Covariant form factors for spin-1 particles

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Outline

Light-Front Motivations

- **Ligh-Front is the Ideal Framework to Describe Hadronic Bound States**
- **Constituent Picture and Unanbiguous Partons Content of the Hadronic System**
- **Light-Front Wavefunctions: Representation of Composite Systems in QFT**
- **Invariant Under Boosts**
- **Light-Front Vacuum is Trivial**
- **After Integrate in k^-: Bethe-Salpeter Amplitude (Wave Function)**
- **LF Lorentz Invariant Hamiltonian:** $P^2 = P^+ P^- - P_\perp^2$

Light-Front Coordinates and Elect. Current

Four-Vector $\Longrightarrow \quad x^\mu = (x^0, x^1, x^2, x^3) \;=\; (x^+, x^-, x_\perp)$

$x^+ = t + z \qquad x^+ = x^0 + x^3 \quad \Longrightarrow$ **Time**

$x^- = t - z \qquad x^- = x^0 - x^3 \quad \Longrightarrow$ **Position**

Metric Tensor and Scalar product

$$x \cdot y = x^\mu y_\mu = x^+ y_+ + x^- y_- + x^1 y_1 + x^2 y_2 = \frac{x^+ y^- + x^- y^+}{2} - \vec{x}_\perp \vec{y}_\perp$$

$$p^+ = p^0 + p^3 \;,\; p^- = p^0 - p^3 \;,\; p^\perp = (p^1, p^2)$$

$$p^\mu x_\mu = \frac{p^+ x^- + p^- x^+}{2} - \vec{p}_\perp \vec{x}_\perp, \quad x^+, x^-, \vec{x_\perp} \quad \Longrightarrow p^+, p^-, \vec{p}_\perp$$

- **Dirac Matrix and Electromagnetic Current**

$\gamma^+ = \gamma^0 + \gamma^3 \;\Longrightarrow$ **Electr. Current** $J^+ = J^0 + J^3$

$\gamma^- = \gamma^0 - \gamma^3 \;\Longrightarrow$ **Electr. Current** $J^- = J^0 - J^3$

$\gamma^\perp = (\gamma^1, \gamma^2) \;\Longrightarrow$ **Electr. Current** $J^\perp = (J^1, J^2)$

$p^{-} \implies$ **Light-Front Energy**

$$p^2 = p^+ p^- - (\vec{p}_\perp)^2 \implies \quad p^- = \frac{(\vec{p}_\perp)^2 + m^2}{p^+}$$

On-shell

Bosons $\implies \quad S_F(p) = \frac{1}{p^2 - m^2 + \imath\epsilon}$

Fermions $\implies \quad S_F(p) = \frac{\not{p} + m}{p^2 - m^2 + \imath\epsilon} + \frac{\gamma^+}{2p^+}$

Review Papers:

- Phys. Rept. 301, (1998) 299-486, Brodsky, Pauli and Pinsky
- A. Harindranath, Pramana, Journal of Indian Academy of Sciences Physics Vol.55, Nos 1 & 2, (2000) 241.
- An Introduction to Light-Front Dynamics for Pedestrians Avaroth Harindranath

Light-Front book organizers: James Vary and Frank Wolz,(1997)

General expression for the spin-1 electromagnetic current

Plus and minus components

$$
\begin{aligned}
J^{\pm}_{\lambda'\lambda} &= (p'^{\pm} + p^{\pm})[F_1(q^2)(\epsilon_{\lambda'}.\epsilon_{\lambda}) - \frac{F_2(q^2)}{2m_v^2}(q.\epsilon_{\lambda'})(q.\epsilon_{\lambda})] \\
&- F_3(q^2)((q.\epsilon_{\lambda'})\epsilon^{\pm}_{\lambda} - (q.\epsilon_{\lambda})\epsilon^{\pm}_{\lambda'}) \ .
\end{aligned}
$$

- $\Longrightarrow$ F_1, F_2 **and** F_3 **: Covariant electromagnetic form factors**
- m_v**, Vector bound state mass**
- **Ref. Gilman, Ronald A. and Gross, Franz;
J. Phys. G 28 (2002) R37–R116.**

Frame

- **Breit frame**
- $\Longrightarrow \quad p^{\mu} = (p_0, -q/2, 0, 0)$, **Initial state**
- $\Longrightarrow \quad p'^{\mu} = (p_0, q/2, 0, 0)$ **Final state**

Transfer momentum $===>> \quad q^{\mu} = (0, q, 0, 0)$

- **Polarization in the cartesian basis**

$\epsilon^{\mu}_{x} = (-\sqrt{\eta}, \sqrt{1+\eta}, 0, 0), \epsilon^{\mu}_{y} = (0, 0, 1, 0), \epsilon^{\mu}_{z} = (0, 0, 0, 1)$

for the vector meson in the initial state

- **And in the final state**

$\epsilon'^{\mu} = (\sqrt{\eta}, \ \sqrt{1+\eta}, 0, 0), \ \epsilon'^{\mu}_{y} = (0, 0, 1, 0), \ \epsilon'^{\mu}_{z} = (0, 0, 0, 1)$

- **With,** $\eta = \frac{q^2}{4m_{\rho}^2}$

Matrix elements of the electromagnetic current are (explicity)

$$
\begin{aligned}
J^{+}_{xx} &= (p^{+}+p^{'+})\left[F_1(Q^2)\epsilon_{x'}.\epsilon_x - \frac{F_2(Q^2)}{2m_V^2}(q.\epsilon_{x'}q.\epsilon_x)\right] \\
&\quad -F_3(Q^2)\left(q.\epsilon_{x'}\epsilon^{+}_{x} - q.\epsilon_x\epsilon^{+}_{x}\right)
\end{aligned}
$$

$$
\begin{aligned}
\epsilon'_x.\epsilon_x &= \left(\sqrt{\eta}. - \sqrt{\eta} - \sqrt{1+\eta}\sqrt{1+\eta}\right) = -(1+2\eta)\,, \\
(0,q,0,0).\epsilon_x &= -q\sqrt{1+\eta}, \\
(0,q,0,0).\epsilon'_x &= -q\sqrt{1+\eta}, \\
((0,q,0,0).\epsilon'_x)\,\epsilon^{+}_x &= -q\sqrt{1+\eta}(-\sqrt{\eta}) = q\sqrt{\eta}\sqrt{1+\eta}, \\
((0,q,0,0).\epsilon_x)\,\epsilon'^{+}_x &= -q\sqrt{1+\eta}(\sqrt{\eta}) = -q\sqrt{\eta}\sqrt{1+\eta}, \\
(q.\epsilon'_x)(q.\epsilon_x) &= q^2(1+\eta)
\end{aligned}
$$

Relation between the electromagnetic current, J^+_{ji}, and the covariant form factors

- **Plus component of the electromagnetic current**

$$
\begin{aligned}
J^+_{xx} &= 2p^+\left(-2F_1(1+2\eta) - \frac{F_2}{2m_v^2}q^2(1+\eta)\right) - F_3 2q\sqrt{\eta}\sqrt{1+\eta}, \\
J^+_{yy} &= -2p^+F_1, \\
J^+_{zz} &= -2p^+F_1, \\
J^+_{zx} &= -F_3 q\sqrt{1+\eta}, \\
J^+_{xz} &= F_3 q\sqrt{1+\eta}, \\
J^+_{yx} &= J^+_{xy} = J^+_{zy} = J^+_{yz} = 0.
\end{aligned}
$$

- **Electromagentic form factors in terms of matrix elements of the current**

$$
\begin{aligned}
F_1^+ &= -\frac{J_{yy}^+}{2p^+} = -\frac{J_{zz}^+}{2p^+}, \\
F_2^+ &= \frac{m_v^2}{p^+ q^2 (1+\eta)} \left[J_{yy}^+ (1+2\eta) - J_{xx}^+ + J_{zx}^+ 2\sqrt{\eta} \right], \\
F_3^+ &= -\frac{J_{zx}^+}{q\sqrt{1+\eta}} = \frac{J_{xz}^+}{q\sqrt{1+\eta}} .
\end{aligned}
$$

Relation between the electromagnetic current, J_{ji}^-, and the covariant form factors

- **Minus component of the electromagnetic current**

$$
\begin{aligned}
J_{xx}^- &= 2p^- \left(-F_1(1+2\eta) - \frac{F_2}{2m_v^2} q^2 (1+\eta) \right) - F_3 2q\sqrt{\eta}\sqrt{1+\eta}, \\
J_{yy}^- &= -2p^- F_1, \\
J_{zz}^- &= -2p^- F_1, \\
J_{zx}^- &= F_3 q\sqrt{1+\eta}, \\
J_{xz}^- &= -F_3 q\sqrt{1+\eta}, \\
J_{yx}^- &= J_{xy}^- = J_{zy}^- = J_{yz}^- \; = \; 0.
\end{aligned}
$$

- **Electromagnetic form factors in terms of matrix elements of the current**

$$
\begin{aligned}
F_1^- &= -\frac{J_{yy}^-}{2p^-} = -\frac{J_{zz}^-}{2p^-}, \\
F_2^- &= \frac{m_v^2}{p^- q^2(1+\eta)}\left[J_{yy}^-(1+2\eta) - J_{xx}^- + J_{zx}^- 2\sqrt{\eta}\right], \\
F_3^- &= -\frac{J_{zx}^-}{q\sqrt{1+\eta}} = \frac{J_{xz}^-}{q\sqrt{1+\eta}}.
\end{aligned}
$$

- **Basis**
- **In spherical basis**

$$
\begin{aligned}
\epsilon^{\mu}_{+-} &= -(+)\frac{\epsilon^{\mu}_{x} + (-)\imath\epsilon^{\mu}_{y}}{\sqrt{2}} \\
\epsilon^{\mu}_{0} &= \epsilon^{\mu}_{z} \; .
\end{aligned}
$$

- **Matrix elements electromagnetic current (plus)**

$$J^{+}_{ji} = \frac{1}{2}\begin{pmatrix} J^{+}_{xx} + J^{+}_{yy} & \sqrt{2}J^{+}_{zx} & J^{+}_{yy} - J^{+}_{xx} \\ -\sqrt{2}J^{+}_{zx} & 2J^{+}_{zz} & \sqrt{2}J^{+}_{zx} \\ J^{+}_{yy} - J^{+}_{xx} & -\sqrt{2}J^{+}_{zx} & J^{+}_{xx} + J^{+}_{yy} \end{pmatrix}$$

- **Matrix elements electromagnetic current (minus)**

$$J^{-}_{ji} = \frac{1}{2}\begin{pmatrix} J^{-}_{xx} + J^{-}_{yy} & \sqrt{2}J^{-}_{zx} & J^{-}_{yy} - J^{-}_{xx} \\ -\sqrt{2}J^{-}_{zx} & 2J^{-}_{zz} & \sqrt{2}J^{-}_{zx} \\ J^{-}_{yy} - J^{-}_{xx} & -\sqrt{2}J^{-}_{zx} & J^{-}_{xx} + J^{-}_{yy} \end{pmatrix}$$

- **Light-front matriz electromagnetic current:**

$$I^{\pm}_{m'm} = \begin{pmatrix} I^{\pm}_{11} & I^{\pm}_{10} & I^{\pm}_{1-1} \\ -I^{\pm}_{10} & I^{\pm}_{00} & I^{\pm}_{10} \\ I^{\pm}_{1-1} & -I^{\pm}_{10} & I^{\pm}_{11} \end{pmatrix}$$

- **Relation (by Melosh matriz)**

$$R_M.J^{\pm}.R_M = I^{\pm} \quad (VIP!!)$$

$$R_M = \begin{pmatrix} \frac{1+\cos\theta}{2} & -\frac{\sin\theta}{2} & \frac{1-\cos\theta}{2} \\ \frac{\sin\theta}{\sqrt{2}} & \cos\theta & -\frac{\sin\theta}{2} \\ \frac{1-\cos\theta}{2} & \frac{\sin\theta}{\sqrt{2}} & \frac{1+\cos\theta}{2} \end{pmatrix} .$$

- **With**

$\cos\theta = \frac{1}{\sqrt{1+\eta}}, \sin\theta = \frac{\sqrt{\eta}}{\sqrt{1+\eta}}$ **and** $\eta = \frac{\vec{p}^2}{m^2}$, **if** $p_x < 0$, $\theta < 0$

- **Relations between the matrix elements of the current in the Cartesian spin basis, J^+**

$$I_{11}^+ = \frac{J_{xx}^+ + (1+\eta)J_{yy}^+ - \eta J_{zz}^+ - 2\sqrt{\eta}J_{zx}^+}{2(1+\eta)},$$

$$I_{10}^+ = \frac{\sqrt{2\eta}J_{xx}^+ + \sqrt{2\eta}J_{zz}^+ - \sqrt{2}(\eta-1)J_{zx}^+}{2(1+\eta)},$$

$$I_{1-1}^+ = \frac{-J_{xx}^+ + (1+\eta)J_{yy}^+ + \eta J_{zz}^+ + 2\sqrt{\eta}J_{zx}^+}{2(1+\eta)},$$

$$I_{00}^+ = \frac{-\eta J_{xx}^+ + J_{zz}^+ - 2\sqrt{\eta}J_{zx}^+}{(1+\eta)}$$

• **Relations between the matrix elements of the current in the Cartesian spin basis, J^-**

$$I_{11}^- = \frac{J_{xx}^- + (1+\eta)J_{yy}^- - \eta J_{zz}^- - 2\sqrt{\eta}J_{zx}^-}{2(1+\eta)},$$

$$I_{10}^- = \frac{\sqrt{2\eta}J_{xx}^- + \sqrt{2\eta}J_{zz}^- - \sqrt{2}(\eta-1)J_{zx}^-}{2(1+\eta)},$$

$$I_{1-1}^- = \frac{-J_{xx}^- + (1+\eta)J_{yy}^- + \eta J_{zz}^- + 2\sqrt{\eta}J_{zx}^-}{2(1+\eta)},$$

$$I_{00}^- = \frac{-\eta J_{xx}^- + J_{zz}^- - 2\sqrt{\eta}J_{zx}^-}{(1+\eta)}$$

Model

- **Full Model**

$$\Gamma^{\mu}(k,p) = \gamma^{\mu} - \frac{m_v}{2}\left(k^{\mu} + k'^{\mu}\right) D_v^{-1}(k)$$

- m_v is the vector spin- 1 particle mass, $D_v(k) = (p \cdot k + m_v m - \imath\epsilon)$, and $k' = k - p$.
- $\Gamma(k,p) = \gamma^{\mu}$

$$J^{\pm}_{ji} = \imath \int \frac{d^4k}{(2\pi)^4} \frac{Tr\left[\Gamma\Gamma\right]^{\pm}_{ji} \Lambda(k,p_f)\Lambda(k,p_i)}{((k-p_i)^2 - m^2 + \imath\epsilon)(k^2 - m^2 + \imath\epsilon)((k-p_f)^2 - m^2 + \imath\epsilon)}$$

- Dirac trace:

$$Tr\left[\Gamma\Gamma\right]^{\pm}_{ji} = Tr\left[\epsilon_j \cdot \Gamma(k,p_f)(\not{k} - \not{p}_f + m)\gamma^{\pm}(\not{k} - \not{p}_i + m)\epsilon_i \cdot \Gamma(k,p_i)(\not{k} + m)\right]$$

- Regularization function: $\Lambda(k,p) = N/[(k-p)^2 - m_R^2 + \imath\epsilon]^2$ which is chosen to turn the loop integration finite.

Wave function, Spin-1 Particles

$$\Phi_i\left(x,\vec{k}_\perp\right) = \frac{N}{(1-x)^2\left(m_\rho^2-M_0^2\right)\left(m_\rho^2-M_R^2\right)^2} \times \vec{\epsilon}_i \cdot \left[\vec{\gamma} - \frac{\vec{k}}{\frac{M_0}{2}+m}\right]$$

- **where $x = k^+/p^+$, and, the free quark-antiquark mass squared is**

$$M_0^2 = \frac{k_\perp^2+m^2}{x} + \frac{(\vec{p}-\vec{k})_\perp^2+m^2}{1-x} - p_\perp^2 .$$

- **The function M_R^2 is,**

$$M_R^2 = \frac{k_\perp^2+m^2}{x} + \frac{(\vec{p}-\vec{k})_\perp^2+m_R^2}{1-x} - p_\perp^2$$

- **Polarization state is given by $\vec{\epsilon}_i$**
- $N \Longrightarrow$ **Normalization factor, $G_0(Q^2=0)=1$**

Angular condition

- **Light-front we have the angular condition equation for $I^{+}_{m'm}$**
- **Plus component of the electromagnetic current**

$$\begin{aligned}\Delta^{(+)}(Q^2) &= (1+2\eta)I^{+}_{11} + I^{+}_{1-1} - \sqrt{8\eta}I^{+}_{10} - I^{+}_{00} \\ &= (1+\eta)(J^{+}_{yy} - J^{+}_{zz}) = 0;\end{aligned}$$

- **Minus component of the electromagnetic current**

$$\begin{aligned}\Delta^{(-)}(Q^2) &= (1+2\eta)I^{-}_{11} + I^{-}_{1-1} - \sqrt{8\eta}I^{-}_{10} - I^{-}_{00} \\ &= (1+\eta)(J^{-}_{yy} - J^{-}_{zz}) = 0;\end{aligned}$$

Ref. I.L. Grach, L.A. Kondratyuk, Sov.J.Nucl.Phys.38 (1984) 198
I.L. Grach, L.A. Kondratyuk, M. Strikman, Phys.Rev.Lett.62 (1989) 387

- **Angular Condition: Violation!!**

$$q_x \Longrightarrow J^+_{yy} = J^+_{zz} \begin{cases} Parity \\ + \\ Rotations \end{cases}$$

- **Equal time:** $\Delta^{\pm}(Q^2) = 0$, **is true!!**

- **Light-front:** $\Delta^{\pm}(Q^2) = 0$, **is not true!!**

- **With the covariant form factors**

$$\begin{aligned} J^{\pm}_{yy} &= -2p^{\pm}F^{\pm}_{1}, \\ J^{\pm}_{zz} &= -2p^{\pm}F^{\pm}_{1}. \end{aligned}$$

$$\begin{aligned} \Delta^{\pm}(Q^2) &= (1+2\eta)(J^{\pm}_{yy} - J^{\pm}_{zz}) \\ &= (1+2\eta)(-2p^{\pm}F^{\pm}_{1} + 2p^{\pm}F^{\pm}_{1}) = 0. \end{aligned}$$

▪

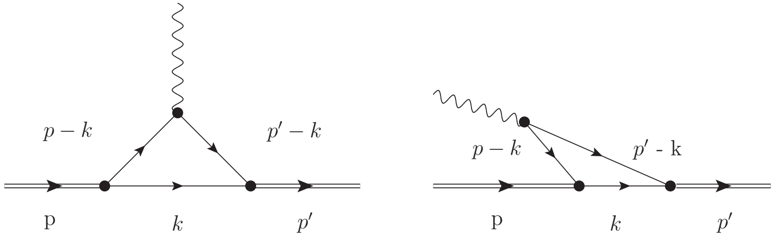


Fig. 1. **Feynman diagrams for the valence contribution (left panel) and the non-valence contribuition (right panel) for the electromagnetic current.**

Pole Dislocation Method

$p^+ \quad \Longrightarrow \quad p'^+ = p^+ + \delta$

Boson Eletromagnetic Current

Breit Frame $\Longrightarrow \;\; q^- = 0,\ q^+ \Longrightarrow 0_+,\ \vec{q}_\perp \neq 0$

$J^+ = J^- +$ **restoration covariance term**

$J_\perp \propto q^+ \Rightarrow 0$

J. de Melo, Sales and T.Frederico Nucl. Phys. B631, (1998) 574.

Ward-Takahashi Identity $\Longrightarrow$ **Pair Contribuition**

Naus, de Melo and Frederico

Few-Body Syst. 24, 1998, 99-107

- **Chang e Yan, Phys. Rev. D7 (73) 1147, Phys. Rev. D7 (73) 1780.**
- **Sawicki, Phys. Rev. D44 (91) 433, Phys. Rev. D46 (92) 474.**

Prescriptions

$$\begin{cases} FFS\ (Frederico, Frankfurt, Strikman) \\ GK\ (Grach, Kondratyku) \\ CCKP\ (Coester, Chung, Keister, Polyzou) \\ BH\ (Brodsky, Hiller) \\ KA\ (Karmanov) \end{cases} \quad \textbf{vs} \quad \textbf{COVARIANT}$$

- **Breit Frame** $\Longrightarrow P^+ = P'^+,\ P^- = P'^-,\ \vec{P'}_\perp = -\vec{P}_\perp = \vec{q}/2$

- $J^+_\rho = \begin{cases} 4\ \ Current\ \ Elements \\ 3\ \ Form\ \ Factors\ \ G_0\ ,\ G_1\ and\ G_2 \end{cases}$

- **Ref. J.P.B.C. de Melo, T. Frederico, Phys. Rev.C55 (1997) 2043**

Electromagnetic form factors: G_0, G_1 *and* G_2

- **I.L. Grach, L.A. Kondratyuk prescrition**

====>> **Eliminate the $I_{00}^{\pm}$ component of the electromagnetic current**

$$
\begin{aligned}
G_0^{GK} &= \frac{1}{3}[(3-2\eta)I_{11}^{\pm} + 2\sqrt{2\eta}I_{10}^{\pm} + I_{1-1}^{\pm}] \\
&= \frac{1}{3}[J_{xx}^{\pm} + (2-\eta)J_{yy}^{\pm} + \eta J_{zz}^{\pm}], \\
G_1^{GK} &= 2[I_{11}^{\pm} - \frac{1}{\sqrt{2\eta}}I_{10}^{\pm}] = J_{yy}^{\pm} - J_{zz}^{\pm} - \frac{J_{zx}^{\pm}}{\sqrt{\eta}}, \\
G_2^{GK} &= \frac{2\sqrt{2}}{3}[\sqrt{2\eta}I_{10}^{\pm} - \eta I_{11}^{\pm} - I_{1-1}^{\pm}] = \frac{\sqrt{2}}{3}[J_{xx}^{\pm} - (1+\eta)J_{yy}^{\pm} + \eta J_{zz}^{\pm}] \ .
\end{aligned}
$$

Ref. (GK) I.L. Grach, L.A. Kondratyuk, Sov. J. Nucl. Phys., 38 (1984) 198

CCKP

$$G_0^{CCKP} = \frac{1}{3(1+\eta)}[(\frac{3}{2}-\eta)(I_{11}^{+}+I_{00}^{+})+5\sqrt{2\eta}I_{10}^{+}+(2\eta-\frac{1}{2})I_{1-1}^{+}]$$
$$= \frac{1}{6}[2J_{xx}^{+}+J_{yy}^{+}+3J_{zz}^{+}]$$
$$G_1^{CCKP} = \frac{1}{(1+\eta)}[I_{11}^{+}+I_{00}^{+}-I_{1-1}^{+}-\frac{2(1-\eta)}{\sqrt{2\eta}}I_{10}^{+}] = -\frac{J_{zx}^{+}}{\sqrt{\eta}}$$
$$G_2^{CCKP} = \frac{\sqrt{2}}{3(1+\eta)}[-\eta I_{11}^{+}-\eta I_{00}^{+}+2\sqrt{2\eta}I_{10}^{+}-(\eta+2)I_{1-1}^{+}] =$$
$$\frac{\sqrt{2}}{3}[J_{xx}^{+}-J_{yy}^{+}]$$

Ref. **Chung, Polyzou, Coester, Keister, Phys. Rev. C37 (1988) 2000**

Brodsky-Hiller - (BH) - I_{11}^{+}

$$
\begin{aligned}
G_0^{BH} &= \frac{1}{3(1+2\eta)}[(3-2\eta)I_{00}^{+} + 8\sqrt{2\eta}I_{10}^{+} + 2(2\eta-1)I_{1-1}^{+}] \\
&= \frac{1}{3(1+2\eta)}[J_{xx}^{+}(1+2\eta) + J_{yy}^{+}(2\eta-1) + J_{zz}^{+}(3+2\eta)] \\
G_1^{BH} &= \frac{2}{(1+2\eta)}[I_{00}^{+} - I_{1-1}^{+} + \frac{(2\eta-1)}{\sqrt{2\eta}}I_{10}^{+}] \\
&= \frac{1}{(1+2\eta)}[\frac{J_{zx}^{+}}{\sqrt{\eta}}(1+2\eta) - J_{yy}^{+} + J_{zz}^{+}] \\
G_2^{BH} &= \frac{2\sqrt{2}}{3(1+2\eta)}[\sqrt{2\eta}I_{10}^{+} - \eta I_{00}^{+} - (\eta+1)I_{1-1}^{+}] \\
&= \frac{\sqrt{2}}{3(1+2\eta)}[J_{xx}^{+}(1+2\eta) - J_{yy}^{+}(1+\eta) - \eta J_{zz}^{+}]
\end{aligned}
$$

Ref. **Brodsky, Hiller, Phys. Rev. D46 (1992) 2141**

FFS

$$G_0^{FFS} = \frac{1}{3(1+\eta)}[(\frac{3}{2}-\eta)(I_{11}^{+}+I_{00}^{+})+5\sqrt{2\eta}I_{10}^{+}+(2\eta-\frac{1}{2})I_{1-1}^{+}]$$

$$= \frac{1}{6}[2J_{xx}^{+}+J_{yy}^{+}+3J_{zz}^{+}]$$

$$G_1^{FFS} = G_1^{CCKP}\ ,\ G_2^{FFS}=G_2^{CCKP}$$

Ref. **Frankfurt, Frederico, Strikman, Phy. Rev. C48 (1993) 2182**

Karmanov

$$
\begin{aligned}
G_0^{KA} &= \frac{1}{3}\left[2(1-\eta)I_{11}^{+} + 4\sqrt{2\eta}I_{10}^{+} + I_{00}^{+}\right] \\
&= \frac{1}{3}[J_{xx}^{+} + J_{yy}^{+}(1-2\eta) + (2\eta+1)J_{zz}^{+}] \\
G_1^{KA} &= \left[2I_{11}^{+} - \sqrt{\frac{2}{\eta}}I_{10}^{+}\right] = \left[J_{yy}^{+} - \frac{J_{zx}^{+}}{\sqrt{\eta}} - J_{zz}^{+}\right] \\
G_2^{KA} &= \frac{2\sqrt{2}}{3}\left[(1+\eta)I_{11}^{+} - \sqrt{2\eta}I_{10}^{+} - I_{00}^{+}\right] \\
&= \frac{\sqrt{2}}{3}\left[J_{xx}^{+} + (1+\eta)J_{yy}^{+} - (2+\eta)J_{zz}^{+}\right]
\end{aligned}
$$

Ref.: **V. Karmanov, Nucl. Physics A608 (1996) 316**

Vertex by Parts

- **Vertex** $\Gamma(\gamma^{\mu}, \gamma^{\nu})$

$$Tr[gg]_{ji} = Tr[\not\epsilon_f^{\alpha}(\not k - \not p' + m)\gamma^{+}(\not k - \not p + m)\not\epsilon_i^{\alpha}(\not k + m)]$$

- **+Z (Pair Terms)**

$$Tr_{xx}^{+Z} = \frac{-k^{-}\ \eta}{2}\ Tr[\gamma^{-}(\not k - \not p' + m)\gamma^{+}(\not k - \not p + m)\not\epsilon_i^{\alpha}\gamma^{-}\gamma^{+}]$$
$$Tr_{yy}^{+Z} = k^{-}(k^{+} - p^{+})^{2} = 0$$
$$Tr_{zz}^{+Z} = \frac{k^{-}}{2}\ Tr[\gamma^{-}(\not k - \not p' + m)\gamma^{+}(\not k - \not p + m)\not\epsilon_i^{\alpha}\gamma^{-}\gamma^{+}]$$
$$Tr_{zx}^{+Z} = \frac{-k^{-}\sqrt{\eta}}{2}\ Tr[\gamma^{-}(\not k - \not p' + m)\gamma^{+}(\not k - \not p + m)\not\epsilon_i^{\alpha}\gamma^{-}\gamma^{+}]$$

- **Cont.**

$$Tr[gg]^{+Z}_{ji} = \frac{k^-}{2}\; R_{gg}$$

$$R_{gg} = Tr[\not\epsilon^{\alpha}_{f}(\not k - \not p' + m)\gamma^{+}(\not k - \not p + m)\not\epsilon^{\alpha}_{i}\gamma^{+}]$$

- **Fact:** $\rightarrow k^{-(m+1)}\ (p^{+} - k^{+})^{n}$

  No Pair Terms Contribution if $m < n$

Ref. Nucl. Phys. A660 (1999) 219, De Melo, Frederico, Naus and Sauer

- **Simplification**: $[\gamma^\mu, \gamma^\nu]$ **Dirac Trace**:

$$\begin{aligned} Tr[gg]^{+Z}_{xx} &= -\eta \; Tr[gg]^{+Z}_{zz} \\ Tr[gg]^{+Z}_{zx} &= -\sqrt{\eta} \; Tr[gg]^{+Z}_{zz} \\ Tr[gg]^{+Z}_{zz} &= R_{gg} \end{aligned}$$

- **Also:**

$$Tr[gg]^{+Z}_{yy} = 4k^-(p^+ - k^+)^2 = 0!!$$

- **Ref.** de Melo, Naus and Frederico; Phys.Rev.C 59 (1999) 2278
de Melo, Frederico, Naus and Sauer; Nucl.Phys.A 660 (1999) 219

- **Pair Terms**

$$
\begin{aligned}
J^{+Z}_{xx} &= \lim_{\delta^+ \to 0} \int d^3K \frac{Tr[J^{+Z}_{xx}]}{[1][2][4][5][6][7]} \neq 0 \\
J^{+Z}_{zx} &= \lim_{\delta^+ \to 0} \int d^3K \frac{Tr[J^{+Z}_{zx}]}{[1][2][4][5][6][7]} \neq 0 \\
J^{+Z}_{zz} &= \lim_{\delta^+ \to 0} \int d^3K \frac{Tr[J^{+Z}_{zz}]}{[1][2][4][5][6][7]} \neq 0 \\
J^{+Z}_{yy} &= \lim_{\delta^+ \to 0} \int d^3K \frac{Tr[J^{+Z}_{yy}]}{[1][2][4][5][6][7]} = 0
\end{aligned}
$$

- **Basis $I^+_{m'm}$:**

$$
\begin{aligned}
I^{+Z}_{11} &= 0, \; I^{+Z}_{10} = 0 \\
I^{+Z}_{1-1} &= 0, \; I^{+Z}_{00} = (1+\eta)J^+_{zz} \neq 0
\end{aligned}
$$

- **Pair Term Contribution:** **only: I^{+Z}_{00} !!**
- **Inna Grach:** **Elimination** I^+_{00}

.

$$G_0^{GK} = \frac{1}{3}[J_{xx}^{+} + 2J_{yy}^{+} - \eta J_{yy}^{+} + \eta J_{zz}^{+}]$$

$$G_1^{GK} = J_{yy}^{+} - J_{zz}^{+} - \frac{J_{zx}^{+}}{\sqrt{\eta}}$$

$$G_2^{GK} = \frac{\sqrt{2}}{3}[J_{xx}^{+} + J_{yy}^{+}(-1-\eta) + \eta J_{zz}^{+}]$$

- **Pair Terms Combination**

$$G_0^{GK\ (+Z)} = \frac{1}{3}\left(J_{xx}^{(+Z)}[gg] + \eta J_{zz}^{+Z}[gg]\right) = \frac{1}{3}\left(-\eta J_{zz}^{+Z}[gg] + \eta J_{zz}^{+Z}[gg]\right) = 0$$

$$G_1^{GK\ (+Z)} = \left(-J_{zz}^{+Z}[gg] - \frac{J_{zx}^{+Z}[gg]}{\sqrt{\eta}}\right) = -J_{zz}^{+Z}[gg] + \sqrt{\eta}\frac{J_{zz}^{+Z}[gg]}{\sqrt{\eta}} = 0$$

$$G_2^{GK\ (+Z)} = \frac{\sqrt{2}}{3}\left(J_{xx}^{+Z}[gg] + \eta J_{zz}^{+Z}[gg]\right) = \frac{\sqrt{2}}{3}\left(-\eta J_{zz}^{+} + \eta J_{zz}^{+Z}\right) = 0$$

Cross term with γ^μ and derivatives

- $\Gamma(\gamma^\mu, 2k - p')$

$$Tr[dg]_{ji} = \epsilon'_j \cdot (2k - p')\ Tr[(\not k - \not p' + m)\gamma^+(\not k - \not p + m)\not\epsilon_i(\not k + m)]$$

- **Terms with $m \geq n$:**

$$Tr[dg]^Z_{ji} = \epsilon'^+_j \epsilon^+_i R_{dg} - 4m\ k^- k^+ \epsilon'^+_j\ \vec{\epsilon}_{i\perp} \cdot \vec{q}_\perp$$

- $R_{dg} = 4m\ k^- \left(k^-(k^+ - p^+) + (\vec{k}_\perp - \vec{p}'_\perp) \cdot (\vec{k}_\perp - \vec{p}_\perp) + q_\perp \cdot k_\perp + m^2 \right)$
- **The Z-modes to yy is zero** $\rightarrow$ $\epsilon^+_y = 0$ **and** $\epsilon'^+_y = 0$

- **Term with k^- and $k^{-2}(k^+ - p^+)$**
- **Interval $0 < k^+ - p^+ < \delta^+$ with γ^μ and derivative couplings:**

$$J_{ji}^{+Z}[dg] = \lim_{\delta^+ \to 0_+} \int [d^4k]^Z \frac{Tr[dg]_{ji}^Z}{\{1\}\{2\}\{3\}} \frac{1}{\{4\}^2} \frac{1}{\{5\}^2} \frac{m_v}{2(p' \cdot k + m\, m_v - i\epsilon)}$$

- **Zero of $\{3\}$ is dislocated by using $p'^+ = p^+ + \delta^+$**
- **Cauchy integration in k^- with k^+ in the interval $0 < k^+ - p^+ < \delta^+$**
- **Two poles: one from the dislocated denominator $\{3\} = 0$, and**

$$k^- = \frac{1}{p^+}\left(2\vec{p}\,'_\perp \cdot \vec{k}_\perp - k^+ p^- - 2\, m\, m_v + \imath\epsilon\right) \; .$$

- **The residue from the zero of $\{3\}$ is $\mathcal{O}[(\delta^+)^2]$**
- **The residue from the pole gives a contribution $\mathcal{O}[(\delta^+)^0]$.**

After integration in k^-, one has that:

$$J_{ji}^{+Z}[dg] \sim \mathcal{O}[\delta^+] \; .$$

Direct term with derivative couplings

$$Tr[dd]_{ji} = \left[A_{dd}\ \frac{k^-}{2} + B_{dd}\right] \epsilon'_j \cdot (2k - p')\ \epsilon_i \cdot (2k - p)\ .$$

$$\begin{aligned} A_{dd} &= Tr[(\not k - \not p' + m)\gamma^+(\not k - \not p + m)\gamma^+] = 8(p^+ - k^+)^2 \\ B_{dd} &= Tr\left[(\not k - \not p' + m)\gamma^+(\not k - \not p + m)\left(\frac{\gamma^-}{2}k^+ - \vec{\gamma}_\perp.\vec{k}_\perp + m\right)\right]\ . \end{aligned}$$

$$\begin{aligned} J^{+Z}_{ji}[dd] &= \lim_{\delta^+\to 0_+}\int [d^4k]^Z \frac{Tr[dd]_{ji}}{\{1\}\{2\}\{3\}\{5\}^2\{6\}^2} \\ &\times \frac{m_v^2}{4(p\cdot k + m\ m_v - i\epsilon)(p'\cdot k + m\ m_v - i\epsilon)} \end{aligned}$$

$\Longrightarrow\ J^{+Z}_{ij}[dd] \sim\ \mathcal{O}[\delta^+]$

$I^{+Z}_{11} = 0\ ,\ I^{+Z}_{10} = 0\ ,\ I^{+Z}_{1-1} = 0$ **and** $I^{+Z}_{00} = (1+\eta)J^{+Z}_{zz}$ **with** $\lim_{\delta^+\to 0_+} J^{+Z}_{zz} \neq 0$

- **Zero-mode contributions to the matrix elements of the current**

$$J^{+Z}_{yy} = 0, J^{+Z}_{xx} = -\eta J^{+Z}_{zz} \text{ and } J^{+Z}_{zx} = -\sqrt{\eta} J^{+Z}_{zz},$$

- **Only from valence contribuitions as**

$$J^{+Z}_{zz} = J^{+V}_{yy} - J^{+V}_{zz},$$

$===>>$ **Is a consequence of the angular condition**

- **Final relations for the matrix elements of the plus component of the current, computed solely in terms of valence matrix elements**

$$\begin{aligned} J^{+}_{xx} &= J^{+V}_{xx} - \eta \left(J^{+V}_{yy} - J^{+V}_{zz} \right) \\ J^{+}_{zx} &= J^{+V}_{zx} - \sqrt{\eta} \left(J^{+V}_{yy} - J^{+V}_{zz} \right) \end{aligned}$$

- **With the relations above, we have**

$$G_0^{GK(+Z)} = \frac{1}{3}\left[J_{xx}^{(+Z)} + \eta J_{zz}^{+Z}\right] = \frac{1}{3}\left[-\eta J_{zz}^{+Z} + \eta J_{zz}^{+Z}\right] = 0,$$

$$G_1^{GK(+Z)} = \left[-J_{zz}^{+Z}[gg] - \frac{J_{zX}^{+Z}}{\sqrt{\eta}}\right] = -J_{zz}^{+Z} + \sqrt{\eta}\frac{J_{zz}^{+Z}}{\sqrt{\eta}} = 0,$$

$$G_2^{GK(+Z)} = \frac{\sqrt{2}}{3}\left(J_{xx}^{+Z} + \eta J_{zz}^{+Z}\right) = \frac{\sqrt{2}}{3}\left[-\eta J_{zz}^{+} + \eta J_{zz}^{+Z}\right] = 0,$$

- **Prove** <u>GK **Prescription is free of the zero modes contribuitions!!**</u>

Ref.

J.P.B.C. de Melo, T. Frederico, Phys. Lett. B 708 (2012) 87
J.P.B.C. de Melo, Phys. Lett. B788 (2019) 152

The elimination of zero-modes for the matrix elements of the current $I^+_{m'm}$, leads to the following

$$I_{11}^{+Z} = 0, \quad I_{10}^{+Z} = 0, \quad I_{1-1}^{+Z} = 0,$$

and

$$I_{00}^{+Z} = (1+\eta) J_{zz}^{+Z} = (1+\eta)\left(J_{yy}^{+V} - J_{zz}^{+V}\right)$$

No Zero Modes or Pair Terms Contribution with Inna Grach prescp.!!

Ref.

J.P.B.C. de Melo, T. Frederico, Phys. Rev.C55 (1997) 2043
J.P.B.C. de Melo, T. Frederico, Phys. Lett. B 708 (2012) 87
J.P.B.C. de Melo, Phys. Lett. B788 (2019) 152
J.P.B.C. de Melo, 2309.07890 (2023) [hep-ph]

- **Similar Results are found by Ji, Bakker and Choi**
- **Phy.Rev.D65 (2002) 116001**
- **Phy.Rev.D70 (2004) 053015**

CCKP prescription elimination of the zero modes contribuition

$$
\begin{aligned}
G_0^{CCKP} \\
&= \frac{1}{3(1+\eta)}\left[\left(\frac{3}{2}-\eta\right)\left(I_{11}^{+}+I_{00}^{+}\right)+5\sqrt{2\eta}I_{10}^{+}+\left(2\eta-\frac{1}{2}\right)I_{1-1}^{+}\right] \\
&= \frac{1}{6}\left[2J_{xx}^{+}+J_{yy}^{+}+3J_{zz}^{+}\right]=\frac{1}{3}\left[J_{xx}^{+V}+(2-\eta)J_{yy}^{+V}+\eta J_{zz}^{+V}\right] \quad = \quad G_0^{GK} \\
G_1^{CCKP} &= \frac{1}{(1+\eta)}\left[I_{11}^{+}+I_{00}^{+}-I_{1-1}^{+}-\frac{2(1-\eta)}{\sqrt{2\eta}}I_{10}^{+}\right]=-\frac{J_{zx}^{+}}{\sqrt{\eta}}= \\
&= \left[J_{yy}^{+V}-J_{zz}^{+V}-\frac{J_{zx}^{+V}}{\sqrt{\eta}}\right]=G_1^{GK}, \\
G_2^{CCKP} &= \frac{\sqrt{2}}{3(1+\eta)}\left[-\eta I_{11}^{+}-\eta I_{00}^{+}+2\sqrt{2\eta}I_{10}^{+}-(\eta+2)I_{1-1}^{+}\right]= \\
&= \frac{\sqrt{2}}{3}\left[J_{xx}^{+}-J_{yy}^{+}\right]=\frac{\sqrt{2}}{3}\left[J_{xx}^{+V}-(1+\eta)J_{yy}^{+V}+\eta J_{zz}^{+V}\right] \quad = \quad G_2^{GK}
\end{aligned}
$$

BH prescription elimination of the zero modes contribuition

$$
\begin{aligned}
G_0^{BH} &= \frac{1}{3(1+2\eta)}\left[(3-2\eta)I_{00}^{+} + 8\sqrt{2\eta}I_{10}^{+} + 2(2\eta-1)I_{1-1}^{+}\right] \\
&= \frac{1}{3(1+2\eta)}\left[J_{xx}^{+}(1+2\eta) + J_{yy}^{+}(2\eta-1) + J_{zz}^{+}(3+2\eta)\right] \\
&= \frac{1}{3}\left[J_{xx}^{+V} + (2-\eta)J_{yy}^{+V} + \eta J_{zz}^{+V}\right] = G_0^{GK}, \\
G_1^{BH} &= \frac{2}{(1+2\eta)}\left[I_{00}^{+} - I_{1-1}^{+} + \frac{(2\eta-1)}{\sqrt{2\eta}}I_{10}^{+}\right] \\
&= \frac{1}{(1+2\eta)}\left[-\frac{J_{zx}^{+}}{\sqrt{\eta}}(1+2\eta) - J_{yy}^{+} + J_{zz}^{+}\right] = \left[J_{yy}^{+V} - \frac{J_{zx}^{+V}}{\sqrt{\eta}} - J_{zz}^{+V}\right] = G_1^{GK} \\
G_2^{BH} &= \frac{\sqrt{2}}{3(1+2\eta)}\left[\sqrt{2\eta}I_{10}^{+} - \eta I_{00}^{+} - (\eta+1)I_{1-1}^{+}\right] \\
&= \frac{\sqrt{2}}{3(1+2\eta)}\left[J_{xx}^{+}(1+2\eta) - J_{yy}^{+}(1+\eta) - \eta J_{zz}^{+}\right] = \frac{\sqrt{2}}{3}\left[J_{xx}^{+V} - (1+\eta)J_{yy}^{+V} + \eta J_{zz}^{+V}\right] \\
&= G_2^{GK}.
\end{aligned}
$$

FFS prescription elimination of the zero modes contribuition

$$
\begin{aligned}
G_0^{FFS} & \\
&= \frac{1}{3(1+\eta)}\left[(2\eta+3)I_{11}^{+} + 2\sqrt{2\eta}I_{10}^{+} - \eta I_{00}^{+} + (2\eta+1)I_{1-1}^{+}\right] \\
&= \frac{1}{3}\left[J_{xx}^{+} + 2J_{yy}^{+}\right] \\
&= \frac{1}{3}\left[J_{xx}^{+V} + (2-\eta)J_{yy}^{+V} + \eta J_{zz}^{+V}\right] = G_0^{GK}, \\
G_1^{FFS} &= G_1^{CCKP} = G_1^{GK}, \\
G_2^{FFS} &= G_2^{CCKP} = G_2^{GK},
\end{aligned}
$$

Karmanov prescription elimination of the zero modes contribuition

$$G_0^{KA} = \frac{1}{3}\left[2(1-\eta)I_{11}^+ + 4\sqrt{2\eta}I_{10}^+ + I_{00}^+\right]$$
$$= \frac{1}{3}\left[J_{xx}^+ + J_{yy}^+(1-2\eta) + (2\eta+1)J_{zz}^+\right]$$
$$= \frac{1}{3}\left[J_{xx}^{+V} + (2-\eta)J_{yy}^{+V} + \eta J_{zz}^{+V}\right] = G_0^{GK}$$
$$G_1^{KA} = \left[2I_{11}^+ - \sqrt{\frac{2}{\eta}}I_{10}^+\right] = \left[J_{yy}^+ - \frac{J_{zx}^+}{\sqrt{\eta}} - J_{zz}^+\right]$$
$$= \left[J_{yy}^{+V} - \frac{J_{zx}^{+V}}{\sqrt{\eta}} - J_{zz}^{+V}\right] = G_1^{GK}$$
$$G_2^{KA} = \frac{2\sqrt{2}}{3}\left[(1+\eta)I_{11}^+ - \sqrt{2\eta}I_{10}^+ - I_{00}^+\right]$$
$$= \frac{\sqrt{2}}{3}\left[J_{xx}^+ + (1+\eta)J_{yy}^+ - (2+\eta)J_{zz}^+\right]$$
$$= \frac{\sqrt{2}}{3}\left[J_{xx}^{+V} - (1+\eta)J_{yy}^{+V} + \eta J_{zz}^{+V}\right] = G_2^{GK}$$

Other way to see: Symmetry Broken

$$\begin{aligned}
\delta[G_0^{BH} - G_0^{GK}] &= -\frac{(3-2\eta)}{3(1+2\eta)}\left[(1+2\eta)I_{11}^{+} + I_{1-1}^{+} - \sqrt{8\eta}I_{10}^{+} - I_{00}^{+}\right] \\
&= -\frac{(3-2\eta)}{3(1+2\eta)}\Delta(Q^2), \\
\delta[G_1^{BH} - G_1^{GK}] &= -\frac{2}{(1+2\eta)}\left[(1+2\eta)I_{11}^{+} + I_{1-1}^{+} - \sqrt{8\eta}I_{10}^{+} - I_{00}^{+}\right] \\
&= -\frac{2}{(1+2\eta)}\Delta(Q^2), \\
\delta[G_2^{BH} - G_2^{GK}] &= \frac{2\sqrt{2}\eta}{3(1+2\eta)}\left[(1+2\eta)I_{11}^{+} + I_{1-1}^{+} - \sqrt{8\eta}I_{10}^{+} - I_{00}^{+}\right] \\
&= \frac{2\sqrt{2}\eta}{3(1+2\eta)}\Delta(Q^2)\ .
\end{aligned}$$

$$\delta[G_0^{CCKP} - G_0^{GK}] = -\frac{(3-2\eta)}{6(1+\eta)}\left[(1+2\eta)I_{11}^{+} + I_{1-1}^{+} - \sqrt{8\eta}I_{10}^{+} - I_{00}^{+}\right]$$
$$= -\frac{(3-2\eta)}{6(1+\eta)}\Delta(Q^2)\ ,$$
$$\delta[G_1^{CCKP} - G_1^{GK}] = -\frac{1}{(1+\eta)}\left[(1+2\eta)I_{11}^{+} + I_{1-1}^{+} - \sqrt{8\eta}I_{10}^{+} - I_{00}^{+}\right]$$
$$= -\frac{1}{(1+\eta)}\Delta(Q^2)\ ,$$
$$\delta[G_2^{CCKP} - G_2^{GK}] = \frac{\sqrt{2}\eta}{3(1+\eta)}\left[(1+2\eta)I_{11}^{+} + I_{1-1}^{+} - \sqrt{8\eta}I_{10}^{+} - I_{00}^{+}\right]$$
$$= \frac{\sqrt{2}\eta}{3(1+\eta)}\Delta(Q^2)\ .$$

$$
\begin{aligned}
\delta[G_0^{KA} - G_0^{GK}] &= \frac{1}{3}\left[I_{00}^{+} - I_{1-1}^{+} - 2\sqrt{2\eta} I_{10}^{+} - I_{11}^{+}(1+2\eta)\right] \\
&= -\frac{\Delta(Q^2)}{3}, \\
\delta[G_1^{KA} - G_1^{GK}] &= 0, \\
\delta[G_2^{KA} - G_2^{GK}] &= \frac{-2\sqrt{2}}{3}\left[I_{00}^{+} - I_{1-1}^{+} + \sqrt{8\eta} I_{10}^{+} - (2\eta+1) I_{11}^{+}\right] \\
&= \frac{2\sqrt{2}}{3}\Delta(Q^2).
\end{aligned}
$$

$$\delta[G_0^{FFS} - G_0^{GK}] = \frac{\eta}{3(1+\eta)}\left[(1+2\eta)I_{11}^{+} + I_{1-1}^{+} - \sqrt{8\eta}I_{10}^{+} - I_{00}^{+}\right]$$
$$= -\frac{\eta}{3(1+\eta)}\Delta(Q^2) \ ,$$
$$\delta[G_1^{FFS} - G_1^{GK}] = -\frac{1}{(1+\eta)}\left[(1+2\eta)I_{11}^{+} + I_{1-1}^{+} - \sqrt{8\eta}I_{10}^{+} - I_{00}^{+}\right]$$
$$= -\frac{1}{(1+\eta)}\Delta(Q^2) \ ,$$
$$\delta[G_2^{FFS} - G_2^{GK}] = \frac{\sqrt{2}\eta}{3(1+\eta)}\left[(1+2\eta)I_{11}^{+} + I_{1-1}^{+} - \sqrt{8\eta}I_{10}^{+} - I_{00}^{+}\right]$$
$$= \frac{\sqrt{2}\eta}{3(1+\eta)}\Delta(Q^2) \ .$$

Elimination of Zero Modes

- **VIP:**

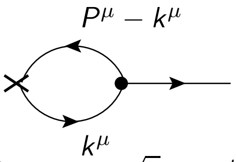
$$
\begin{aligned}
J^{+z}_{xx} &= -\eta J^{+z}_{zz} \\
J^{+z}_{zx} &= -\sqrt{\eta} J^{+z}_{zz} \\
J^{+z}_{yy} &= 0\ .
\end{aligned}
$$

- **Also**:

$$
J^{+Z}_{zz} = J^{+V}_{yy} - J^{+V}_{zz}
$$

- **J.P.B.C. de Melo and T. Frederico, Phys. Lett. B708, (2012) 87**
- **J.P.B.C. de Melo,Phys. Lett. B 768, (2019) 152**

Decay Constant



$$< 0|J^\mu(0)|p,\lambda >= \imath\sqrt{2} f_V M \epsilon^\mu_\lambda$$

- $\epsilon^\mu_\lambda \implies$ **Polarization**$\rightarrow \ \epsilon^+_z \ = \ 1$

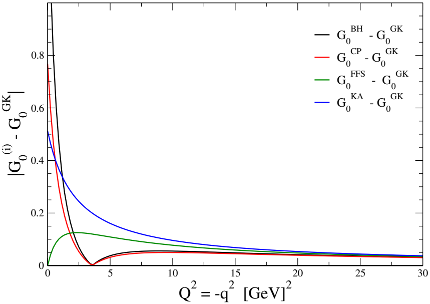
Dirac Trace:

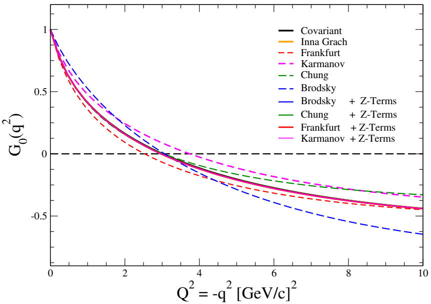
$$Tr[O^+] \ = \ [-4k^{+2} + 4k^2_\perp + 4k^+ p^+ + 4m^2] - \frac{m_V}{2} \frac{[4m(2k^+ - p^+)(k^- \ - \ k^+)]}{\frac{p^+k^- + p^-k^+}{2} + m_v m - \imath\epsilon}$$

- **Because the denominator:** **Zero mode is cancel out!!**

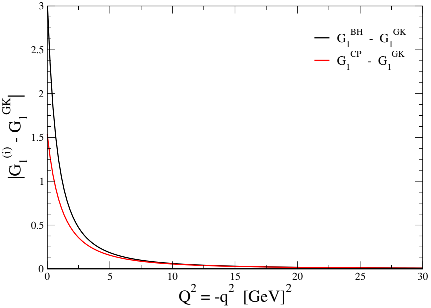
Observables: Charge, Magnetic, Quadrupole, Radius

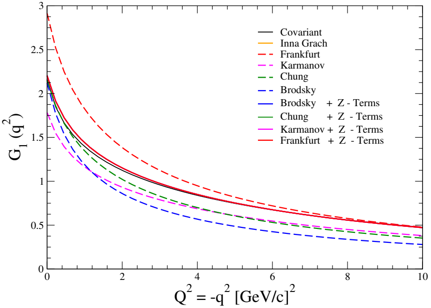
$$
\begin{aligned}
G_0(0) &= 1, \ \text{(charge normalization)}, \\
\mu_\rho &= G_1(0) \\
Q_{2\rho} &= \lim_{Q^2\to 0} 3\sqrt{2}\ \frac{G_2(Q^2)}{Q^2} \\
< r_\rho^2 > &= \lim_{Q^2\to 0} \frac{-6\left[G_0(Q^2)-1\right]}{Q^2} = -6\ \frac{dG_0(Q^2)}{dQ^2}\bigg|_{Q^2=0}
\end{aligned}
$$

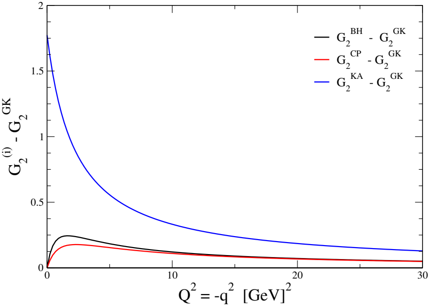




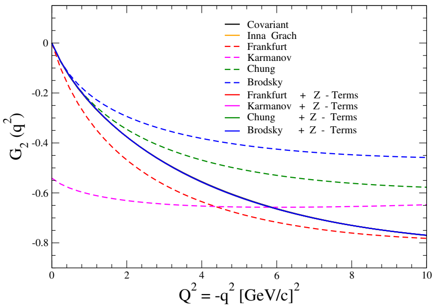
- $m_q = 0.430\ GeV,\ m_\rho = 0.775\ GeV,\ m_R = 3.0\ GeV$
- Fixed by the exp. $f_\rho = 153 \pm 8\ MeV$ (PDG)


$|G_1^{(i)} - G_1^{GK}|$
$G_1^{BH} - G_1^{GK}$
$G_1^{CP} - G_1^{GK}$
$Q^2 = -q^2$ [GeV]^2

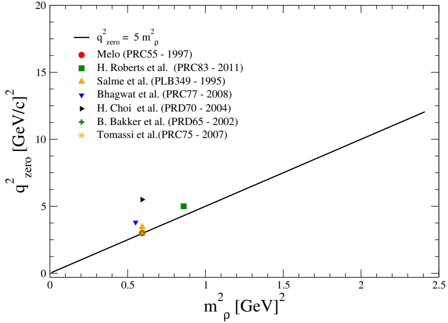

$G_1(q^2)$
Covariant
Inna Grach
Frankfurt
Karmanov
Chung
Brodsky
Brodsky + Z - Terms
Chung + Z - Terms
Karmanov + Z - Terms
Frankfurt + Z - Terms
$Q^2 = -q^2$ [GeV/c]^2

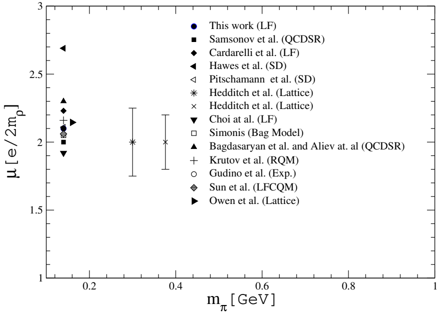


$G_2^{BH} - G_2^{GK}$
$G_2^{CP} - G_2^{GK}$
$G_2^{KA} - G_2^{GK}$
$G_2^{(i)} - G_2^{GK}$
$Q^2 = -q^2$ [GeV]^2
0
0.5
1
1.5
2
10
20
30



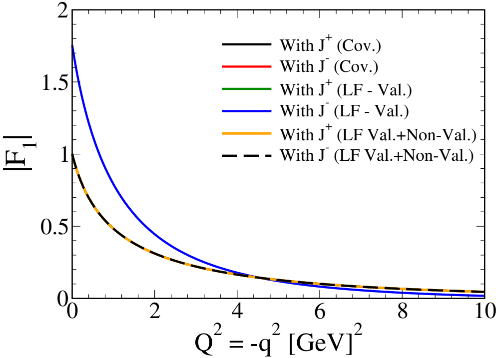
Covariant
Inna Grach
Frankfurt
Karmanov
Chung
Brodsky
Frankfurt + Z - Terms
Karmanov + Z - Terms
Chung + Z - Terms
Brodsky + Z - Terms
$G_2(q^2)$
$Q^2 = -q^2$ [GeV/c]^2
0
-0.2
-0.4
-0.6
-0.8
2
4
6
8
10



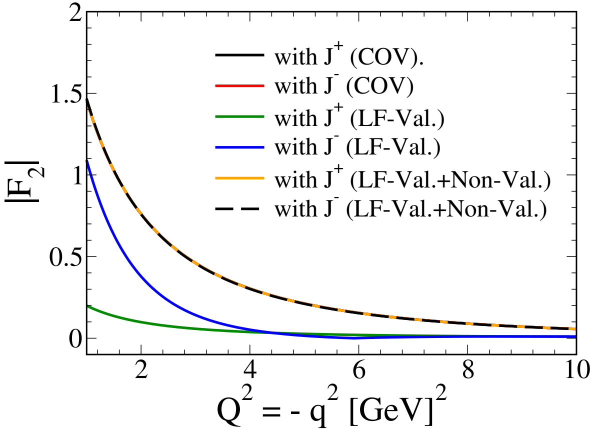


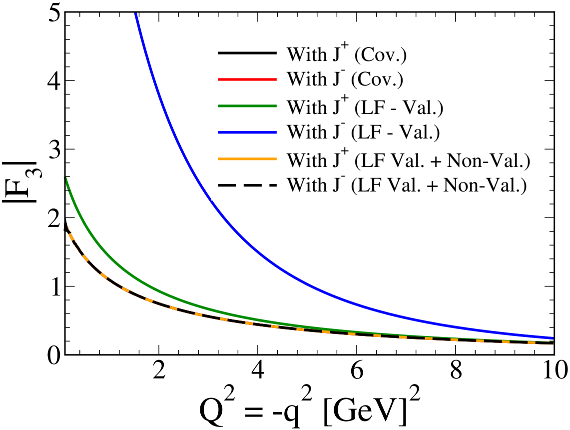
(Left) Charge electromagnetic form factor zero, $G_0(q^2_{zero}) = 0$ in function of the rho meson mass, in the present model and compared with another's models in the literature.
(Right) Magnetic moment for the rho meson compared with another models in the literature, (QCDSR) QCD sum rules, (LF) Light-Front approach, (SD) Schwinger-Dyson, Lattice calculations, Bag model, including with some experimental analysis.

Covariant Electromagnetic Form Factors



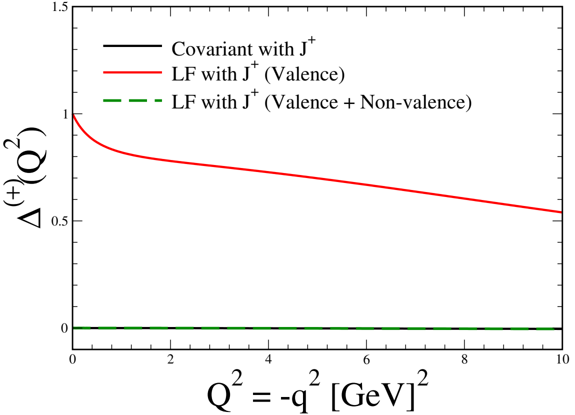
- $m_q = 0.430\ GeV,\ m_\rho = 0.775\ GeV,\ m_R = 3.0\ GeV$
- **Fixed by the exp.** $f_\rho = 153 \pm 8\ MeV$ **(PDG)**


with J+ (COV).
with J- (COV)
with J+ (LF-Val.)
with J- (LF-Val.)
with J+ (LF-Val.+Non-Val.)
with J- (LF-Val.+Non-Val.)
|F2|
Q2 = - q2 [GeV]2
0
0.5
1
1.5
2
2
4
6
8
10

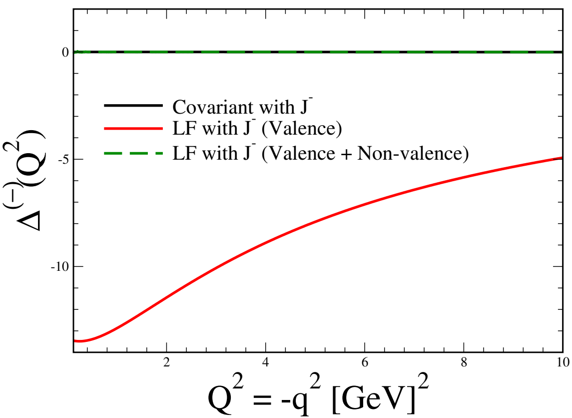


5
4
3
2
1
0
2
4
6
8
10
|F3|
Q2 = -q2 [GeV]2
With J+ (Cov.)
With J- (Cov.)
With J+ (LF - Val.)
With J- (LF - Val.)
With J+ (LF Val. + Non-Val.)
With J- (LF Val. + Non-Val.)

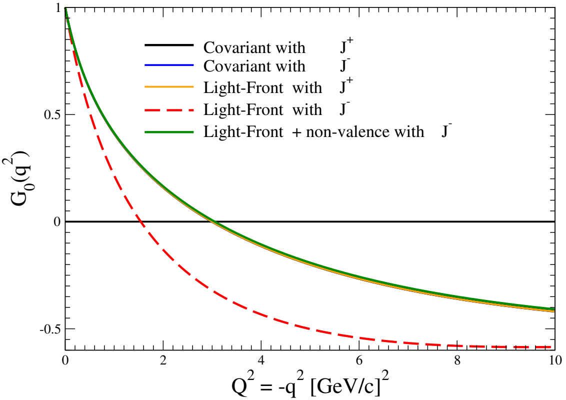
Angular Condition (with the plus component of e.m)



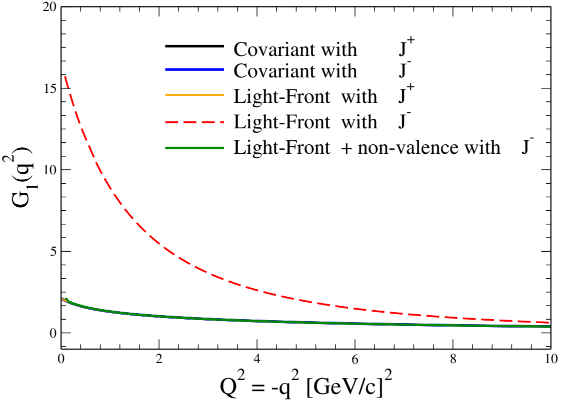
Angular Condition (with the minus component of e.m)



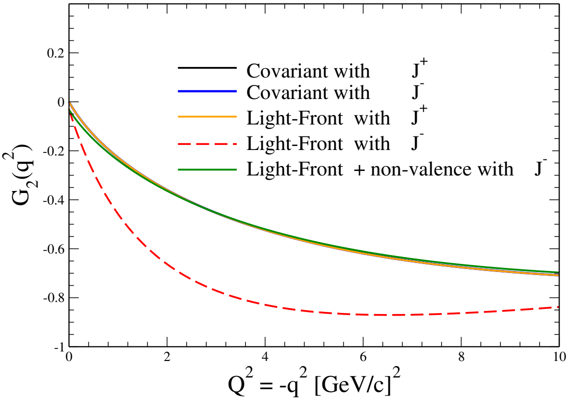
Charge Electromagnetic form factor: $G_0(q^2)$



Magnetic Electromagnetic form factor: $G_1(q^2)$



Quadrupole Electromagnetic form factor: $G_2(q^2)$



	f_ρ [MeV]	μ $[e/2m_\rho]$	Q_d $[e/m_\rho^2]$	$< r^2 >$ $[fm^2]$
This work	153.66	2.10	-0.898	0.267
Pichowsky (1999)	153.95	2.69	-0.055	0.61
Jaus (2003)	-	1.83	-0.330	-
Aliev (2009)	-	2.4 ± 0.4	0.85 ± 0.15	-
Biernat (214)	-	2.20	-0.47	-
Choi (2004)	-	1.92	-0.430	-
Melikhov (2002)	-	2.35	-0.364	-
Samsonov (2003)	-	2.00 ± 0.3	-	-
Pitschmann (2013)	-	2.11	-0.850	0.26
Krutov (2016,2018)	152 ± 8	2.16 ± 0.03	-	0.56 ± 0.04
Sun (2017)	-	2.06	-0.323	0.52
Hawes (1999)	-	2.69	-0.055	0.61
Cardarelli (1995)	-	2.26	-0.367	0.35
Bhagwat (2008)	-	2.01	-0.41	0.54
Roberts (2011)	-	2.11	-0.85	0.31
Serrano (2015)		2.57	-1.05	0.67
Gudiño (2015)	-	2.1±0,5	-	-
Simonis (2016)	-	2.06	-	-
Simonis (2018)	-	2.17		
Owen (2015)		2.145	-0.733	0.670
Shultz (2015)	-	2.17	-0.540	0.30
PDG	153 ± 8	-	-	-

- de Melo, Phys. Lett. B788 (2019) 152

Remarks

- Light-front approach correctly describes hadronic bound states
- Take New Informations about Bound States
- Breaking of the rotational invariance has to be evaluated
- The inclusion of zero modes or pair terms is crucial
- The break of the rotational symmetry for J^{-}_{ji} case is very pronounced
- We can see that with the inclusion of pair terms, we have the covariance restored
- Next: Full Spin-1 vertex for J^{-} and others meson with S=1

Summary

- **Light-Front** $\Longrightarrow \begin{cases} Bound\ \ States \\ Covariance \end{cases}$
- **Rotational Invariance Broken**$\Longrightarrow k^{-}$ **Problematic**
- **Terms** $\begin{cases} -\ Good\ \ /\ \ Covariant \\ -\ Z\ \ \ Terms\ \ /\ \ Non-Valence\ \ Components \end{cases}$
- **Electromagnetic Current:**
- $\begin{cases} -\ \mathrm{Present}\ \ \ \mathrm{Work}:\ J^{+}\ \ \ Component \\ -\ \mathrm{Future}\ \ \ \mathrm{Works}:\ \ J^{-}\ \ \ and\ \ \ J_{\perp} \\ -\ \mathrm{Kaon}\ \ \mathrm{K}^{*},\mathrm{S}=1 \end{cases}$
- **Take New Informations about Bound States**
- $\begin{cases} -\ \mathrm{Correlations}\ \ \ |q\bar{q}> \\ -\ \mathrm{Rho}\ \ \mathrm{Meson}\ \ \mathrm{Decay} \\ -\ \mathrm{Deuteron} \end{cases}$

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