \int_0^\infty ds \frac{\rho_s(s)}{\gamma - s + i\epsilon} = \frac{B(\gamma)}{\gamma A^2(\gamma) - B^2(\gamma) + i\epsilon}
$$

Fermion Schwinger-Dyson equation

$$
\rho_A(\gamma) = R\mathcal{K}_{0A}^{\xi}(\gamma, \overline{m}_0^2, m_g^2)
$$

+
$$
\int_0^{\infty} ds \mathcal{K}_A^{\xi}(\gamma, s, m_g^2) \rho_v(s) - [m_g \to \Lambda]
$$

$$
\rho_B(\gamma) = R \overline{m}_0 \mathcal{K}_{0B}^{\xi}(\gamma, \overline{m}_0^2, m_g^2)
$$

+
$$
\int_0^{\infty} ds \mathcal{K}_B^{\xi}(\gamma, s, m_g^2) \rho_s(s) - [m_g \to \Lambda]
$$

• Driving term:

$$
\mathcal{K}^{\xi}_{0A(0B)}=K_{A(B)}+m_{g}^{-2}\bar{K}^{\xi}_{A(B)}
$$

• Kernel:
\n
$$
\mathcal{K}_{A}^{\xi}(\gamma, s, m_g^2) = K_A(\gamma, s, m_g^2) \Theta(s - (\overline{m}_0 + m_g)^2)
$$
\n
$$
+ m_g^{-2} \overline{K}_{A}^{\xi}(\gamma, s, m_g^2) \Theta(s - (\overline{m}_0 + \sqrt{\xi}m_g)^2)
$$

Connection Formulas

$$
f_A(\gamma) = 1 + \int_0^\infty ds \frac{\rho_A(s)}{\gamma - s}
$$

\n
$$
f_B(\gamma) = m_B + \int_0^\infty ds \frac{\rho_B(s)}{\gamma - s}
$$

\n
$$
d(\gamma) = \left[\gamma f_A^2(\gamma) - \pi^2 \gamma \rho_A^2(\gamma) - f_B^2(\gamma) + \pi^2 \rho_B^2(\gamma) \right]^2
$$

\n
$$
= \left[\gamma f_A^2(\gamma) - \pi^2 \gamma \rho_A^2(\gamma) - f_B^2(\gamma) + \pi^2 \rho_B^2(\gamma) \right]^2
$$

\n
$$
+ \frac{\rho_A(\gamma)}{d(\gamma)} \left[\gamma f_A^2(\gamma) - \pi^2 \gamma \rho_A^2(\gamma) - f_B^2(\gamma) + \pi^2 \rho_B^2(\gamma) \right]^2
$$

\n
$$
+ 4\pi^2 \left[\gamma \rho_A(\gamma) f_A(\gamma) - \rho_B(\gamma) f_B(\gamma) \right]^2
$$

\n
$$
+ \frac{\rho_B(\gamma)}{d(\gamma)} \left[\gamma f_A^2(\gamma) - \pi^2 \gamma \rho_A^2(\gamma) - f_B^2(\gamma) + \pi^2 \rho_B^2(\gamma) \right]
$$

Phenomenological Model

Duarte, Frederico, WP, Ydrefors PRD 105, 114055 (2022) We can calibrate the model to reproduce Lattice Data for $M(p^2)$

Lattice data from: Oliveira, Silva, Skullerud and Sternbec, PRD 99 (2019) 094506

IN PROGRESS

- **BSE** in Minkowski space: Quark self-energies (simple model); \checkmark
- Quark self-energies: SDE in Minkowski space & chiral symmetry breaking; \checkmark

[D. Duarte et al PRD105, 114055 (2022)]

Pion FF and T-even TMDs; \checkmark

FUTURE

- \blacktriangleright **T-odd TMDs, GTMDs (DGLAP&ERBL)**
- **Fragmentation Functions** \blacktriangleright
- **Dressed Quarks & Gluons, different gauges** \blacktriangleright
- Confinement & quark-gluon vertex \blacktriangleright
- \blacktriangleright kaon, D, B, rho..., and the nucleon,

Light-Cone 2023: Hadrons and Symmetries

https://indico.in2p3.fr/event/29047/

18 - 22 de set, de 2023 Rio de Janeiro, Brazil Fuso horário America/Sao Paulo

Digite o seu termo de pesquisc Q

Visão Geral

Invited Speakers

Registration

Tabela de Horários

Zoom

Chamada para Resumos (Abstracts)

Lista de Contribuição

Livro de Resumos

Lista de participantes

Proceedings

Committees

Past Workshops

Practical informations

Contact

 \boxtimes lightcone2023@gmail.com

The Light-Cone 2023 (LC2023) will be hosted by the Brazilian Center for Physics Research - CBPF from September 18 to 22, 2023 in the city of Rio de Janeiro, Brazil.

Registration will be open by May 1 to August 31, 2023.

The Brazilian Center for Physics Research (CBPF) is one of the most important institutions in Brazil, where both theoretical and experimental research are developed, offers an ideal and stimulating environment to achieve the goals of the conference. thanks to their active community acting in many aspects of the Standard model and beyond.

The conference continues a series that started in 1991 and, since then, takes place at least once a year under the supervision of the International Light Cone **Advisory Committee.**

The scientific goal of the Light Cone conference series is to continuously update the knowledge in light-front theory, its intersections with Euclidean Lattice and continuum approaches towards the

phenomenological applications to describe hadrons and nuclei. Light-front theory provides a suitable framework for the calculations of decay rates, scattering amplitudes, correlations, spin effects, phases, distributions and other

THANK YOU!

THANKS TO

IRP/CNRS - SUBATOMIC PHYSICS: FROM THEORY TO APPLICATIONS

CAPES/COFECUB project

Backup slides

Generalized Stietjes transform and the LF valence wave function Carbonell, TF, Karmanov PLB769 (2017) 418 (bosons) Generall can bliefes transformed to the BS and lies transform and the LF valence wave function

$$
\Psi_i(\gamma, z; \kappa^2) = \int_0^\infty d\gamma' \frac{g_i(\gamma', z; \kappa^2)}{[\gamma + \gamma' + m^2 z^2 + (1 - z^2)\kappa^2]^2}
$$

$$
\gamma = k_\perp^2 \quad z = 2x-1
$$

BS amplittude

⁄ 1

⁄ Œ

UNIQUENESS OF THE NAKANISHI REPRESENTATION *Feynman Integrals* (Gordon and Breach, New York, 1971).

PHENOMENOLOGICAL APPLICATIONS from the valence wf → BSA!

) + *^L*(*s*)

49 *Generalized Stietjes transform and the LF valence wave function II* Carbonell, TF, Karmanov PLB769 (2017) 418

$$
f(\gamma) \equiv \int_{0}^{\infty} d\gamma' L(\gamma, \gamma') g(\gamma') = \int_{0}^{\infty} d\gamma' \frac{g(\gamma')}{(\gamma' + \gamma + b)^2}
$$

denoted symbolically as $f = \hat{L} g$.

Externed of the LF projected pion BSE with NIR \sim recover of the μ projecties profit bold with that \mathcal{S} K. Kusaka, K. Simpson, and A. G. Williams, \mathcal{S} \triangleright Kernel of the LF projected pion BSE with NIR $\mathbf{1}$ (2008), and private communication.

the tricky powers are *j* = 2*,* 3, even if *n >* 3 is cho- \triangleright end-point singularities in the k- integration (zero-modes) \overline{a} \bullet \blacktriangleright [12] W. de Paula, T. Frederico, R. Pimentel, G. Salm`e and

> T.M. Yan, Phys. Rev. D 7, 1780 (1973). integral, suitable for our purposes, M. Viviani, in preparation.

$$
\mathcal{I}(\beta, y) = \int_{-\infty}^{\infty} \frac{dx}{\left[\beta x - y \mp i\epsilon\right]^2} = \pm \frac{2\pi i \delta(\beta)}{\left[-y \mp i\epsilon\right]}
$$

 \rightarrow Kernel with delta and its derivativel namely the singular contribution to *Lij* , given by \rightarrow Kernel with delta and its derivative!

We also need (1*/*2) @*I*(*, y*)*/*@*y*, easily deduced from Eq.

End-point singularities– more intuitive: can be treated by the pole-dislocation method de Melo et al. NPA631 (1998) 574C, PLB708 (2012) 87 0 ities– more intuitive: ca
.631 (1998) 574C PI B *d d* by the pole n (*z*⁰ *z*) Gegenbauer ones, with indexes equal to 5*/*2*,* 7*/*2*,* 7*/*2*,* 7*/*2 for *gi*(⁰

Light-front amplitudes 51 the valence probability *pval* and decay constant *ffi*.

σ arnal has similar magnitude with LACD form factor \sim 50% **Kernel has similar magnitude with LQCD form-factor ~ 50%**

Figure 4: Left: Integrand of the full form factor vs z for fixed values of Q^2 . In the main frame are shown the rigure 4: Lett: Integrand of the full form factor vs z for fixed values of Q . In the main frame are shown the
results for $z \ge 0$ and in the inset the results for the full interval are visible. Right: The corresponding results for $z \ge 0$ and in the inset the results for the full interval are visible. Right: The corresponding results
for the valence form factor. In the main frame the results obtained by using the complete formula are sh for the valence form factor. In the main frame the results obtained by using the complete formula are shown.
In the inset the results for the asymptotic formula are displayed. For the visibility the results for $Q^2 > 0$ h been multiplied by a factor of 10 . μ is the results for the asymptotic formula are displayed. For the visionity the results for ϵ $>$ ϵ have altiplied by a factor of 10. tactor
ts for for the valence form factor. In the main frame the results obtained by using the complete form
In the inset the results for the asymptotic formula are displayed. For the visibility the results f *^f*⇡ ⁼ *^N^c* $\mathcal{L}_{\mathcal{A}}$

(4.1), and the truncation of the series used to compute ↵*MS* in Eq. (4.1). At the **EVOLUTION**

perfect of the procedure determines the relation of the relation between the relationships between the relationships L. Chang, J. Papavassiliou, C. D. Roberts, J. Rodríguez-Quintero, and S. M. Schmidt, Concern- σ . Rounguez-Quintero, and S. M. Schimar, Concerning pion parton distributions, Eur. Phys. J. A 58, 10 (2022) , arXiv:2112.09210 [hep-ph].

[6] A. Deur, S. J. Brodsky, and G. F. de Teramond, The $[0]$ T. Bear, S. 3. Broasny, and S. 1. de Teramond, The QCD Running Coupling, Nucl. Phys. **90**, 1 (2016), aI Λ IV. IOU4. UOUOZ [Hep-pH]. $\frac{q}{2}$ i ($\frac{q}{2}$) $\frac{q}{2}$ ($\frac{q}{2}$), $\frac{q}{2}$ ($\frac{q}{2}$), $\frac{q}{2}$ ($\frac{q}{2}$), $\frac{q}{2}$ α 11111100 α 0000 α [α p α].

\overline{I} \overline{O} \overline{I} \overline{I} \overline{I} LO evolution with effective α $ECI \Omega = E f_{\text{scat}}^{\text{in}} C \text{cm}^{\text{in}} = I \text{cm}$ LCLO – Eliective Coupling Let Ω uden Errelatio **ECLO** = Effective Coupling Leading Order Evolution

 $S_{\rm eff}$ (see Fig.). So the pion parton distribution distributions, $K_{\rm eff}$ $= 0.330 + 0.030$ ($\mathcal{Y}^{\mathbf{0}}$ discovering $\mathcal{Y}^{\mathbf{0}}$ able that the parameter which fits the Bjorken sum rule data is set independently by $Q_0 \ = \ 0.330 \ \pm 0.030 \ \ \mathrm{GeV}$ point of the QCD e↵ective charge as a function of *Q*²