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Electroweak structure of composite hadronic systems

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Outline:

I. A snapshot on LF dynamics

II. BS amplitude & LF wave Function

III. Mandelstam formula

IV. Pion space- & time-like electromagnetic form-factor

V. Nucleon space- & time-like electromagnetic form-factors

VI. Quasi-Potential & Light-Front B.-S. Equation

VII. Two-boson systems: E.M. current operator & WTI

VIII. Two-Fermion systems: Yukawa model

IX. LF three-boson dynamics & ladder 4d B.-S. equation

X. Conclusions and Perspectives

I. A Snapshot on Light-Front dynamics: $x^+ = t+z=0$

S.J. Brodsky et al. / Physics Reports 301 (1998) 299–486

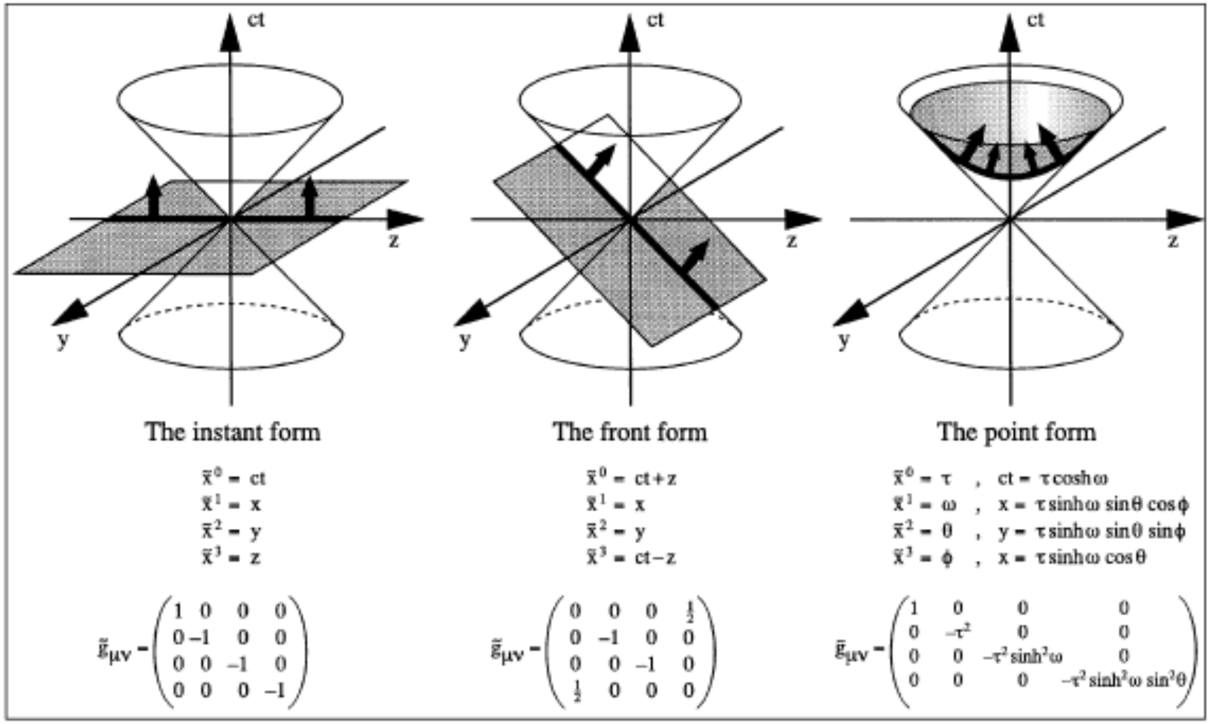
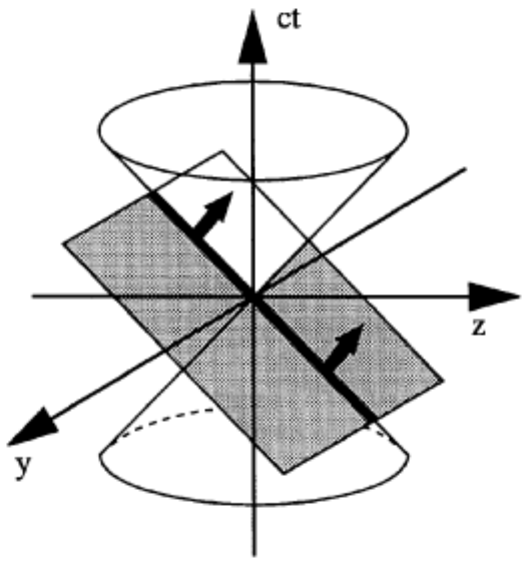


Fig. 1. Dirac's three forms of Hamiltonian dynamics.

Properties of LF quantization



1. **Trivial vacuum - perturbative (except for zero modes);**
2. **Maximal number of 7 kinematical transf. (3 boosts + 1 rot. + 3 transl.)**
3. **Truncation in the Fock-space not stable under rotations around transverse directions (non-kinematical boosts).**

Light-Front time x^{+}: $\tau = t + z/c$

Generator of LF time translations

$$\hat{p} = (\hat{p}^{+}, \hat{p}^{-}, \hat{\mathbf{p}}_{\perp}) \qquad \hat{p}^{\pm} = \hat{p}^{0} \pm \hat{p}^{3}$$

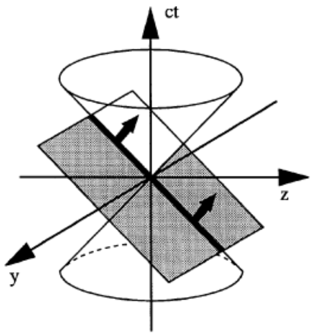
Kinematical generators of translations

Mass square Fock-space operator

$$\boxed{H_{LF} = \hat{p}^{-}\hat{p}^{+} - \hat{\mathbf{p}}_{\perp}^{2}}$$

Particles: eigenstates

$$\boxed{H_{LF}|p\rangle = \mathcal{M}_{H}^{2}|p\rangle}$$



Properties of LF quantization

"Two-particle state" (valence component with two-particles):

$$|p\rangle = \sum_{n\geq 2} \int [dx_i]\,[d^2\mathbf{k}_{i\perp}]\;\psi_{n/p}(x_i, \mathbf{k}_{i\perp}, \lambda_i)\big|n : x_i p^+, x_i \mathbf{p}_\perp + \mathbf{k}_{i\perp}\big\rangle$$

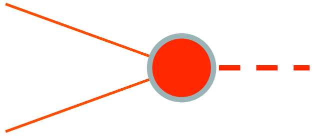
$$\int [dx_i]\,[d^2\mathbf{k}_{i\perp}] \equiv \prod_{i=1}^{n} \int dx_i \frac{d^2\mathbf{k}_{i\perp}}{2(2\pi)^3}\,\delta\Big(1 - \sum_{j=1}^{n} x_j\Big)\, 16\pi^3\,\delta^{(2)}\Big(\sum_{j=1}^{n} \mathbf{k}_{\perp j}\Big)$$

Fock-amplitudes: $\psi_{n/p}(x_i, \mathbf{k}_{i\perp}) = \big\langle n : x_i, \mathbf{k}_{i\perp}\big|p\big\rangle$

- ***Relation with covariant quantum field theory formulation ?***

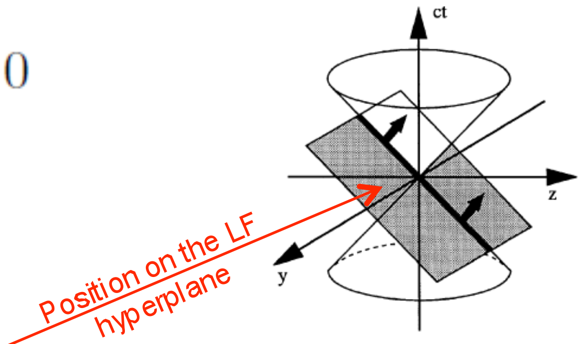
II. Bethe-Salpeter amplitude & LF wave-function

H.A. Bethe, E.E. Salpeter, Phys. Rev. 84 (1951) 1232



$$\boxed{\Phi(x_1^\mu, x_2^\mu, p^\mu) = \langle 0 | T\left\{\varphi(x_1^\mu)\varphi(x_2^\mu)\right\} |p\rangle}$$

$$X^\mu = (x_1^\mu + x_2^\mu)/2,\ x^\mu = x_1^\mu - x_2^\mu \qquad X^\mu = 0$$



$$\boxed{\tilde{\Phi}(x, p) = \langle 0 | T\left\{\varphi(x^\mu/2)\varphi(-x^\mu/2)\right\} |p\rangle =}$$

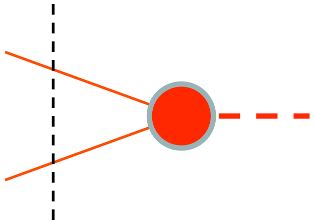
$$= \theta(x^+)\ \langle 0|\varphi(\tilde{x}/2)\ e^{-i\hat{p}^- x^+/2}\ \varphi(-\tilde{x}/2)|p\rangle\ e^{ip^- x^+/4}\ + \cdots$$

$$\boxed{= \theta(x^+)\ \sum_n e^{-i(p_n - p/2)^- x^+/2} \langle 0|\varphi(\tilde{x}/2)|p_n\rangle\ \langle p_n|\varphi(-\tilde{x}/2)|p\rangle\ + \cdots}$$

$$\boxed{\hat{p}\ |p_n\rangle\ =\ p_n\ |p_n\rangle}$$

Full complexity of the Fock-space is necessary to describe the BS amplitude!

Valence component & BS amplitude @ equal LF times



$$\lim_{x^+\to 0_+} \tilde{\Phi}(x,p) = \langle 0|\varphi(\tilde{x}/2)\ \varphi(-\tilde{x}/2)|p\rangle$$

$$= \sum_n e^{-i(p_n-p/2)^- x^+/2}\langle 0|\varphi(\tilde{x}/2)|p_n\rangle\ \langle p_n|\varphi(-\tilde{x}/2)|p\rangle$$

0

$$= \langle 0|\varphi(\tilde{x}/2)\varphi(-\tilde{x}/2)|p\rangle$$ *valence wave function*

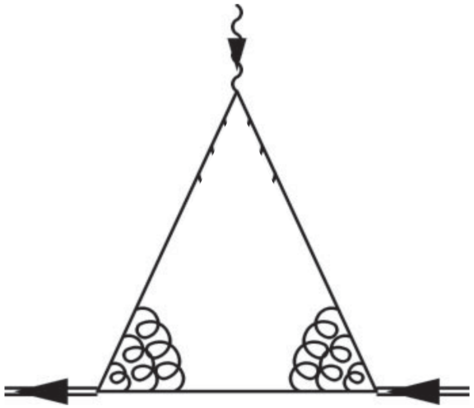
$$\boxed{\psi_{n=2/p}(x,\mathbf{k}_\perp) = p^+\ x\ (1-x)\ \int_{-\infty}^{\infty}\frac{dk^-}{2\pi}\ \Phi(k,p)}$$

Fourier transf.

Is it possible from the valence wf retrieve the full BS amplitude? Yes!

How to reveal the Fock-space structure with electron scattering?

III. Principles: Mandelstam formula - impulse approximation



STRATEGY 1

- Chargeless particle exchange & point-like vertices: *Current conservation;*

- Input BS amplitude (e.g., use a model or solve BS equation or *use a systematic 3D reduction to LF*);

STRATEGY 2

- Phenomenology uses a naive LF reduction (elimination of the relative LF time) → integration over $\boldsymbol{k}^-$ and use further constraints on the valence wave function (*it can be done systematically if the underlying field theory is given*);
- Applications to pion and nucleon form factor (*lessons for the systematical reduction to LF*).

Frame for evaluating form-factors?

Frames for computation of e.m. form factors with LF wave functions

$q^+=0$ ***Drell-Yan frame*** : one-body e.m. current operator diagonal Fock-space

$\mathbf{q}_\perp=0$ ***Lev-Pace-Salme frame***: one-body e.m. current operator non-diagonal F.-S.
PRL83 (1999) 5250

Frame	Advantages	Disadvantages
Drell-Yan	Diagonal Fock-space	Dominated by valence wf Angular conditions (a.c.) **J>1/2** for $\mathbf{I}^+$ Violation a.c. with truncation F-S No TL processes
Lev-Pace-Salme	Trivial angular condition *Space- and time-like ff*	Non-diagonal Fock-space Truncation beyond the valence $\mathbf{I}^+$ and $\mathbf{I}_\perp$

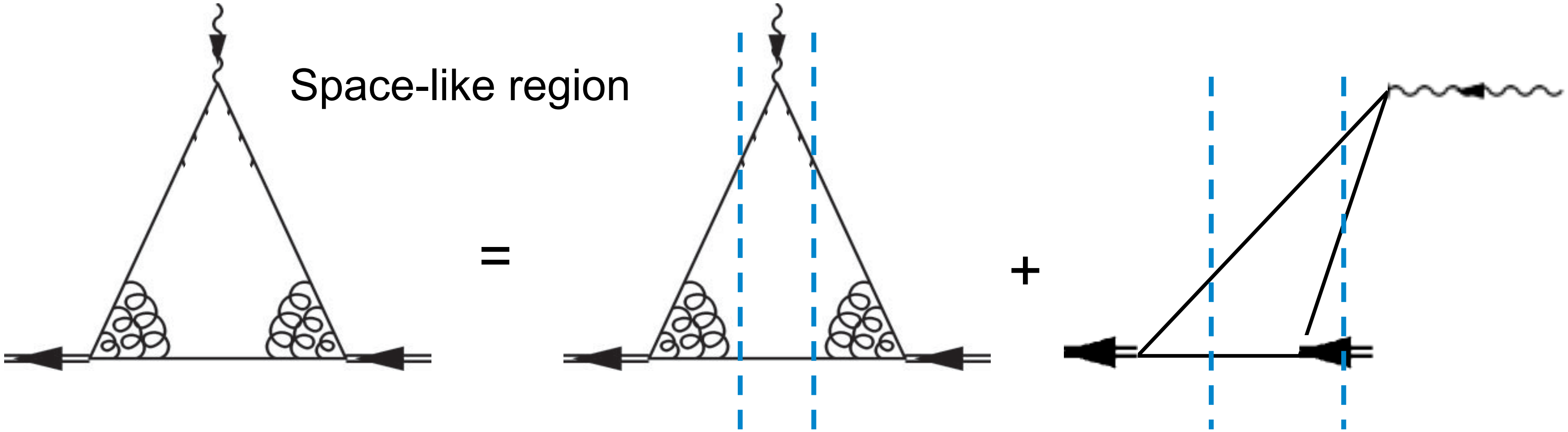
$q^+ > 0$ frame: Sawicki, Brodsky,Ji, Bakker, de Melo, Salme, Pace,TF, Miller,Tiburzi…

Nonvalence vertices→ two-body currents!

GPD (two-body syst.): Tiburzi & MillerPRD67(03)054014; 054015

GPD (pion): Ji, Mischenko, Radyuskin PRD73(06)114013;
TF,Pace,Pasquini,Salme PRD80(09)054021.

Naive elimination of the relative LF time in the Mandestam impulse approximation:



$$q^2 = (p' - p)^2 = \omega^2 - |\mathbf{q}|^2 = -Q^2 < 0 \qquad \boxed{\mathbf{q}_\perp = 0}$$

$$p'^+ - p^+ = q^+ \qquad p'^- - p^- = \frac{M_N^2}{p'^+} - \frac{M_N^2}{(p'^+ - q^+)} = q^- = \frac{q^2}{q^+}$$

$$\boxed{p^+ = \frac{q^+}{2}\left[1 - \sqrt{1 - 4\frac{M^2}{q^2}}\right] \qquad p'^+ = \frac{q^+}{2}\left[1 + \sqrt{1 - 4\frac{M^2}{q^2}}\right]}$$

$Q^2 >> M^2$ → triangle diagram is suppressed ! ($p^+ << p'^+$)

Dominance of the pair diagram & two-body currents !

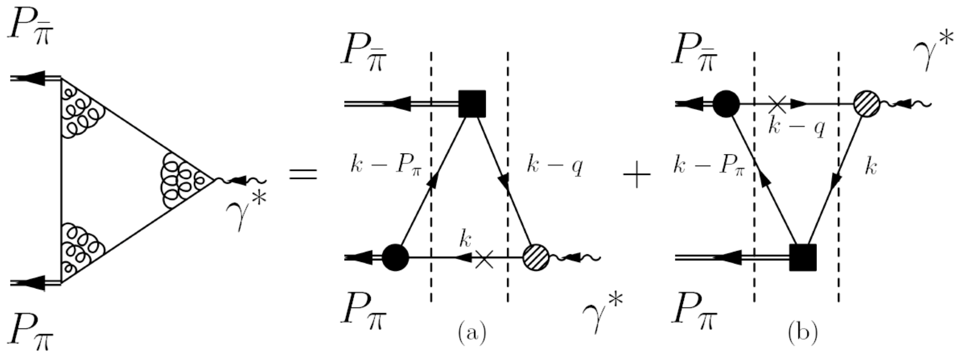
Sawicki, Brodsky, Hwang, Bakker, Ji, Choi, de Melo, Pace, Salme…

IV. Pion SL and TL E.M. form factor

de Melo et al, PLB 581(04) 75, PRD73(06) 074013

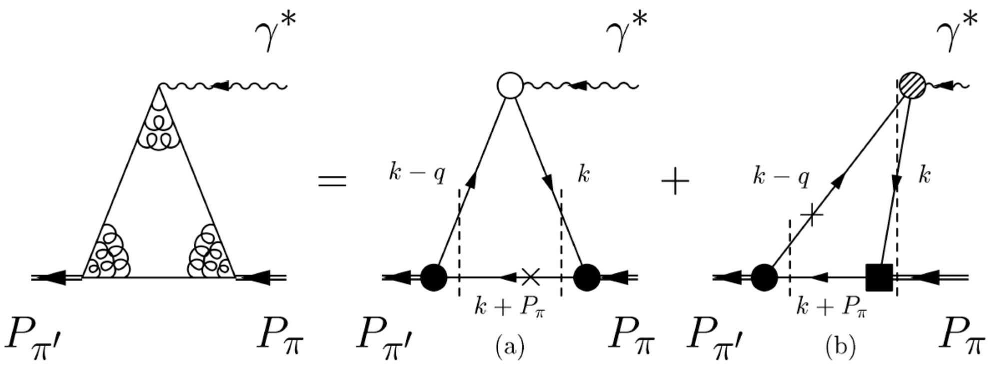
$$q^+ > 0\ ,\quad \mathbf{q}_\perp = 0$$

Time-like region (Fig. 1)



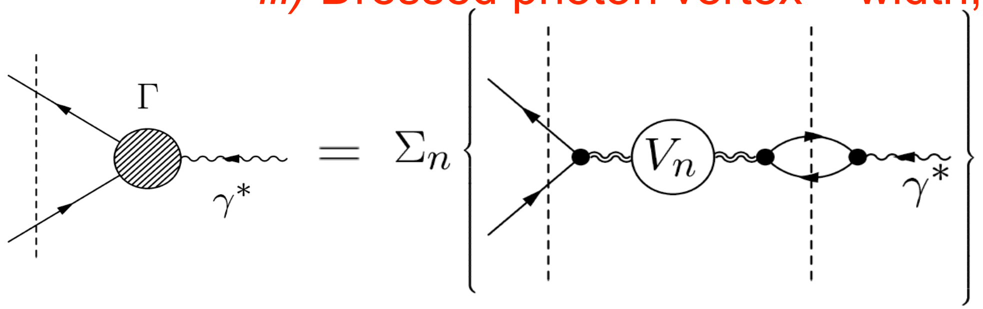
$0 < k^+ < P_\pi^+$ $\qquad$ $P_\pi^+ < k^+ < q^+$

Space-like region (Fig. 2)

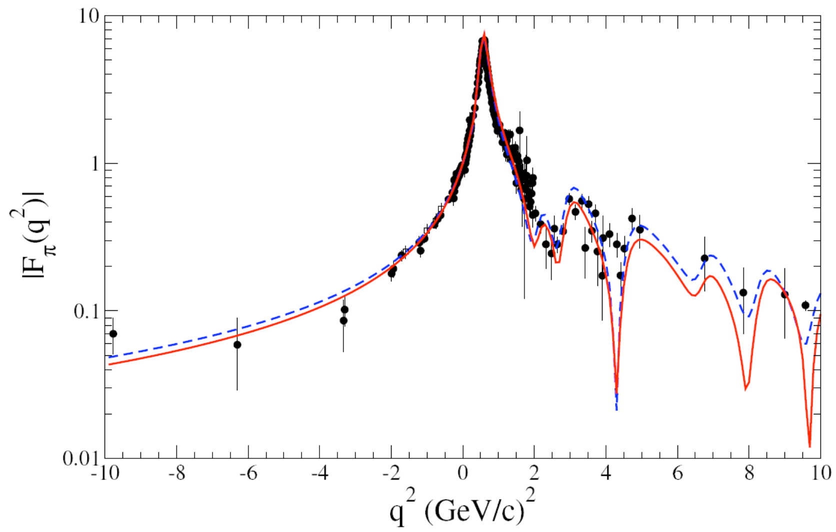


Ingredients: *i)* Pion nonvalence vertex = constant ; *ii)* pion massless;

iii) Dressed photon vertex ~ width, wf & spin structure;



Vector mesons:
wf: T.F, HC Pauli, SG Zhou, PRD66(02)
Spin struc.: [Jaus PRD 41 (1990) 3394)]



● → Experimental EM form factor from the collection by R. Baldini et al. (Eur. Phys. J. C11 (1999) 709, and private comm.)

V. Nucleon SL and TL E.M. form factor

de Melo et al PLB 671(09) 153

$$\bar{U}^{\sigma'}_{N'} I^{\mu}(q^2) U^{\sigma}_{N}$$
$$= 3N_c \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \sum \{ \bar{\Phi}^{\sigma'}(k_1, k_2, k'_3, P'_N)$$
$$\times S^{-1}(k_1) S^{-1}(k_2) \mathcal{I}^{\mu}(k_3, q) \Phi^{\sigma}(k_1, k_2, k_3, P_N) \},$$

Nucleon BS amplitude:

$$\Phi^{\sigma}(k_1, k_2, k_3, P_N) = \Lambda(k_1, k_2, k_3) \mathcal{U}^{\sigma}(k_1, k_2, k_3, P_N)$$

Nucleon spin-coupling model: de Araújo et al Phys. Lett. B478 (01)86

$$\mathcal{U}^{\sigma} = [\mathcal{S}(123) + \mathcal{S}(312) + \mathcal{S}(321)] \chi_{\tau_N} U^{\sigma}_{N}$$

with χ_{τ_N} the nucleon isospin state and

$$\mathcal{S}(ij\ell) = \imath [S(k_i) \tau_y \gamma^5 S_C(k_j) C] \otimes S(k_\ell).$$

$$S_C(k) = C S^T(k) C^{-1}$$

*Kinematics for **time-like** nucleon e.m. form factors*

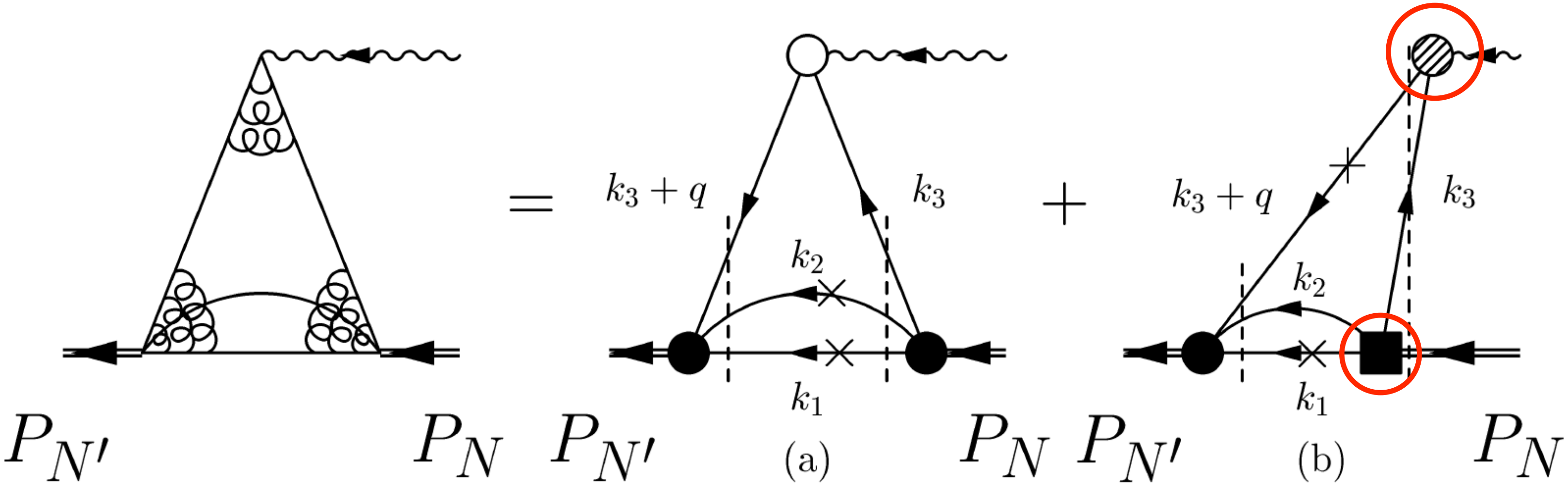
$$q^2 = (p' + p)^2 = \omega^2 - |\mathbf{q}|^2 > 0$$

$$\boxed{\mathbf{q}_\perp = 0}$$

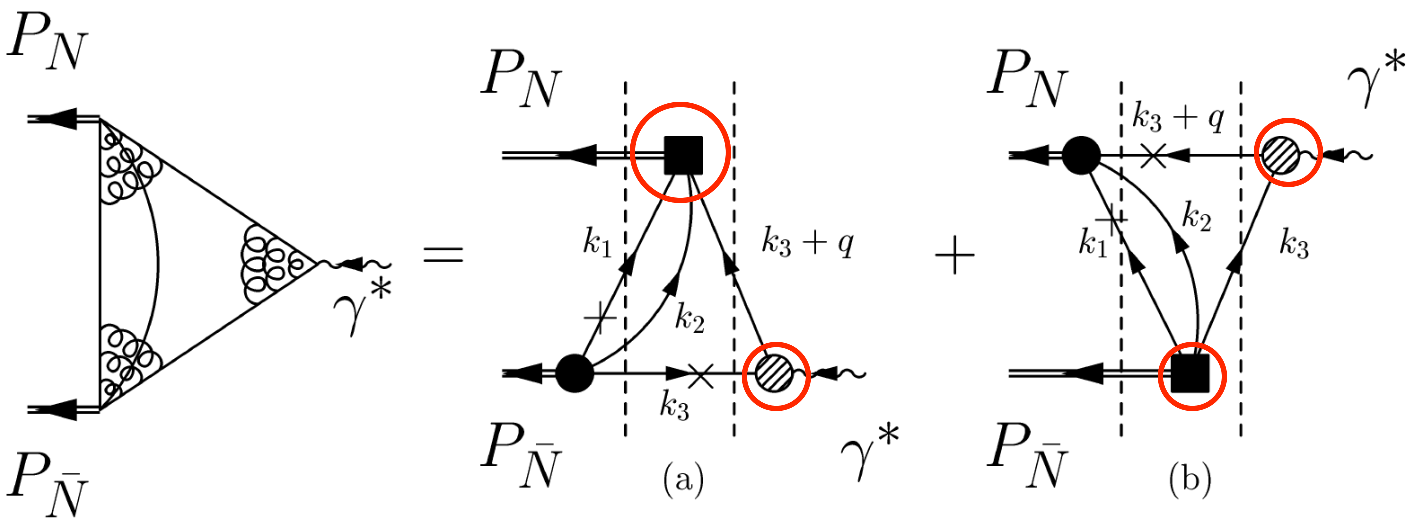
$$p'^{+} + p^{+} = q^{+} \qquad p'^{-} + p^{-} = \frac{M_N^2}{p'^{+}} + \frac{M_N^2}{(q^{+} - p'^{+})} = q^{-} = \frac{q^2}{q^{+}}$$

$$\boxed{p^{+} = \frac{q^{+}}{2}\left[1 - \sqrt{1 - 4\frac{M_N^2}{q^2}}\right] \qquad p'^{+} = \frac{q^{+}}{2}\left[1 + \sqrt{1 - 4\frac{M_N^2}{q^2}}\right]}$$

Space-like region



Time-like region



Ingredients:

S.J. Brodsky, F. Schlumpf, Phys. Lett. B 329 (1994) 111;
S.J. Brodsky, F. Schlumpf, Prog. Part. Nucl. Phys. 34 (1995) 69.

$$\Psi_N(\tilde{k}_1, \tilde{k}_2, P_N) = \mathcal{N} \frac{P_N^+ (9m^2)^{7/2}}{(\xi_1 \xi_2 \xi_3)^p [\beta^2 + M_{0N}^2(k_1, k_2, k_3)]^{7/2}} \qquad \beta = 0.645 \text{ GeV}$$

$$\mu_p^{\text{th}} = 2.87 \pm 0.02 \ \ (\mu_p^{\text{exp}} = 2.793) \ \text{and} \ \ \mu_n^{\text{th}} = -1.85 \pm 0.02 \ (\mu_n^{\text{exp}} = -1.913)$$

$$\Lambda_{\text{NV}}^{\text{SL}} = [g_{1,2}]^2 [g_{N,\bar{3}}]^{7/2-2} \left[\frac{k_{12}^+}{P_N'^+}\right] \left[\frac{P_N'^+}{k_{\bar{3}}^+}\right]^r \left[\frac{P_N^+}{k_{\bar{3}}^+}\right]^r \qquad g_{A,B} = (m_A m_B)/[\beta^2 + M_0^2(A,B)]$$

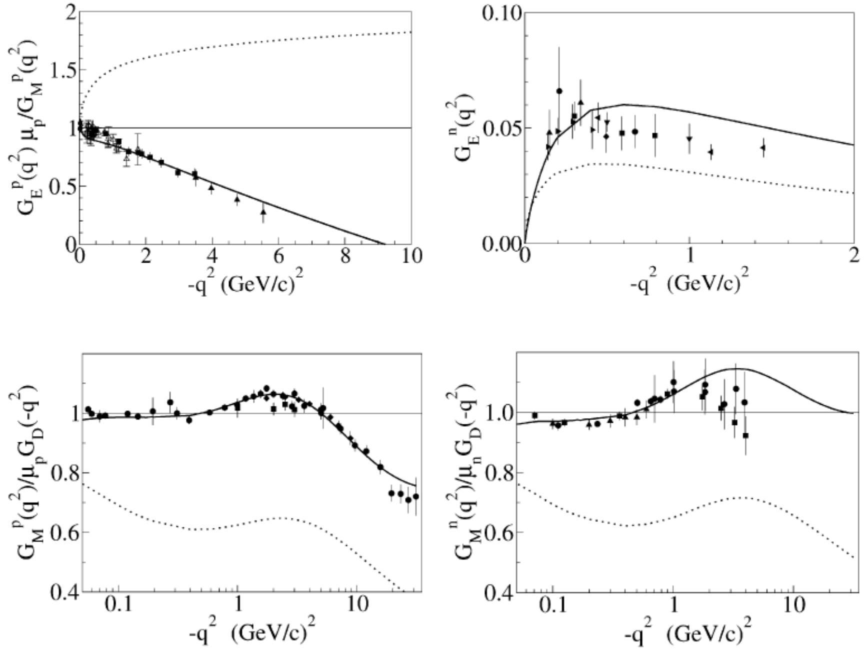
$$\Lambda_{\text{NV}}^{\text{TL}} = 2[g_{\bar{1},\bar{2}}]^2 [g_{N,\overline{12}}]^{7/2-2} \left[\frac{-k_{12}^+}{P_{\bar{N}}^+}\right] \left[\frac{P_{\bar{N}}^+}{k_3'^+}\right]^r \left[\frac{P_N^+}{k_3'^+}\right]^r$$

r=0.17 dipole type decay

Dressed photon vertex:

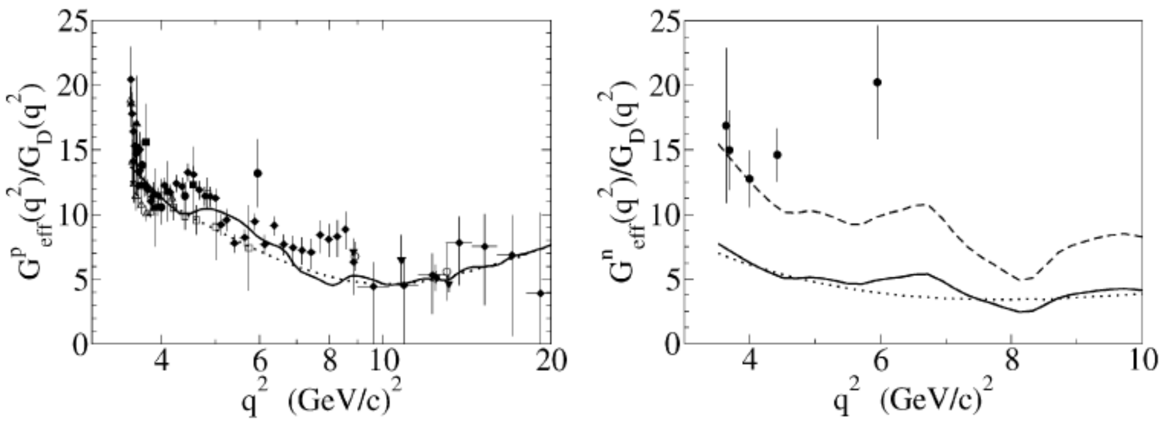
$$\Gamma^\mu_{IS}(k,q) = \frac{1}{6}\,\theta(p^+ - k^+)\,\theta(k^+)\gamma^\mu + \theta(q^+ + k^+)\,\theta(-k^+)\,\underbrace{\left[\frac{Z}{6}\gamma^\mu + \Gamma^\mu_{VMD}(k,q,IS)\right]}$$

(and the analogous for the isovector current)



Nonvalence contribution for Gep has opposite sign!

$$G_{\mathrm{eff}}^{p(n)}(q^2) = \sqrt{\left(\left|G_M^{p(n)}(q^2)\right|^2 - \eta\left|G_E^{p(n)}(q^2)\right|^2\right)/(1-\eta)}.$$



Puckett et al, PRL 104 (10) 242301

Hall C at Jefferson Lab

PRL **104**, 242301 (2010) PHYSICAL REVI

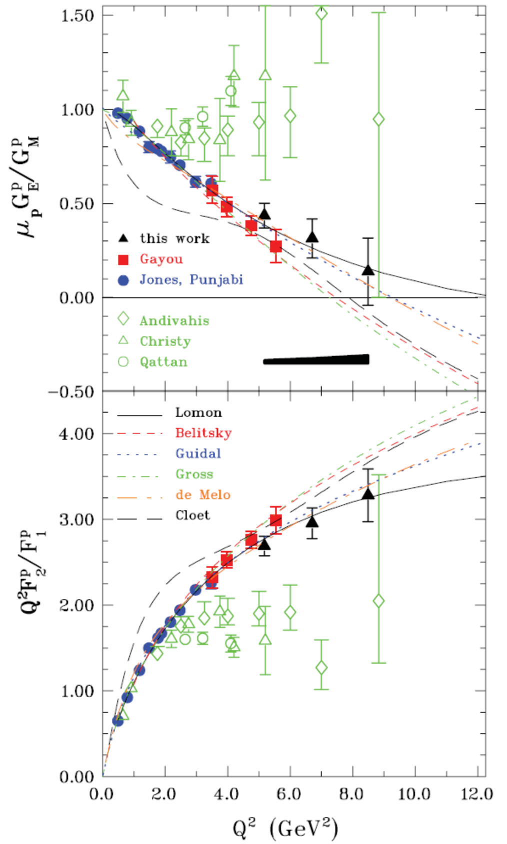


FIG. 3 (color). Upper panel: The proton form factor ratio $\mu_p G_E^p/G_M^p$ from this experiment (filled black triangles), with statistical error bars and systematic error band below the data. Previous experiments are [1] (Jones, Punjabi, Gayou), [3] (Andivahis), [4] (Christy), and [5] (Qattan). Theory curves are [20] (Lomon), [21] (de Melo), [22] (Gross), [23] (Cloët), [24] (Guidal), and [25] (Belitsky). Lower panel: The same data and theory curves as the upper panel, expressed as $Q^2F_2^p/F_1^p$.

VI. Quasi-Potential & LF B.-S. Equation: 2-bosons

OUR AIM:
Derive the dynamics of few-constituents on the LF from a given model for the Bethe-Salpeter equation.

- ***4d Bethe-Salpeter amplitude*** $\leftarrow\rightarrow$ ***Valence state on the LF***

(Kinematical momenta: $k^+ = k^0 + k^3$ and $k_\perp$)

Integration on the "Energy:" $k^- = k^0 - k^3$

"Iterated Resolvents-dynamics of the valence state"- Brodsky, Pauli, Pinsky, Phys. Rept. 301(98)299; Frederico et al NPA737(04)260c

Not complete list of previous works...

LF two-boson/ two-fermion systems: (C-R Ji,Perry,Miller,Karmanov,Carbonell,Brinet Mathiot, Bakker, Amghar,Desplanques...)

Quasi Potential Approach to LF: Sales et al PRC61(00)044003; 63(01)064003
Garsevanishvili et al. Phys. Rep. 458 (08) 247

LF conserved current operators: Kvinikhidze & Blankleider PRD68(03)02581

WTI -QP two-boson/two-fermion - Marinho et al PRD76(07)096001;PRD77(08)116010

LF Dynamics of three-body systems: Bakker, Kondratyuk, Terentev, NPB158(79)497
Zero-Range model & BS eq. - Frederico PLB282(92)409; Carbonell & Karmanov PRC67(03)037001; Marinho & Frederico PoS(LC2008)036; Karmanov & Maris PoS LC2008, 037 (2008), FBS 46, 95 (2009).

LFD of 3-constituents: valence state $\leftarrow\rightarrow$ ***4d Bethe-Salpeter eq. 3-legs***

qqq - Mitra, Ann.Phys. 318(08)845

Non-perturbative renormalization with truncated Fock-space:
Karmanov, Mathiot, Smirnov PRD77(08) 085028...

VI. Quasi-Potential & LF B.-S. Equation: 2-bosons

Starting with a 4-dimensional BS equation for 2→2 scattering amplitude (no self energies/vertex corrections):

$$T = V + VG_0T$$

V is the sum of two-body irreducible diagrams

$$T(K) = W(K) + W(K)\widetilde{G}_0(K)T(K)$$

Woloshyn & Jackson NPB64(1973)269

$$W(K) = V(K) + V(K)\Delta_0(K)W(K) \qquad \Delta_0(K) = G_0(K) - \widetilde{G}_0(K)$$

LF time projection: integration in *k*⁻

Sales, F., Sauer. PRC61(2000)044003

$$\widetilde{G}_0(K) := G_0(K)|g_0^{-1}(K)|G_0(K)$$

$$\left\langle k_1'^- \middle| G_0(K) \middle| k_1^- \right\rangle = -\frac{1}{2\pi}\frac{\delta\left(k_1'^- - k_1^-\right)}{\hat{k}_1^+(K^+ - \hat{k}_1^+)\left(k_1^- - \frac{\hat{\vec{k}}_{1\perp}^2 + m_1^2 - io}{\hat{k}_1^+}\right)\left(K^- - k_1^- - \frac{\hat{\vec{k}}_{2\perp}^2 + m_2^2 - io}{K^+ - \hat{k}_1^+}\right)}$$

$$|G_0(K)| := \int dk_1'^- dk_1^- \left\langle k_1'^- \middle| G_0(K) \middle| k_1^- \right\rangle = \frac{i\theta(K^+ - \hat{k}_1^+)\theta(\hat{k}_1^+)}{\hat{k}_1^+(K^+ - \hat{k}_1^+)\left(K^- - \hat{k}_{1on}^- - \hat{k}_{2on}^- + io\right)}$$

$$k^-_{\text{on}} = \frac{\vec{k}^2_\perp + m^2}{k^+}$$

Valence propagator in global LF time

$$|G(K)| = |G_0(K)| + |G_0(K)T(K)G_0(K)|$$

Valence→ Valence scattering amplitude

$$t(K) := g_0(K)^{-1}|G_0(K)T(K)G_0(K)|g_0(K)^{-1}$$

$$t(K) = w(K) + w(K)g_0(K)t(K)$$

Effective interaction

$$w(K) := g_0(K)^{-1}|G_0(K)W(K)G_0(K)|g_0(K)^{-1}$$

$$W(K) = V(K)\sum_{i=0}^{\infty}[\Delta_0(K)V(K)]^i$$

Bethe-Salpeter amplitude for scattering/bound states

For scattering states: $$|\Psi^+\rangle = |\Psi_0\rangle + G_0(K)V(K)|\Psi^+\rangle$$

and the corresponding homogeneous equation for the bound state

Valence wave function for bound/scattering states: $$|\phi\rangle = || \Psi\rangle$$

Homogeneous equation for the LF valence wave function of a bound state
(projecting the 4-dim BS equation or from the bound-state pole of the 3-dim t-matrix)

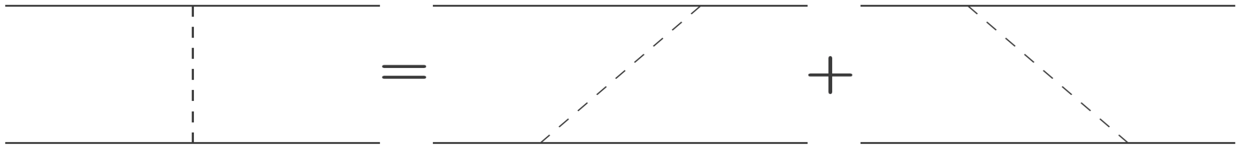
$$|\phi_B\rangle = g_0(K_B)w(K_B)|\phi_B\rangle$$

$$\int dk_1^- \langle k_1^- |\Psi_B\rangle = |\phi_B\rangle$$

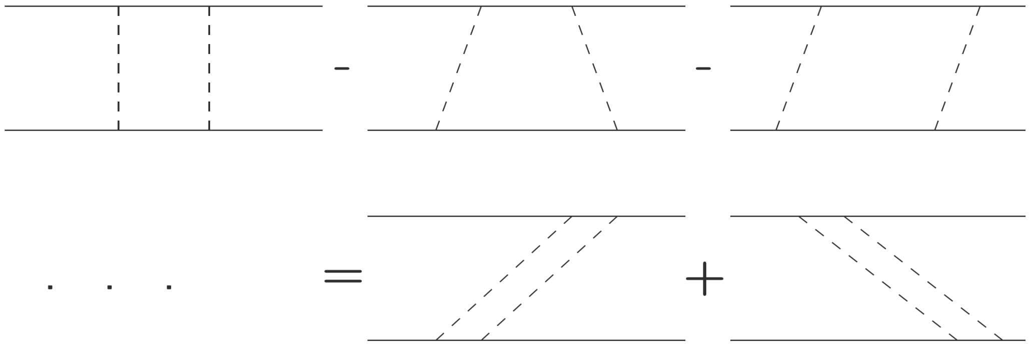
Example: Bosonic Yukawa model

$$\mathcal{L}_I = g_S \phi_1^\dagger \phi_1 \sigma + g_S \phi_2^\dagger \phi_2 \sigma$$

$w^{(1)} =$

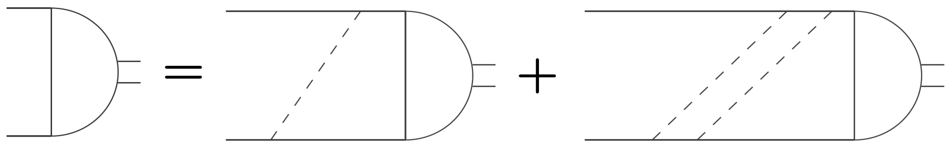


$w^{(2)} =$



Mass^2 eigenvalue eq. & valence wf:

$$\boxed{g(K_\lambda)^{-1} \left|\phi_\lambda\right\rangle = 0}$$



$$\left[g_0^{-1} - w\right]$$

LF Bound state equation

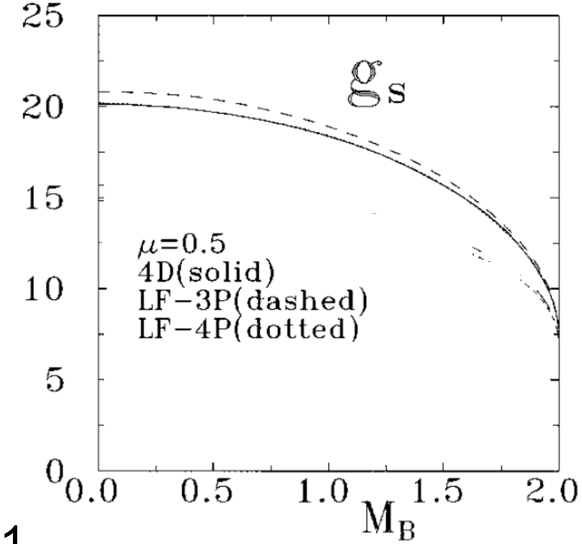
$$\phi_B(y, \vec{k}'_\perp; K) = \frac{\gamma(y, \vec{k}'_\perp; K)}{K^2 - M_0^2}$$

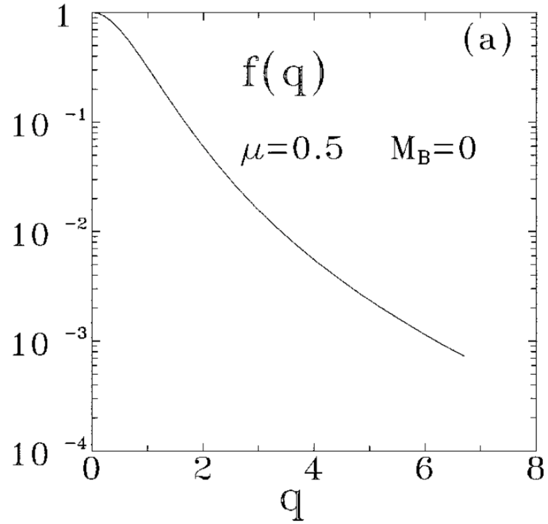
$$\gamma(y, \vec{k}'_\perp; K) = \frac{1}{(2\pi)^3} \int \frac{d^2 k_\perp dx}{2x(1-x)} \gamma(x, \vec{k}_\perp; K) \frac{\mathcal{K}^{(2)}(y, \vec{k}'_\perp; x, \vec{k}_\perp) + \mathcal{K}^{(4)}(y, \vec{k}'_\perp; x, \vec{k}_{1\perp})}{K^2 - M_0^2}$$

$$\mathcal{K}^{(2)}(y, \vec{k}'_\perp; x, \vec{k}_\perp) = g_S^2 \frac{\theta(x-y)}{(x-y)\left(K^2 - \left(M_0^{(3)}\right)^2 + i\epsilon\right)} + \left[x \leftrightarrow y, \vec{k}'_\perp \leftrightarrow \vec{k}_\perp\right]$$

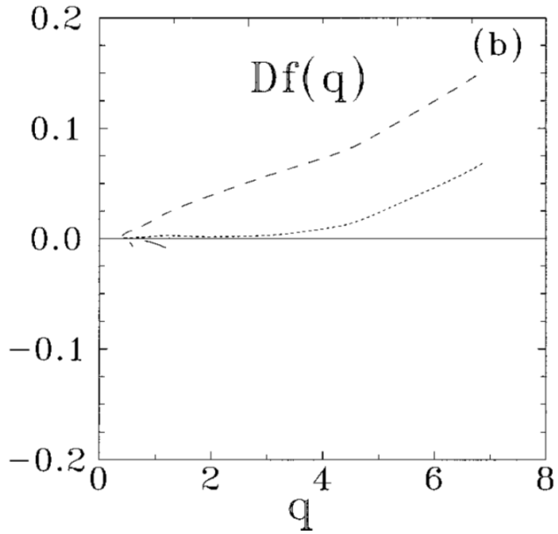
$$\left(M_0^{(3)}\right)^2 = \frac{\vec{k}'^2_\perp + m^2}{y} + \frac{\vec{k}^2_\perp + m^2}{1-x} - \frac{(\vec{k}'_\perp + \vec{k}_\perp)^2 + \mu^2}{x-y}$$

Comparision between LF (3d) and 4d results for bound states





m=1



$$f_{\text{exact}}(\sqrt{\vec{k}_{1\perp}^{2}}) := \int dk_1^- dk_1^+ \langle k_1 | \Psi_B \rangle$$

$$f_{\text{app}}^{(n)}(\sqrt{\vec{k}_{1\perp}^{2}}) = \int dk_1^+ \langle k_1^+ \vec{k}_{1\perp} | \phi_B \rangle_{\text{app}}^{(n)}$$

$$Df(q) = 1 - f_{\text{app}}^{(n)}(q) / f_{\text{exact}}(q)$$

$\Rightarrow$ Hierarchy Equations in LF Fock-space

J. H.O. Sales PhD thesis ITA (unpublished), Frederico et al NPA737(04)260c

- Interaction m.e.'s:

$$\langle q k_\sigma | v | k \rangle = -2(2\pi)^3 \delta(q + k_\sigma - k) \frac{g_S}{\sqrt{q^+ k_\sigma^+ k^+}} \theta(k_\sigma^+)$$

$$\langle q | v | k_\sigma k \rangle = -2(2\pi)^3 \delta(k + k_\sigma - q) \frac{g_S}{\sqrt{q^+ k_\sigma^+ k^+}} \theta(k_\sigma^+)$$

Interaction Fock-space

- Hierarchy Eqs.:

$$g^{(2)}(K) = g_0^{(2)}(K) + g_0^{(2)}(K) v g^{(3)}(K) v g^{(2)}(K),$$

$$g^{(3)}(K) = g_0^{(3)}(K) + g_0^{(3)}(K) v g^{(4)}(K) v g^{(3)}(K),$$

$$g^{(4)}(K) = g_0^{(4)}(K) + g_0^{(4)}(K) v g^{(5)}(K) v g^{(4)}(K),$$

. . .

$$g^{(N)}(K) = g_0^{(N)}(K) + g_0^{(N)}(K) v g^{(N+1)}(K) v g^{(N)}(K),$$

. . .

Subtraction of divergences?

- Truncation: $|\Phi_1, \Phi_2, (N-2)\sigma\rangle$

Iterated resolvents: Brodsky, Pauli, Pinsky, Phys. Rep. **301** (98) 299

Reconstructing 4-d B.S. amplitude from the LF valence wf:

$$\boxed{|\Psi\rangle = G(K)|g(K)^{-1}|\phi\rangle}$$

(i e →0)

$$g(K_\lambda)^{-1}\,|\phi_\lambda\rangle = 0$$

(projecting back to the LF retrieves the valence wf.)

<BS Ampl.| 4d operator |BS Ampl> → <val.|3d operator |val.>

Reverse LF time projection operation: expansion W

$$G(K)|g(K)^{-1} = [1 + \Delta_0(K)W(K)]\,G_0(K)|g_0(K)^{-1}$$

VII. Two-boson systems: E.M. current operator & WTI

$$\langle \Psi_f | \, \mathcal{J}^\mu(Q) \, | \Psi_i \rangle \qquad\qquad Q = K_f - K_i$$

$$\mathcal{J}^\mu(Q) = \mathcal{J}_0^\mu(Q) + \mathcal{J}_I^\mu(Q)$$

Ward-Takahashi in operator form: (Gross & Riska PRC36(1987)1928)

$$\boxed{Q_\mu \mathcal{J}^\mu(Q) = \left[G^{-1}, \hat{e}_1 \right] + (1 \leftrightarrow 2)}$$

$$\langle k_i | \hat{e}_i | p_i \rangle = e_i \delta^4 \left(k_i - p_i - Q \right)$$

$$G^{-1}(K_\lambda) \, | \Psi_\lambda \rangle = 0 \quad \Longrightarrow \quad \boxed{Q_\mu \, \langle \Psi_f | \, \mathcal{J}^\mu(Q) \, | \Psi_i \rangle = 0}$$

Light-front e.m. current operator: valence states

$$\langle \Psi_f | \, \mathcal{J}^\mu(Q) \, | \Psi_i \rangle = \langle \phi_f | \, j^\mu(K_f, K_i) \, | \phi_i \rangle$$

$$|\Psi\rangle = G(K)|g(K)^{-1}|\phi\rangle$$

$$\boxed{j^\mu(K_f, K_i) := g(K_f)^{-1}|G(K_f)\mathcal{J}^\mu(Q)G(K_i)|g(K_i)^{-1}}$$

Gauging method for bound states: Kvinikhidze and Blankleider (PRD68 (2003) 025021)

LF Projection WTI:

$$Q_\mu|G(K_f)\mathcal{J}^\mu(Q)G(K_i)| = |\,[\widehat{e}_1, G]\,| + (1 \leftrightarrow 2)$$

$$\boxed{Q^\mu j_\mu(K_f, K_i) = \left[g^{-1}, \widehat{e}_{LF}\right]}$$

$$\langle k_i^+, \vec{k}_{i\perp} | \widehat{e}_{i,LF} | p_i^+, \vec{p}_{i\perp} \rangle = e_i \delta\left(k_i^+ - p_i^+ - Q^+\right) \delta^2\left(\vec{k}_{i\perp} - \vec{p}_{i\perp} - \vec{Q}_\perp\right)$$

LF charge operator

LF e.m. current is conserved

$$j^\mu(K_f, K_i) = g_0(K_f)^{-1}|G_0(K_f)\,[1 + W(K_f)\Delta_0(K_f)] \times$$

$$\mathcal{J}^\mu(Q)\,[1 + \Delta_0(K_i)W(K_i)]\,G_0(K_i)|g_0(K_i)^{-1}$$

$$\left.\begin{aligned} W^{(n)}(K) &= \sum_{i=1}^{n} W_i(K) \\ W_n &= V(K)[\Delta_0(K)V(K)]^{n-1} \end{aligned}\right\} \quad w^{(n)}(K) = \sum_{i=1}^{n} g_0(K)^{-1}|G_0(K)W_i(K)G_0(K)|g_0(K)^{-1}$$

Truncation in W keeps c.c.? No!

$$\begin{aligned} Q^\mu\langle\phi_f^{(n)}|j_\mu^{(n)}(K_f, K_i)|\phi_i^{(n)}\rangle &= \langle\Psi_f^{(n)}|Q^\mu \mathcal{J}_\mu(Q)|\Psi_i^{(n)}\rangle \\ &= \langle\Psi_f^{(n)}|\left[\widehat{e}, G^{-1}\right]|\Psi_i^{(n)}\rangle \neq 0 \end{aligned}$$

Conserved & truncated LF e.m. current: WTI

Marinho, F., Sauer, PRD 76, 096001(07)

$$Q^\mu j_\mu^{c(n)}(K_f, K_i) = \left[g_n^{-1}, \hat{e}_{LF}\right]$$

$$j^{c\mu(n)} = g_0^{-1}|G_0 \left[\mathcal{J}^\mu(Q) + \sum_{i=1}^{n-1}(W_i\Delta_0\mathcal{J}^\mu(Q) + \mathcal{J}^\mu(Q)\Delta_0 W_i) + \sum_{i=2}^{n-1}\sum_{j=1}^{i-1} W_j\Delta_0\mathcal{J}^\mu(Q)\Delta_0 W_{i-j}\right.$$

$$\left. + W_n\Delta_0\mathcal{J}_0^\mu(Q) + \mathcal{J}_0^\mu(Q)\Delta_0 W_n + \sum_{i=1}^{n-1} W_i\Delta_0\mathcal{J}_0^\mu(Q)\Delta_0 W_{n-i}\right] G_0|g_0^{-1}$$

Message:

Keep in the current all LF two-body irreducible terms consistent with the truncation of the interaction

$\Rightarrow$ ***Conserved e.m. current!***

Current in the Yukawa model for 2-boson systems

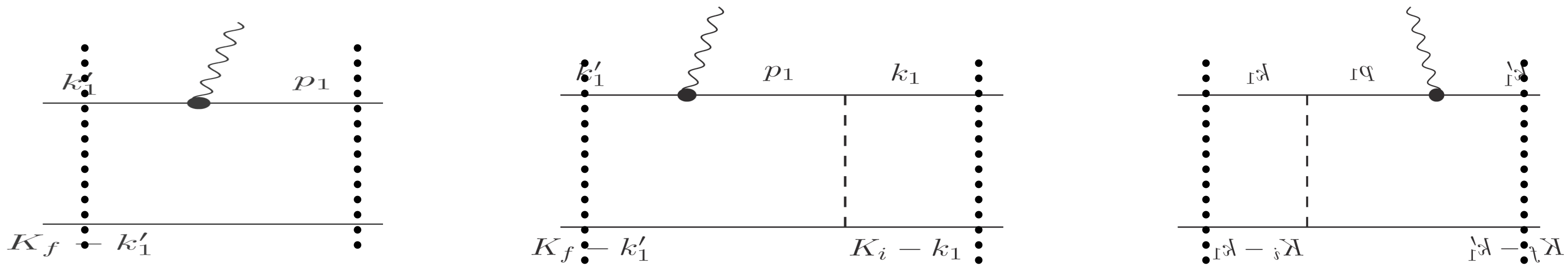
4d Ladder B.-S.

$$\mathcal{L}_I = g_S \phi_1^\dagger \phi_1 \sigma + g_S \phi_2^\dagger \phi_2 \sigma$$

$$\langle k_1 | \mathcal{J}_0^\mu(Q) | p_1 \rangle = -2\pi [e_1 (k_1 + p_1)^\mu \delta^4(k_1 - p_1 - Q)((K_f - k_1)^2 - m_2^2)]$$

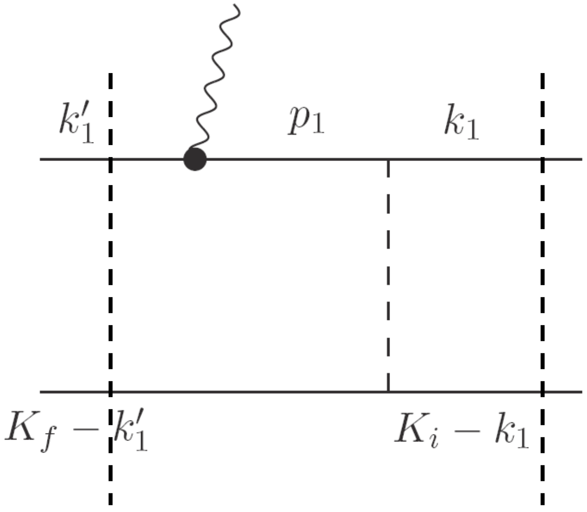
$$j^{c\mu(1)} = j^{c\mu(0)} + g_0^{-1} | G_0 \left[W_1 \Delta_0 \mathcal{J}_0^\mu(Q) + \mathcal{J}_0^\mu(Q) \Delta_0 W_1 \right] G_0 | g_0^{-1}$$

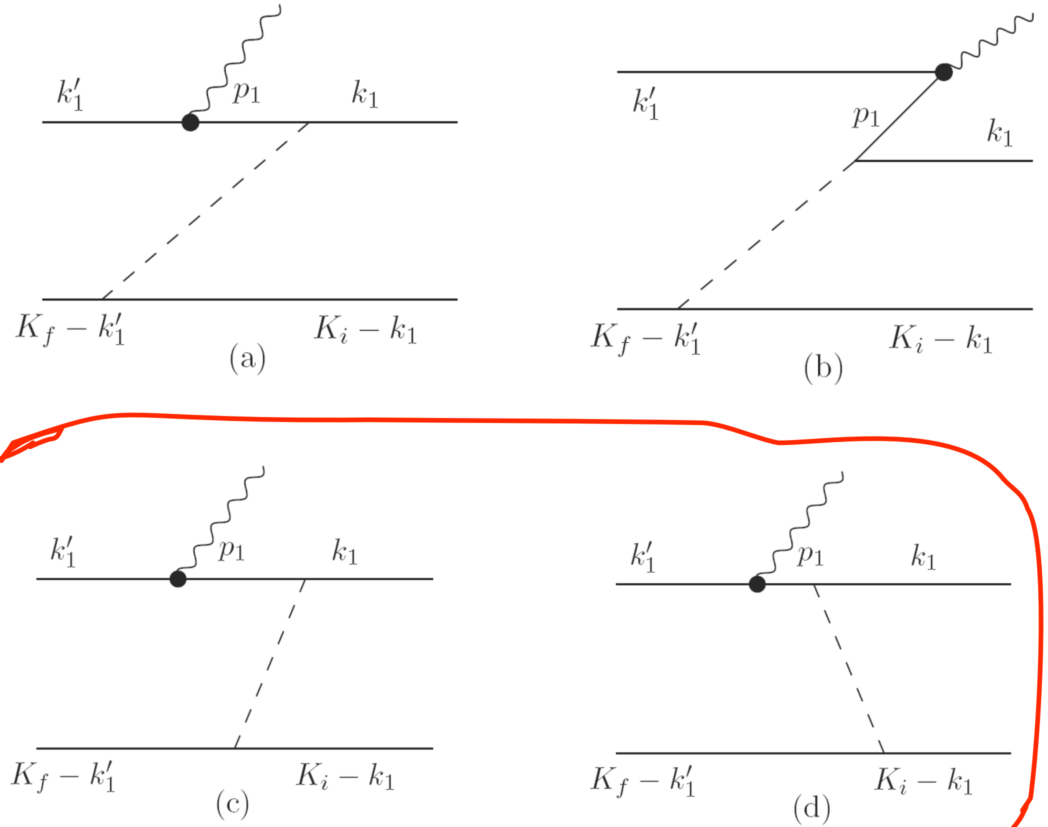
$$j^{c\mu(0)} = g_0^{-1} | G_0 \mathcal{J}_0^\mu(Q) G_0 | g_0^{-1}$$



- 2-body LF reducible terms

LF current in 1st order



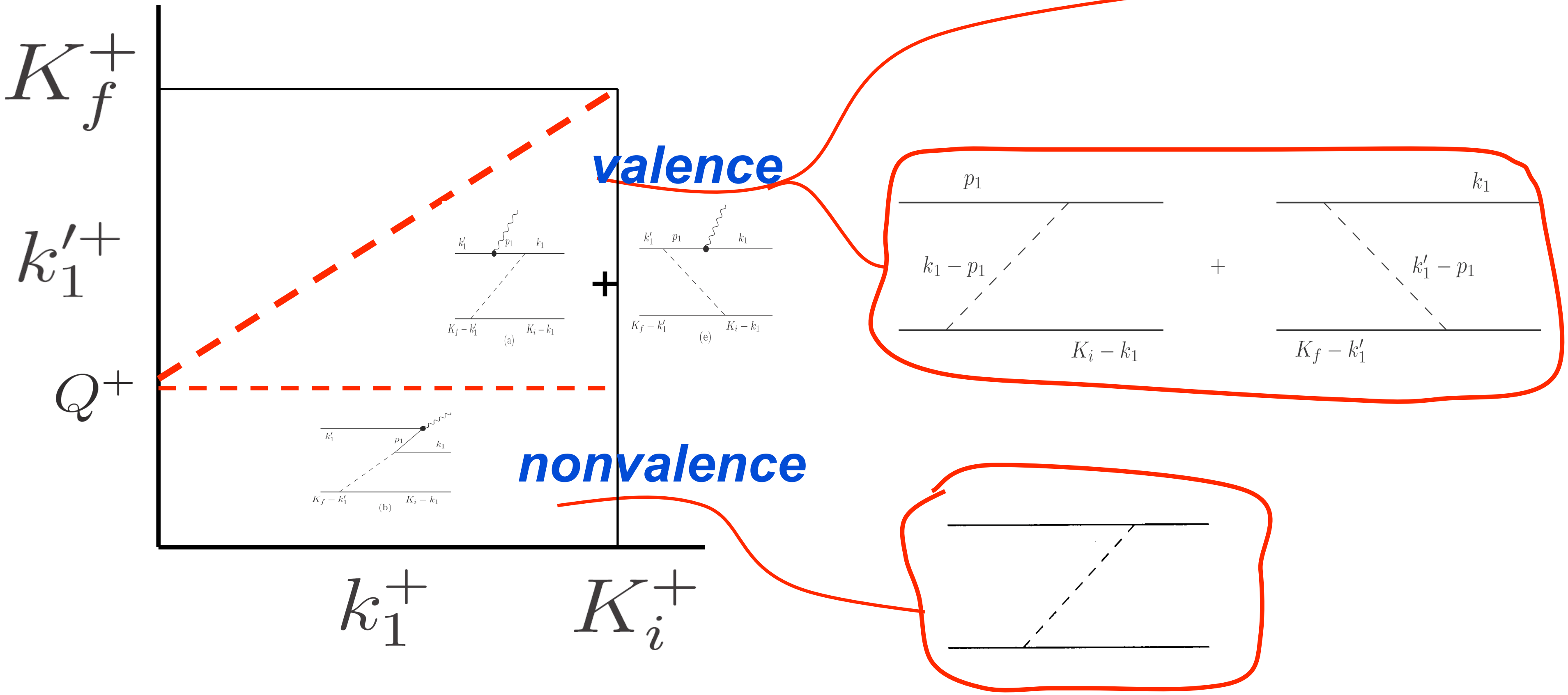


LF 2-body reducible diagrams

$K_i^+ > 0$ and $Q^+ \geq 0$

Kinematical regions:

$$\langle k_1'^+ \vec{k}'_{1\perp} | Q \cdot j^{c(1)} | k_1^+ \vec{k}_{1\perp} \rangle_{(a)+(e)} = \langle k_1'^+ \vec{k}'_{1\perp} | \left[\widehat{e}_{LF}, w^{(1)} \right] | k_1^+ \vec{k}_{1\perp} \rangle \theta \left(k_1'^+ - Q^+ \right)$$



Current conservation is OK!

VIII. Two-Fermion systems: Yukawa model

Starting with a 4-dimensional BS equation for 2→2 scattering amplitude
(no self energies/vertex corr.):

$$T = V + VG_0T$$

V is the sum of two-body irreducible diagrams

$$T(K) = W(K) + W(K)\tilde{G}_0(K)T(K)$$

$$W(K) = V(K) + V(K)\Delta_0(K)W(K) \qquad \Delta_0(K) = G_0(K) - \tilde{G}_0(K)$$

Separation of the instantaneous term in the fermion propagator:

$$\frac{\not{k}_i + m_i}{k_i^2 - m_i^2 + i\varepsilon} = \frac{\not{k}_{ion} + m_i}{k_i^2 - m_i^2 + i\varepsilon} + \frac{\gamma^+}{k_i^+}$$

(Sales et al PRC63(2001)064003)

$$\overline{G}_0 = (\hat{\not{k}}_{1on} + m_1)(\hat{\not{k}}_{2on} + m_2)G_0$$

$$G_0^F = \overline{G}_0 + \Delta G_0^F$$

$$G_0 = \frac{i}{\hat{k}_1^2 - m_1^2 + io}\,\frac{i}{\hat{k}_2^2 - m_2^2 + io}$$

$$|\overline{G}_0(K)| := g_0(K)$$

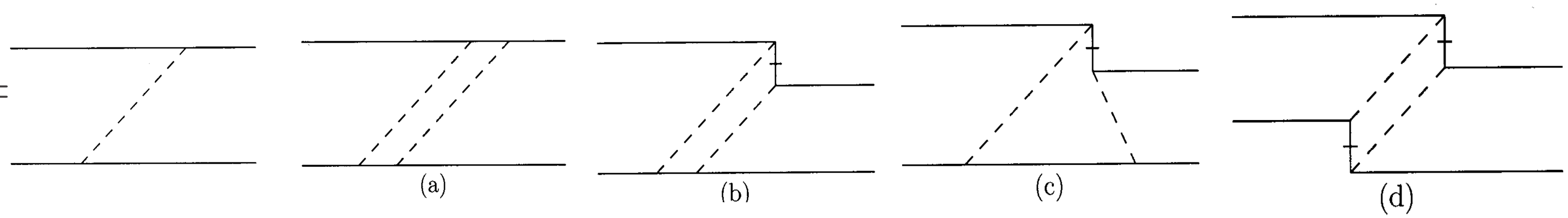
$$\tilde{G}_0 := \overline{G}_0|g_0^{-1}|\overline{G}_0$$

Mass^2 eigenvalue eq. & valence wf:

$$\left[g_0^{-1} - w\right]|\phi_\lambda\rangle = 0$$

Yukawa model: $\mathcal{L}_I = g_S \overline{\Psi} \Psi \sigma$

$$w^{(2)} =$$



Covariant Box-diagram decomposed in LF time-ordered diagrams

B. L. G. Bakker, J. K. Boomsma, C.-R. Ji, Phys. Rev. **D 75**, 065010 (2007)

Fermions: conserved & truncated LF e.m. current & WTI

Marinho, F., Pace,Salme,Sauer, PRD77,116010(2008)

$$j^\mu = g_0^{-1}|\overline{G}_0[1 + W\Delta_0]J^\mu[1 + \Delta_0 W]\overline{G}_0|g_0^{-1}$$

(Sales et al PRC63(2001)064003)

$$\Delta_0 \;:=\; G_0 - \tilde{G}_0$$

$$\tilde{G}_0 := \overline{G}_0|g_0^{-1}|\overline{G}_0$$

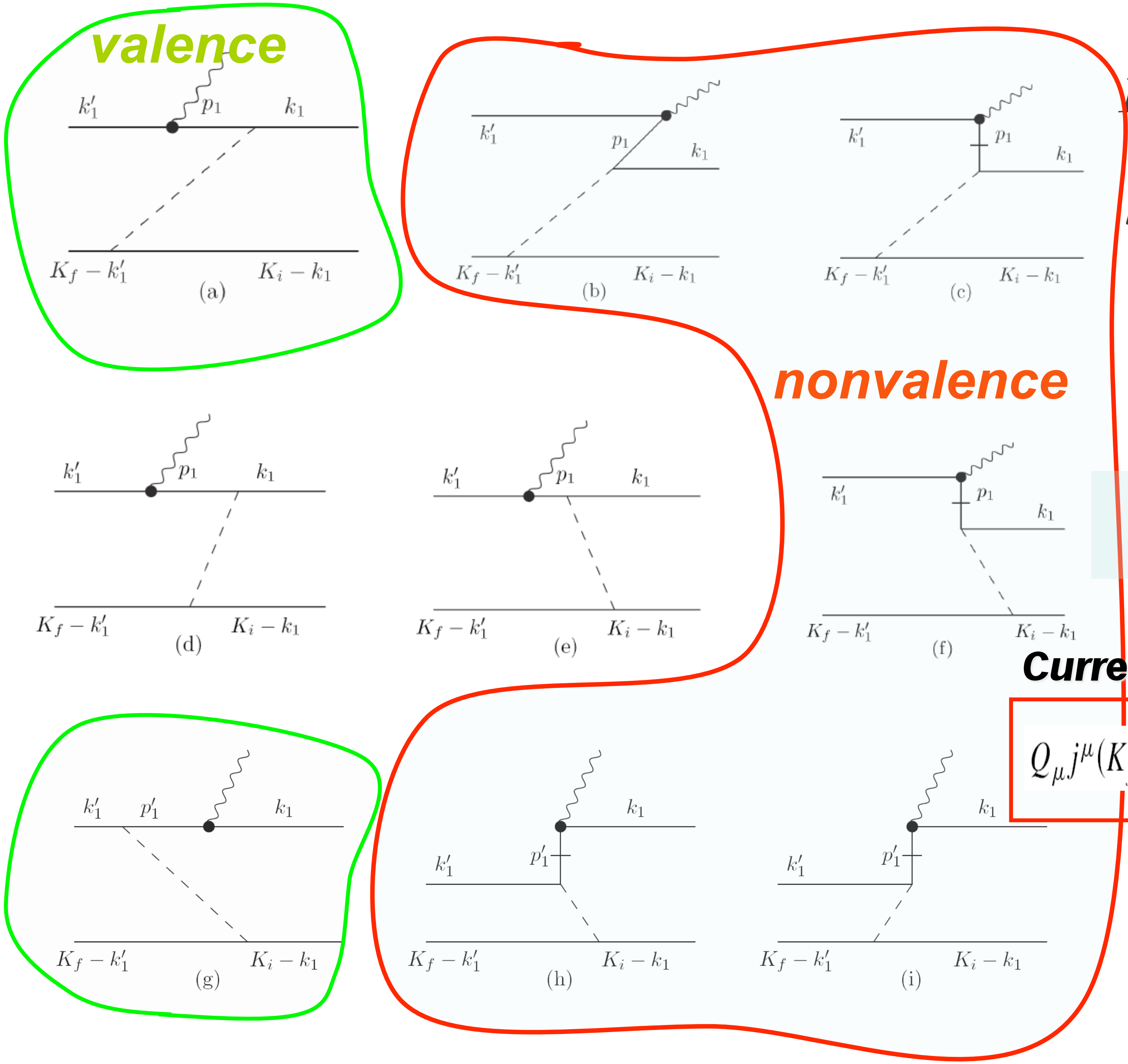
WTI for the conserved current:

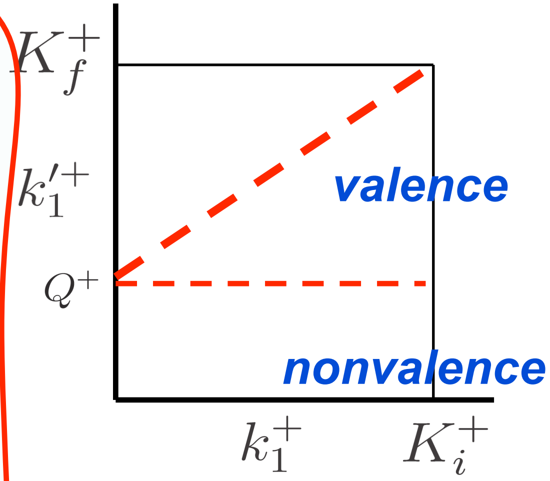
$$Q_\mu j^\mu(K_f, K_i) = g^{-1}(K_f)\hat{Q}^L_{\rm LF} - \hat{Q}^R_{\rm LF} g^{-1}(K_i)$$

Expansion...

$$\boxed{e_1\delta\left(k_1'^+ - k_1^+ - Q^+\right)\delta^2\left(\vec{k}'_{1\perp} - \vec{k}_{1\perp} - \vec{Q}_\perp\right)\Lambda_+(k'_{1on})\frac{m_1}{k_1'^+}\gamma_1^+\Lambda_+(k_{1on})\Lambda_+(k_{2on})}$$

LF current in 1st order





$K_i^+ > 0$ and $Q^+ \geq 0$

Current conservation holds!

$$\boxed{Q_\mu j^\mu(K_f, K_i) = g^{-1}(K_f)\hat{Q}^L_{\mathrm{LF}} - \hat{Q}^R_{\mathrm{LF}} g^{-1}(K_i)}$$

Comments:

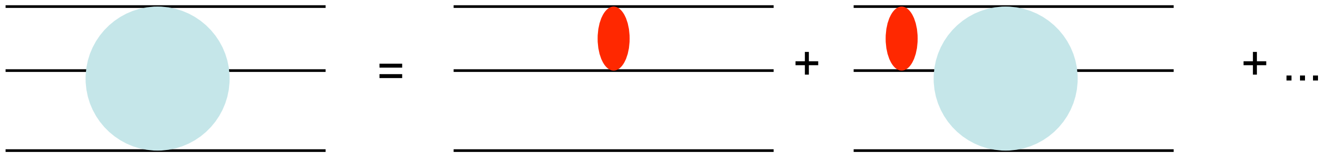
• *Cooke and Miller PRC 66, 034002 (02*): deuteron mass eigenvalues tends to get the correct spin projection degeneracy including up to streched boxes BUT the angular condition in the DY frame is badly violated WITHOUT including the consistent terms in the current…

• *Huang and Polyzou PRC80, 025503 (09):* includes a two-body current within a relativistic LF QM approach…

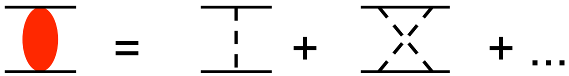
IX. Three-boson systems and ladder 4d BS equation

Marinho, PhD thesis ITA/2007

$$T = V + VG_0T \qquad V = \sum_{i=1}^{3} V_i \quad ; \quad V_i = V_{(2)jk} S_i^{-1}$$



$V_{(2)jk}$ =



$$\langle k_1^-, k_2^- | G_0 | k_1'^-, k_2'^- \rangle = \frac{-i}{(2\pi)^2} \frac{\delta(k_1^- - k_1'^-)}{\hat{k}_1^+ \hat{k}_2^+ (K^+ - \hat{k}_1^+ - \hat{k}_2^+)(k_1^- - \hat{k}_{1on}^-)} \frac{\delta(k_2^- - k_2'^-)}{(k_2^- - \hat{k}_{2on}^-)(K^- - k_1^- - k_2^- - (K - \hat{k}_1 - \hat{k}_2)_{on}^-)}$$

Integration over k- for 1 and 2 → free 3-boson resolvent

$$g_0(\underline{k}_1, \underline{k}_2) = \frac{i\theta(K^+ - k_1^+ - k_2^+)\theta(k_1^+)\theta(k_2^+)}{k_1^+ k_2^+ (K^+ - k_1{}^+ - k_2^+)(K^- - k_{1on}^- - k_{2on}^- - (K - k_1 - k_2)_{on}^-)}$$

$$\underline{k} \equiv (k^+, \vec{k}_\perp)$$

Faddeev decomposition:

$$W(K) = V(K) + V(K)\Delta_0(K)W(K) \qquad \begin{cases} \Delta_0(K) = G_0(K) - \tilde{G}_0(K) \\ \tilde{G}_0(K) := G_0(K)|g_0^{-1}(K)|G_0(K) \end{cases}$$

$$V = \sum_{i=1}^{3} V_i \rightarrow W_i = V_i + V_i\Delta_0 W \qquad W = \sum_{i=1}^{3} W_i$$

$$(1 - V_i\Delta_0)W_i = V_i + V_i\Delta_0(W_j + W_k)$$

$$W_{(2)i} = V_i + V_i\Delta_0 W_{(2)i}$$

$$W_i = W_{(2)i} + W_{(2)i}\Delta_0(W_j + W_k)$$

$$t = \sum_{i=1}^{3} t_i \; ; \qquad w = \sum_{i=1}^{3} w_i$$

$$t_i = w_i + w_i g_0 t$$

$$w_i = g_0^{-1} \; |G_0 W_i G_0| \; g_0^{-1}$$

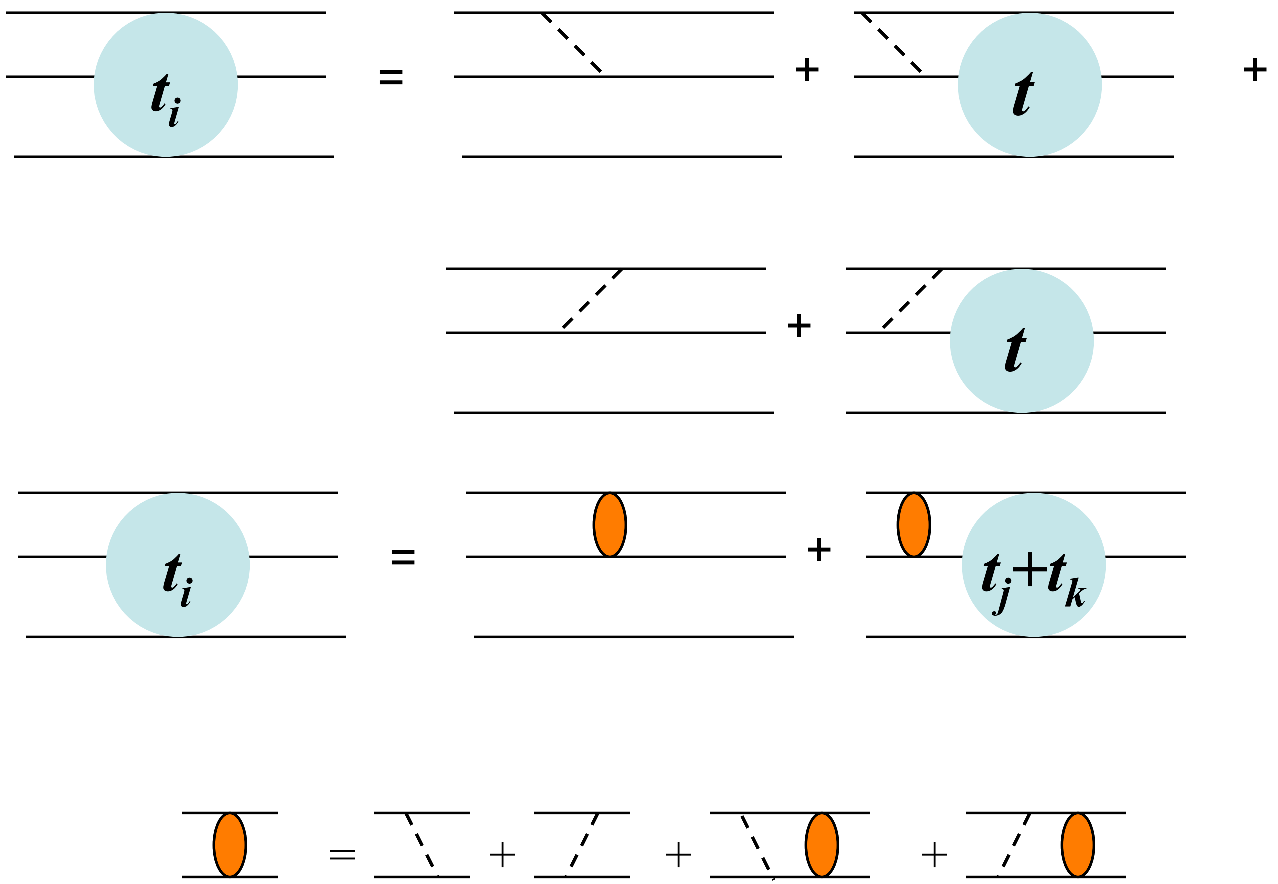
In practice W_i is obtained from a power expansion in V:

$$\boxed{W_i = \boxed{V_i} + \boxed{V_i \Delta_0 (V_i + V_j + V_k)} + V_i \Delta_0 (V_i + V_j + V_k) \Delta_0 (V_i + V_j + V_k) + \ldots}$$

LO NLO

Bosonic Yukawa model: LO

$$w_i^{LO} = g_0^{-1} \, |G_0 V_i G_0| \, g_0^{-1}$$

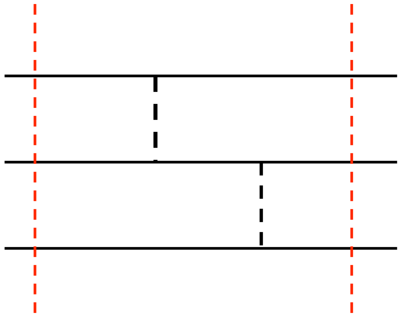


Cluster separability satisfied!

Bosonic Yukawa model: NLO

$$w_i^{NLO} = w_i^{LO} + g_0^{-1} \ |G_0 V_i \Delta_0 (V_i + V_j + V_k) G_0| \ g_0^{-1}$$

three-body reducible diagrams



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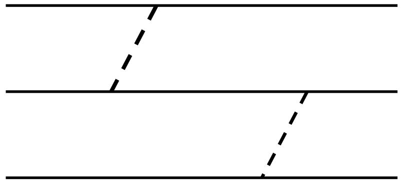

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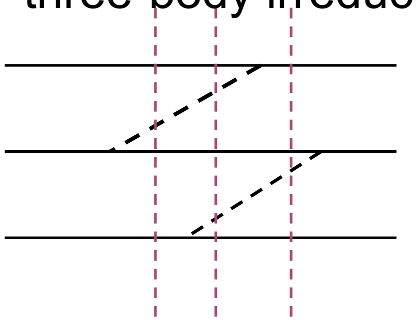
three-body irreducible diagrams = three-body interaction

+



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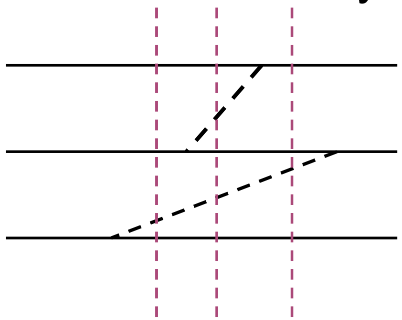

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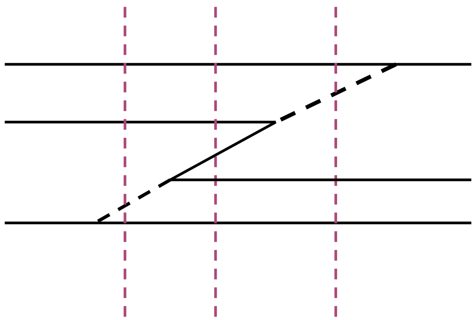

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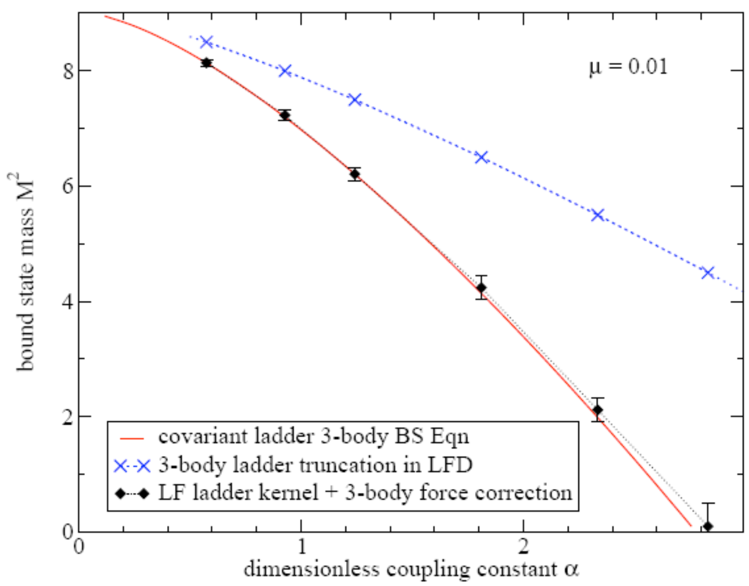
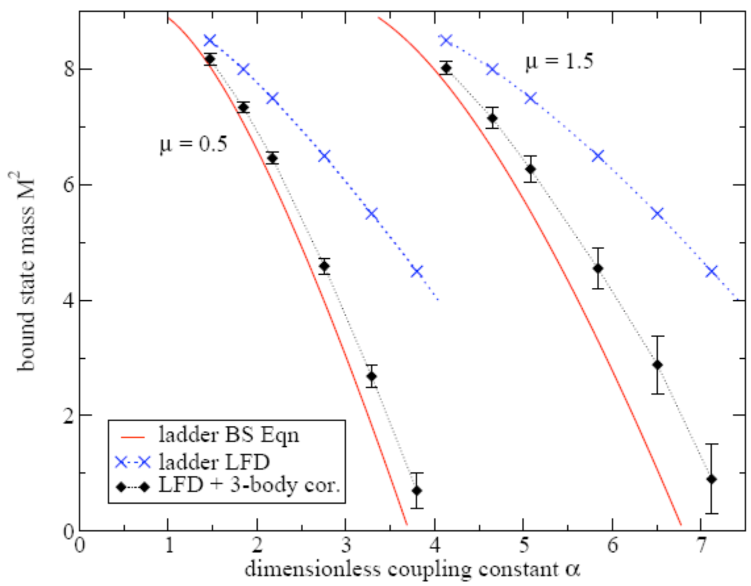
Perturbative contribution of the 3-body interaction to the 3-boson mass
Karmanov & Maris PoS LC2008, 037 (2008), Few Body Syst.46, 95 (2009).





X. Conclusions

- "LF Few-body dynamics": Valence w.f. dynamics
- Quasi-potential Approach to LF → LF dynamics
- 4-d Bethe-Salpeter amplitude ←→ valence w.-f.
- 4-d operators ←→ 3-d operators acting valence w.f.
- Conserved current operator & WTI:

  Conserved LF e.m. current operator expanded systematically and consistent with the mass squared operator;

  Valence and nonvalence contributions to the current required by current conservation;

  Current conservation in LF is a weaker requirement than covariance (covariance under non-kinematical boosts).

  Role of different frames in revealing the Fock-structure (two-body currents)

Perspectives:

- GPD's, conserved current operator for 3-body systems
- 3→3 scattering amplitude B→$pi^-pi^+k^-$
- Applications to deuteron, trinucleon, n-d scattering...
- Excitons, trions ... in graphene!