Solving Bethe–Salpeter equations for the structure of pions, kaons, rho mesons, and for quark-photon vertices in the Euclidean space

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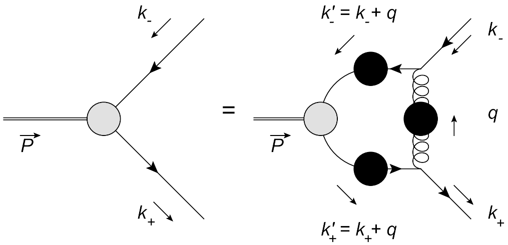
1. Bethe-Salpeter equation in the Euclidean space for pseudoscalar mesons
   - Schwinger-Dyson equation for quark propagators
   - Matrix formulation of the Bethe-Salpeter equation

2. Electromagnetic form factor of pseudoscalar mesons
   - EMFF of pions with an Ansatz vertex
   - Homogeneous BSE for vector mesons
   - Inhomogeneous BSE for the quark-photon vertex
   - EMFFs for pion and kaon with quark-photon vertex solved from the inhomogeneous BSE

Bethe-Salpeter equation (BSE)

In terms of Green's functions, two-body bound state structure is given by the Bethe-Salpeter amplitude (BSA) $\Gamma(k, P)$, determined from the Bethe–Salpeter equation (BSE).



Quark momenta: $k_\pm = k \pm \eta_\pm P$.
BSA: $\Gamma(k, P) = \gamma_5[E(k^2, k\cdot P) + \not{P} F(k^2, k\cdot P) + \not{k} G(k^2, k\cdot P) + [\not{k}, \not{P}] H(k^2, k\cdot P)/2]$.
Inputs:

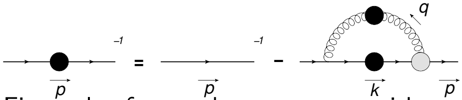
- quark propagators $S_\mathrm{F}(k)$,
- $\mathcal{G}(q^2)$ representing interaction.

In rainbow ladder truncation and Landau gauge, the BSE for pseudoscalar mesons

$$\Gamma(k, P) = -iC_\mathrm{F} g^2 \int d\underline{q} \gamma^\mu S_\mathrm{F}(k'_+) \Gamma(k, P) S_\mathrm{F}(k'_-) \gamma^\nu \left(g_{\mu\nu} - q_\mu q_\nu / q^2\right) \mathcal{G}(q^2). \quad (1)$$

Spacetime metric $g^{\mu\nu} = \mathrm{diag}\{1, -1, -1, \cdots -1\}$. Momentum p^μ is timelike when $p^2 \geq 0$. Euclidean-space momentum $p^4 = -ip^0$ such that $p_\mathrm{E}^2 = -p^2$.

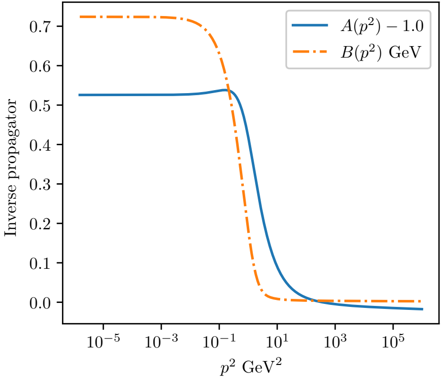
Schwinger-Dyson equation (SDE) for quark propagators



First solve for quark propagators with spacelike momenta from Schwinger–Dyson equation (SDE). In the rainbow-ladder truncation:

$$S_{\mathrm{F}}^{-1}(k_\pm) = Z_2\,(\not{k}_\pm - m_{\mathrm{B}}) + iC_{\mathrm{F}}\,g^2 \int d\underline{q}\,\gamma^\mu$$

$$\times\, S_{\mathrm{F}}(k_\pm + q)\gamma^\nu\,(g_{\mu\nu} - q_\mu q_\nu/q^2)\,\mathcal{G}(q^2).$$



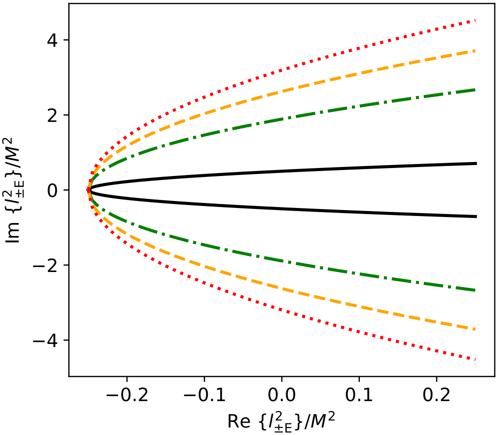
- Renormalization is required for $d = 4$.
- For real and spacelike $k_\pm^2$, the SDE can be solved iteratively for $S_{\mathrm{F}}^{-1}(p) = \not{p}A(p^2) + B(p^2)$ after the Wick rotation.

$$g^2\,\mathcal{G}_{\mathrm{E}}(k_{\mathrm{E}}^2) = \frac{4\pi^2}{\omega^6}\,d_{\mathrm{IR}}\,k_{\mathrm{E}}^2\,e^{-k_{\mathrm{E}}^2/\omega^2} + \frac{8\pi^2\gamma_m}{\ln[e^2 - 1 + (1 + k_{\mathrm{E}}^2/\Lambda_{\mathrm{QCD}}^2)^2]}\,\frac{1 - e^{-k_{\mathrm{E}}^2/(4m_t^2)}}{k_{\mathrm{E}}^2},$$

with $\gamma_m = 12/(33 - 2N_{\mathrm{f}})$ [P. Maris and P. C. Tandy, Phys.Rev.C 60,055214].

Quark propagator with complex-valued momentum

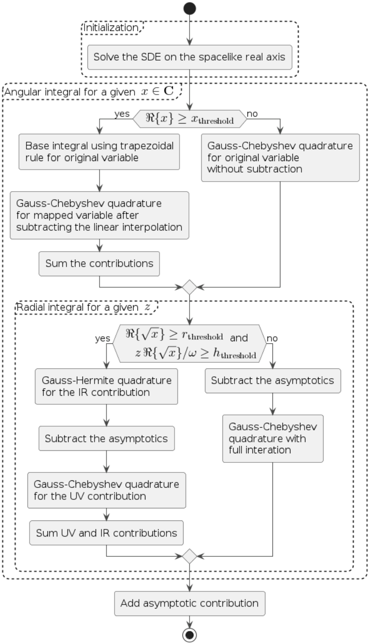
$S_{\mathrm{F}}(p)$ with $p^2 \in \mathbf{C}$ is sampled by the BSE.

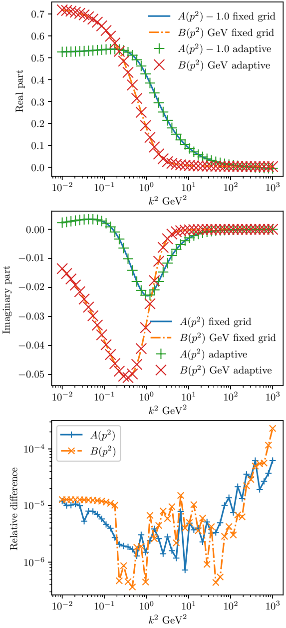


Boundaries in the complex-momentum plan of quark propagators. Regions within these parabolas are sampled by the BSE for the pion with spatial momentum:

black solid line	$0.0\,\mathrm{GeV}$,
green dash-dot line	$0.5\,\mathrm{GeV}$,
yellow dashed line	$0.707\,\mathrm{GeV}$,
red dotted line	$0.866\,\mathrm{GeV}$.

- Integrals in the quark self-energy is numerically difficult to compute for complex-valued p^2.
- Fixed-grid algorithm developed for their accurate and efficient computation [a].





[a] SJ and Ian Cloët, arXiv:2401.11019 [nucl-th]

Matrix formulation of the BSE

After the Wick rotation, the BSE in moving frame of the bound state becomes

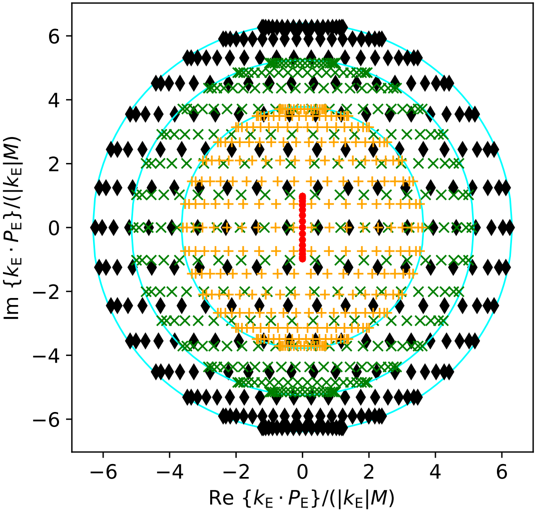
$$\mathbb{G}_{\mathrm{E}}(k_{\mathrm{E}}^2, y, z) = -\frac{g^2\, C_{\mathrm{F}}}{(2\pi)^4} \int_0^{\infty} dl_{\mathrm{E}}\, l_{\mathrm{E}}^3 \int_{-1}^{+1} dz' \sqrt{1-z'^2} \int_{-1}^{1} dy' \int_0^{2\pi} d\phi'\, \mathcal{G}_{\mathrm{E}}((l_{\mathrm{E}}-k_{\mathrm{E}})^2) \times \hat{\mathbb{T}}_{\mathrm{E}}(k_{\mathrm{E}}^2, y, z, l_{\mathrm{E}}^2, y', z', \phi')\, \mathbb{M}_{\mathrm{E}}^{\pm}(l_{\mathrm{E}}^2, y', z')\, \mathbb{M}_{\mathrm{E}}^{-}(l_{\mathrm{E}}^2, y', z')\, \mathbb{G}_{\mathrm{E}}(l_{\mathrm{E}}^2, y', z'), \tag{2}$$

with $\mathbb{G}_{\mathrm{E}}(k_{\mathrm{E}}^2, y, z) = (E_{\mathrm{E}}(k_{\mathrm{E}}^2, y, z), F_{\mathrm{E}}(\cdots), G_{\mathrm{E}}(\dots), H_{\mathrm{E}}(\dots))^{\mathrm{T}}$. [1]

- $\int_0^{2\pi} d\phi\, \mathcal{G}_{\mathrm{E}}((l_{\mathrm{E}}-k_{\mathrm{E}})^2)\, \hat{\mathbb{T}}_{\mathrm{E}}(k_{\mathrm{E}}^2, y, z, l_{\mathrm{E}}^2, y', z', \phi')$ is a complex-valued symmetric matrix with respect to $(k_{\mathrm{E}}^2, y, z) \leftrightarrow (l_{\mathrm{E}}^2, y', z')$.
- $\mathbb{M}_{\mathrm{E}}^{\pm}(l_{\mathrm{E}}^2, y', z')$ correspond to multiplications of quark propagators with BSA.
- Discretized grid for momentum variables (k_{E}^2, y, z) converts the BSE into a matrix eigenvalue problem (non-Hermitian).
- Arnoldi iteration to solve the eigenvalue problem at a given bound state mass.
- Ground state has the largest eigenvalue.

[1] SJ and Ian Cloët, arXiv:2402.00285 [hep-ph]

Angular resolution for the relative momentum of BSA



Angular resolution for the inner product of $k_{\rm E}^j$ with $P_{\rm E}^j$ in the BSA for bound-state spatial momenta of:

red line	$0.0\,\mathrm{GeV}^2$,
orange pluses	$0.5\,\mathrm{GeV}^2$,
green crosses	$0.707\,\mathrm{GeV}^2$,
black diamonds	$0.866\,\mathrm{GeV}^2$.

Model parameters for pions

ω	d_{IR}	Λ_{QCD}	N_{f}	m_t
0.4 GeV	0.859 (GeV)^2	0.234 GeV	4	0.5 GeV

Table: Parameters in the Maris–Tandy model. The IR term is specified by scale ω and strength d_{IR}. Remaining parameters determine the UV term.

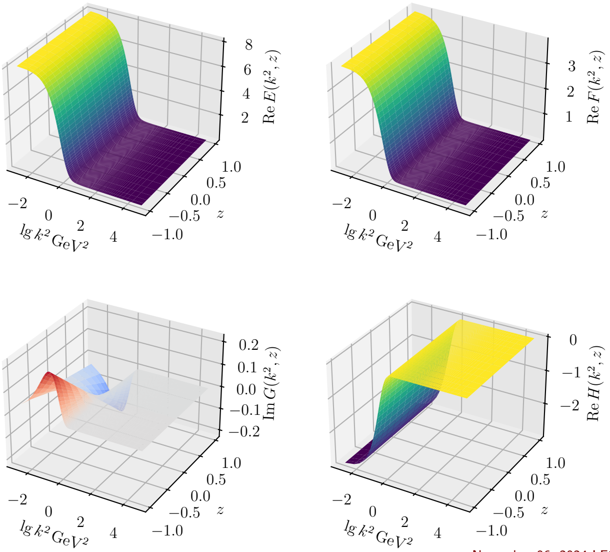
m_{l}	μ^2	Z_2	Z_{m}
3.6964 MeV	361.0 GeV^2	0.98201	0.67048

Table: Parameters for the SDE of light-quark propagators. The renormalized quark mass m_{l} is defined at the renormalization scale μ^2. Renormalization constants are given by Z_2 and Z_m.

Strange quark mass and static observables

M_π	f_π	m_{s}	M_{K}	f_{K}
137.24 MeV	92.22 MeV	84.8574 MeV	495.768 MeV	109.474 MeV

Solution in the pion rest frame



Solution in the kaon rest frame

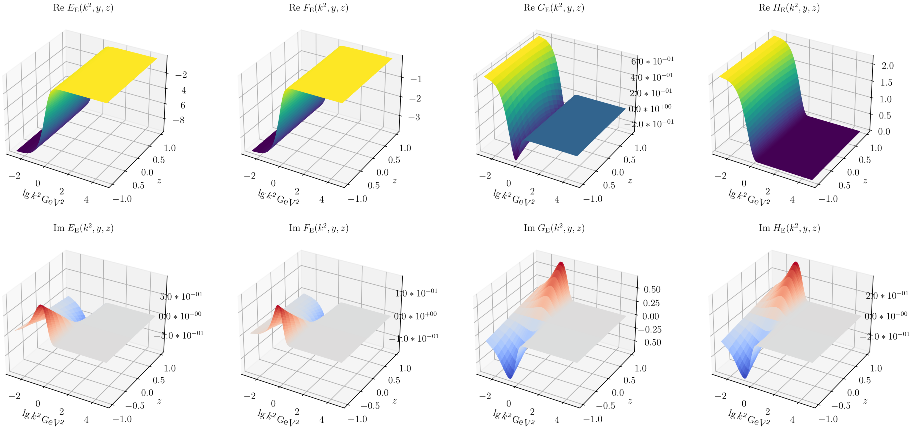
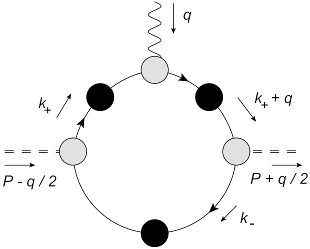


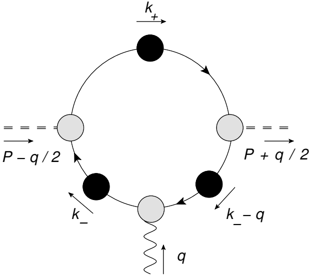
Figure: Scalar functions of the BSA for kaons in the rest frame.

Electromagnetic form factor (EMFF) for pseudoscalar mesons

$$\kappa_{\pm} = \kappa \pm \eta_{\pm}\Pi,$$
$$\kappa'_{\pm} = \kappa' \pm \eta_{\pm}\Pi'.$$

The EMFF in the impulse approximation is given by





$$P^{\mu}F_{+}(q^2) = -ie_q \mathrm{Tr}_{\mathrm{cD}} \int d\underline{l}\, \overline{\Gamma}\,(\kappa', -\Pi')\, S_{\mathrm{F}}(\kappa'_{+})\Gamma^{\mu}_{+\mathrm{EM}}(\kappa_{+}, \kappa'_{+}) S_{\mathrm{F}}(\kappa_{+})\Gamma(\kappa, \Pi) S_{\mathrm{F}}(\kappa_{-}),$$

$$P^{\mu}F_{-}(q^2) = -ie_{\bar{q}} \mathrm{Tr}_{\mathrm{cD}} \int d\underline{l}\, S_{\mathrm{F}}(\kappa'_{-})\overline{\Gamma}(\kappa', -\Pi') S_{\mathrm{F}}(\kappa_{+})\Gamma(\kappa, \Pi) S_{\mathrm{F}}(\kappa_{-})\Gamma^{\mu}_{-\mathrm{EM}}(\kappa'_{-}, \kappa_{-}),$$

with e_q and $e_{\bar{q}}$ being the charges of the valence quark and antiquark in units of the elementary charge [P. Maris and P. C. Tandy, Phys. Rev. C 61, 045202 (2000), Phys. Rev. C 62, 055204 (2000)].

EMFF for the pion with an Ansatz vertex

The quark-photon vertex is given by the Ball–Chiu vertex plus a transverse Ansatz:

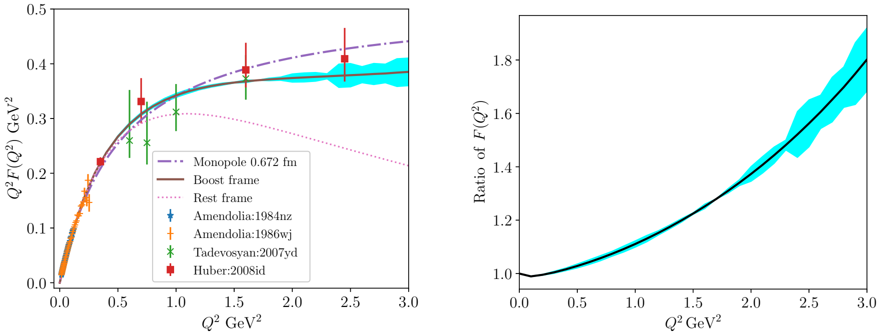
$$\Gamma^\mu_\pm(k,p) = \frac{A(k^2)+A(p^2)}{2}\,\gamma^\mu + \frac{A(k^2)-A(p^2)}{k^2-p^2}\,\frac{t^\mu \not{t}}{2} + \frac{B(k^2)-B(p^2)}{k^2-p^2}\,t^\mu + \Gamma^\mu_{\mathrm{T}}(k,p),$$

$$\Gamma^\mu_{\mathrm{T}}(q-Q/2, q+Q/2) = \left(\gamma^\mu - Q^\mu \not{Q}/Q^2\right)\frac{N_\rho}{1+q^4/\omega^4}\,\frac{f_\rho Q^2}{m_\rho(m_\rho^2-Q^2)}\,e^{-\alpha(m_\rho^2-Q^2)}.$$

N_ρ	ω	f_ρ	m_ρ	α
6.0405	0.66 GeV	201 MeV	0.875 GeV	0.1 $(\mathrm{GeV})^{-2}$

Table: Parameters for the transverse Ansatz of the quark-photon vertex. The normalization N_ρ is determined from the decay constant of the ρ meson f_ρ.

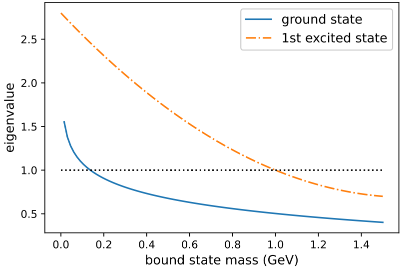
EMFF for the pion with an Ansatz vertex

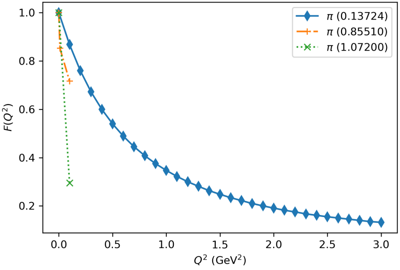


EMFF from the rest-frame pion BSA is significantly less than that with full kinematics for large Q^2.

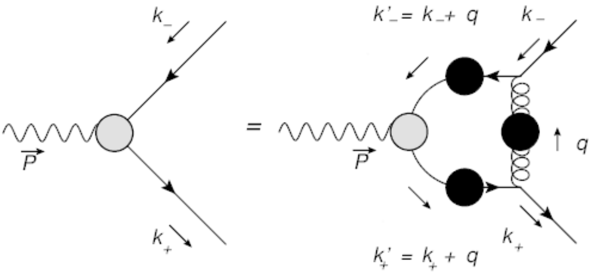
Additional results for the pion

- Animation for the ground-state pion BSA in the moving frame.
- EMFF for excited states of pion.





Homogeneous BSE for vector mesons



The BSE for the Bethe–Salpeter amplitude (BSA) of vector mesons is given by

$$\Gamma^{\mu}(t,Q) = -iC_{\mathrm{F}} \int d\underline{q}\, \gamma^{\lambda} S_{\mathrm{F}}^{+}(k'_{+}) \Gamma^{\mu}(t',Q) S_{\mathrm{F}}^{-}(k'_{-}) \gamma^{\nu} D_{\lambda\nu}(q), \tag{3}$$

with Q^{μ} being the momentum of the bound state flowing into the amplitude. We have defined $k_{\pm} = t \pm \eta_{\pm} Q$, $k'_{\pm} = t' \pm \eta_{\pm} Q$, and redefined $q = t' - t$.

Trace-orthogonal transverse vectors

After defining $\sigma^\mu(q) = (\gamma^\mu \not{q} - \not{q}\gamma^\mu)/2$, $\sigma(t,q) = (\not{t}\not{q} - \not{q}\not{t})/2$, and $\Delta(t,q) = t^2 - (t\cdot q)^2/q^2$, the following set of vectors transverse with respect to q^μ

$$T_1^\mu(t,q) = (t^\mu - q^\mu t\cdot q/q^2)\,\mathbb{1}, \tag{4a}$$
$$T_2^\mu(t,q) = \not{T}_1(t,q)\,T_1^\mu(t,q)/\Delta(t,q) - T_3^\mu(k,p)/3, \tag{4b}$$
$$T_3^\mu(t,q) = \gamma^\mu - q^\mu \not{q}/q^2, \tag{4c}$$
$$T_4^\mu(t,q) = -\sigma(t,q)\,T_1^\mu(t,q)/[2\Delta(t,q)] + T_5^\mu(t,q)/6, \tag{4d}$$
$$T_5^\mu(t,q) = \sigma^\mu(q), \tag{4e}$$
$$T_6^\mu(t,q) = \not{q}\,T_1^\mu(t,q), \tag{4f}$$
$$T_7^\mu(t,q) = [T_1^\mu(t,q) - T_3^\mu(t,q)\not{T}_1(t,q)]/2, \tag{4g}$$
$$T_8^\mu(t,q) = -T_7^\mu(t,q)\not{q}. \tag{4h}$$

is trace orthogonal:

$$\mathrm{Tr}\, T_i^\mu(t,q)\,T_{\mu j}(t,q) = \mathrm{diag}\big\{4\Delta(t,q),\ 8/3,\ 12,\ -2q^2/3,\ -12q^2,\ 4q^2\Delta(t,q),\ -2\Delta(t,q),\ -2q^2\Delta(t,q)\big\}_{ij}. \tag{5}$$

Contribution from quark propagators

Applying Dirac bases in Eq. (4) to decompose the BSA into scalar components:

$$\Gamma^{\mu}(t,Q)=\sum_{j=1}^{8}F_j(t^2,t\cdot Q)\,T_j^{\mu}(t,Q). \tag{6}$$

The Bethe–Salpeter wave function (BSWF) defined as

$$\Psi^{\mu}(t,Q)=S_{\mathrm{F}}^{+}(k_+)\Gamma^{\mu}(t,Q)S_{\mathrm{F}}^{-}(k_-) \tag{7}$$

has the following similar decomposition

$$\Psi^{\mu}(t,Q)=\sum_{j=1}^{8}G_j(t^2,t\cdot Q)\,T_j^{\mu}(t,Q). \tag{8}$$

Scalar functions of the BSWF is related to those of the BSA by

$$G_i(t^2,t\cdot Q)=\sum_{j=1}^{8}\mathbb{M}_{ij}(t^2,t\cdot Q,\eta_+,\eta_-)\,F_j(t^2,t\cdot Q), \tag{9}$$

with $\mathbb{M}_{ij}(t^2,t\cdot Q,\eta_+,\eta_-)$ being a matrix in the component space.

Contribution from quark propagators

Due to the multiplications of two quark propagators, this matrix can be factorized as follows:

$$\mathbb{M}(t^2, t\cdot Q, \eta_+, \eta_-) = \mathbb{M}^+(t^2, t\cdot Q, \eta_+)\,\mathbb{M}^-(t^2, t\cdot Q, \eta_-) \tag{10}$$

with

$$\mathbb{M}^+_{ij}(t^2, t\cdot Q, \eta_+) = \frac{\mathrm{Tr_D}\ T_{i\nu}(t,Q) S_\mathrm{F}^+(k_+) T_j^\nu(t,Q)}{\mathrm{Tr_D}\ T_{i\mu}(t,Q) T_i^\mu(t,Q)}$$

and

$$\mathbb{M}^-_{ij}(t^2, t\cdot Q, \eta_-) = \frac{\mathrm{Tr_D}\ T_{i\nu}(t,Q) T_j^\nu(t,Q) S_\mathrm{F}^-(k_-)}{\mathrm{Tr_D}\ T_{i\mu}(t,Q) T_i^\mu(t,Q)}.$$

Contribution from quark-gluon interactions

Meanwhile the gluon propagator in the Landau gauge is given by

$$D_{\lambda\nu}(q) = (g_{\lambda\nu} - q_\lambda q_\nu/q^2)\,\mathcal{G}(q^2), \tag{11}$$

with $\mathcal{G}(q^2)$ being the Schwinger function. Let us then introduce the scalar composition of the following factors in the BSE:

$$\gamma^\lambda \Psi^\mu(t', Q)\gamma^\nu(g_{\lambda\nu} - q_\lambda q_\nu/q^2) = \sum_{j=1}^{8} H_j(t^2, t\cdot Q, t'^2, t'\cdot Q)\, T_j^\mu(t, Q). \tag{12}$$

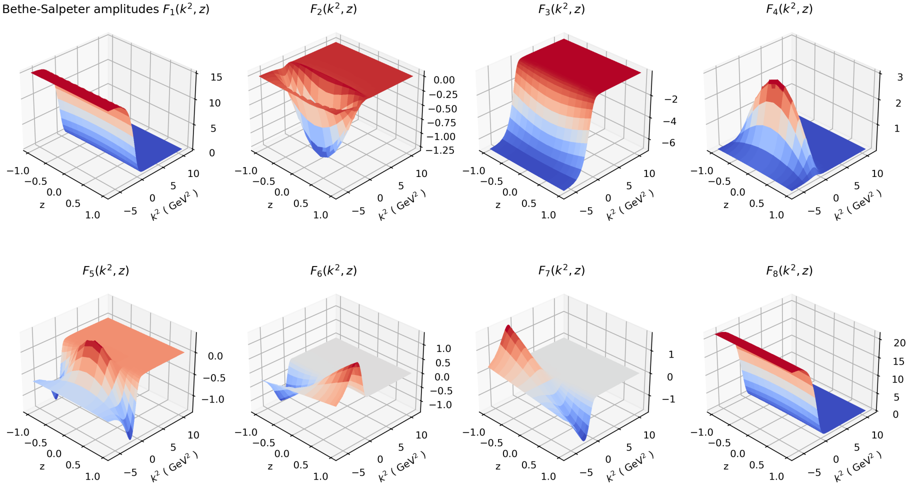
The scalar function $H_i(t^2, t\cdot Q, t'^2, t'\cdot Q)$ after applying trace-orthogonal vectors is $H_i(t^2, t\cdot Q, t'^2, t'\cdot Q) = \sum_{k=1}^{8} \mathbb{T}_{ik}(t^2, t\cdot Q, t'^2, t'\cdot Q)\, G_k(t'^2, t\cdot Q)$, with

$$\mathbb{T}_{ij}(t^2, t\cdot Q, t'^2, t'\cdot Q) = \mathbb{E}_{ij}(t^2, t\cdot Q, t'^2, t'\cdot Q)/[\mathrm{Tr}_{\mathrm{D}}\, T_{i\mu}(t, Q)\, T_i^\mu(t, Q)].$$

$$\mathbb{E}_{jk}(t^2, t\cdot Q, t'^2, t'\cdot Q) = \mathrm{Tr}_{\mathrm{D}}\, T_{j\nu}(t, Q)\,\gamma^\lambda\, T_k^\nu(t', Q)\gamma^\rho(g_{\lambda\rho} - q_\lambda q_\rho/q^2). \tag{13}$$

is symmetric with respect to $(j,\ t^2,\ t\cdot Q) \leftrightarrow (k,\ t'^2,\ t'\cdot Q)$.

BSA for ρ meson in the rest frame



- With same model parameters for the pion, the rho meson mass is 730 MeV.
- Applying the experimental mass of 775 MeV moves the eigenvalue to 1.09.

Inhomogeneous BSE for quark-photon vertex

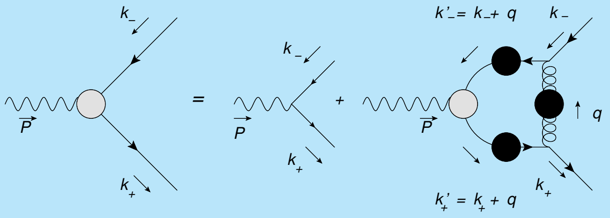


Figure: Inhomogeneous BSE for the quark-vector-boson vertex.

The quark-photon vertex satisfies its inhomogeneous BSE as illustrated

$$\Gamma^{\mu}_{\mathrm{V}}(k_-,k_+) = Z_2\gamma^{\mu} - iC_{\mathrm{F}} \int d\underline{q}\, \gamma^{\lambda} S_{\mathrm{F}}(k'_+)\Gamma^{\mu}_{\mathrm{V}}(k'_-,k'_+)S_{\mathrm{F}}(k'_-)\gamma^{\nu}\, \mathcal{D}_{\lambda\nu}(q), \quad (14)$$

with $k_\pm = t + \eta_\pm Q$ and $k'_\pm = k_\pm + q$.

- The Ball–Chiu vertex gives the solution for the longitudinal part of the quark-photon vertex.

Transverse inhomogeneous term

We therefore convert the inhomogeneous BSE into the corresponding equation for the transverse vertex. Substituting the Ball–Chiu vertex into the equation gives

$$\Gamma^{\mu}_{\mathrm{VT}}(t,Q) = G^{\mu}_{\mathrm{T}}(t,Q) - iC_{\mathrm{F}} \int d\underline{q}\, \gamma^{\lambda} S_{\mathrm{F}}(k'_{+}) \Gamma^{\mu}_{\mathrm{VT}}(t,Q) S_{\mathrm{F}}(k'_{-}) \gamma^{\nu}\, \mathcal{D}_{\lambda\nu}(q), \quad (15a)$$

where we have defined the transverse inhomogeneous term as

$$G^{\mu}_{\mathrm{T}}(t,Q) = Z_2\gamma^{\mu} - \Gamma^{\mu}_{\mathrm{BC}}(k_{-},k_{+}) - iC_{\mathrm{F}} \int d\underline{q}\, \gamma^{\lambda} S_{\mathrm{F}}(k'_{+}) \Gamma^{\mu}_{\mathrm{BC}}(k'_{-},k'_{+}) S_{\mathrm{F}}(k'_{-}) \gamma^{\nu}\, \mathcal{D}_{\lambda\nu}(q). \quad (15b)$$

We could show that $Q_{\mu}\, G^{\mu}_{\mathrm{T}}(t,Q) = 0$. This term can be decomposed into scalar components as

$$G^{\mu}_{\mathrm{T}}(t,Q) = \sum_{j=1}^{8} R_j(t^2, t\cdot Q, Q^2)\, T^{\mu}_{j}(t,Q). \quad (16)$$

Similarly for the transverse part of the quark-photon vertex:

$$\Gamma^{\mu}_{\mathrm{VT}}(t,Q) = \sum_{j=1}^{8} U_j(t^2, t\cdot Q, Q^2)\, T^{\mu}_{j}(t,Q). \quad (17)$$

The scalar functions for the transverse inhomogeneous term are explicitly given by

$$R_i(t^2, t\cdot Q, Q^2) = \overline{R}_i(t^2, t\cdot Q, Q^2) - \frac{iC_\mathrm{F}}{\mathrm{Tr_D}\ T_{i\mu}(t,Q)T_i^\mu(t,Q)} \sum_{j=0}^{8} \sum_{k=1,2,3,6} \int d\underline{q}$$
$$\times\, \mathbb{E}_{ij}(t^2, t\cdot Q, t'^2, t'\cdot Q)\, \mathcal{G}(q^2)\, \mathbb{M}_{jk}(t'^2, t'\cdot Q, \eta_+, \eta_-) r_k(t'^2, t'\cdot Q, Q^2), \quad (18a)$$

with

$$\overline{R}_i(t^2, t\cdot Q, Q^2) = -r_i(t^2, t\cdot Q, Q^2) \quad (18b)$$

for $i \in \{1, 2, 6\}$,

$$\overline{R}_3(t^2, t\cdot Q, Q^2) = Z_2 - r_3(t^2, t\cdot Q, Q^2), \quad (18c)$$

and $\overline{R}_i(t^2, t\cdot Q, Q^2) = 0$ for $i \in \{4, 5, 7, 8\}$.

$$r_1(t^2, t\cdot Q, Q^2) = [B(k_-^2) - B(k_+^2)]/(k_-^2 - k_+^2),$$
$$r_2(t^2, t\cdot Q, Q^2) = \frac{A(k_-^2) - A(k_+^2)}{2(k_-^2 - k_+^2)}\Delta(t, Q),$$
$$r_3(t^2, t\cdot Q, Q^2) = \frac{1}{2}\left[A(k_-^2) + A(k_+^2) + \frac{A(k_-^2) - A(k_+^2)}{3(k_-^2 - k_+^2)}\Delta(t, Q)\right],$$
$$r_6(t^2, t\cdot Q, Q^2) = -[A(k_-^2) - A(k_+^2)]/(2Q^2).$$

Inhomogeneous BSE for the quark-photon vertex

In order to solve for the quark-photon vertex while computing the EMFF, we take the following 2 steps.

1. Iterative solver for spacelike relative momentum.
2. Single-step computation with complex-valued relative momentum.

EMFF for the pion with quark-photon vertex from inhomogeneous BSE

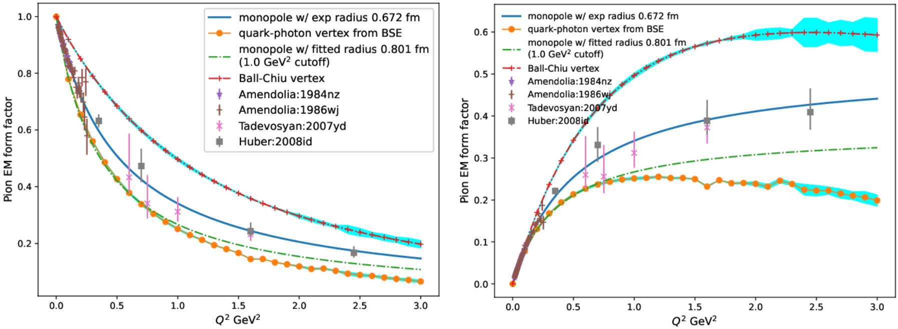
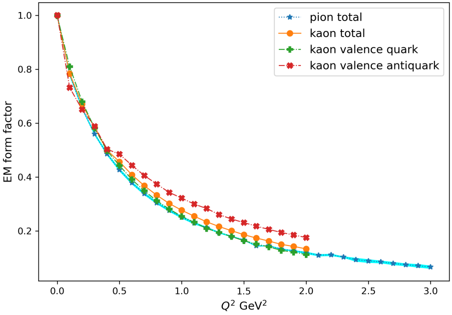
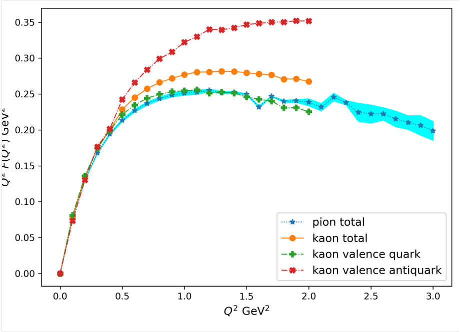


Figure: EM form factor of the pion applying the quark-photon vertex solved from the inhomogeneous BSE. The error band corresponds to the uncertainty in the normalization. The blue solid line and the green dot-dash line correspond to the monopole form an experimental charge radius of 0.672 fm and a fitted radius of 0.801 fm.

EMFFs for the pion and the kaon





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