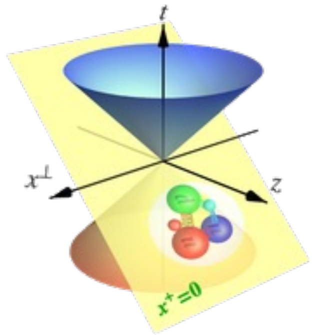
In-medium dressed quark evolution in a light-front Hamiltonian approach

Meijian Li

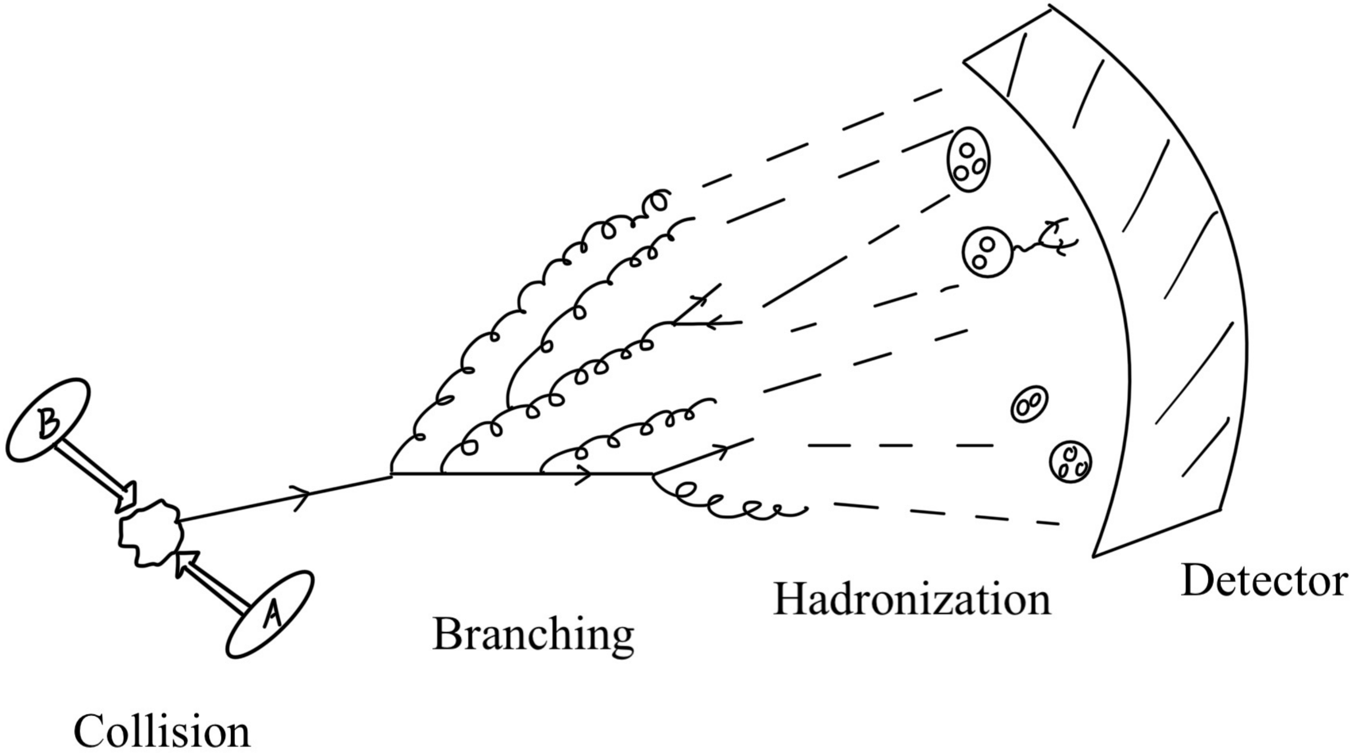
Instituto Galego de Física de Altas Enerxías, Universidade de Santiago de Compostela (IGFAE-USC), Spain

@ LFQCD Seminar, August 14, 2024



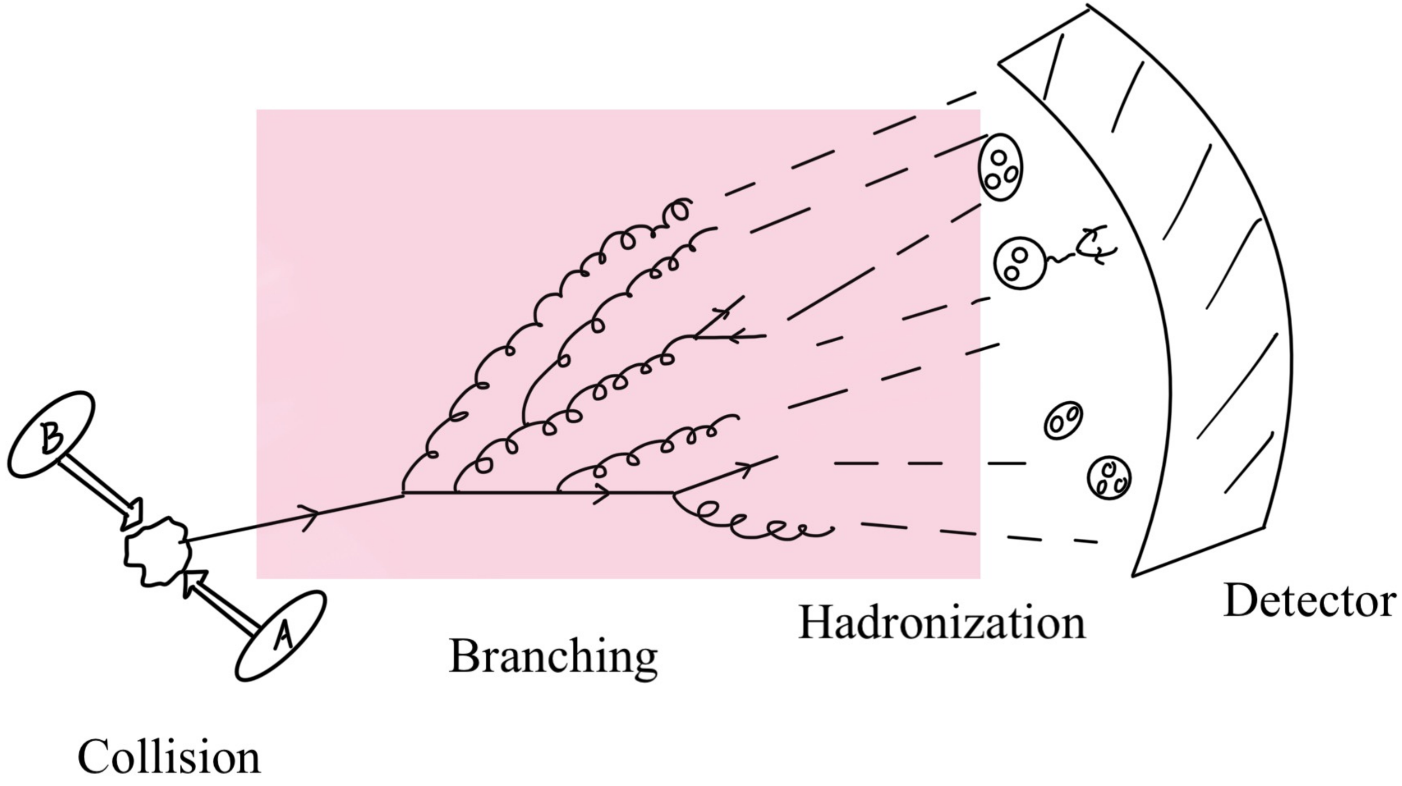
What is a jet?

In high-energy collisions, a jet is a collimated beam of particles produced by the splitting of a common ancestor (quark or gluon).



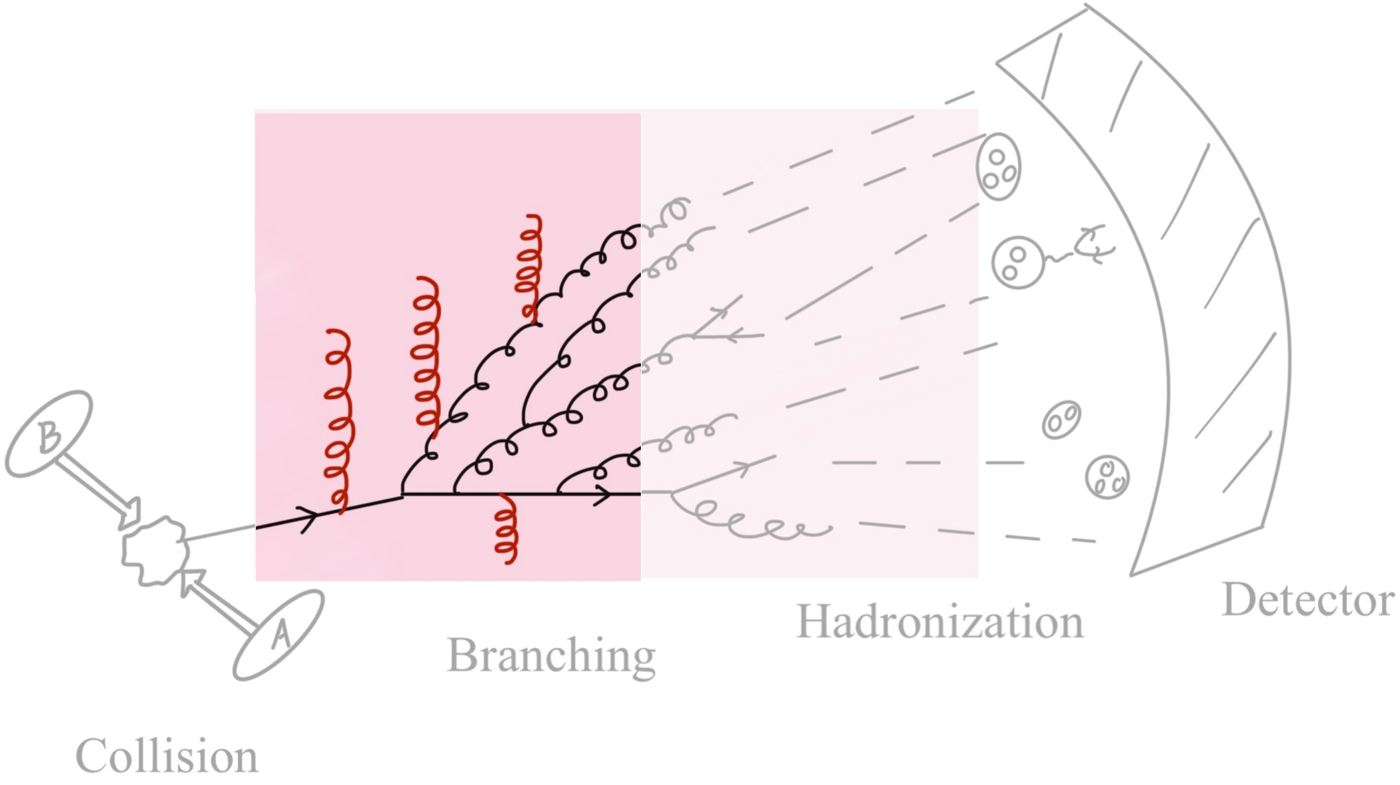
What is a jet?

A probe of matter, a tool to understand interaction.



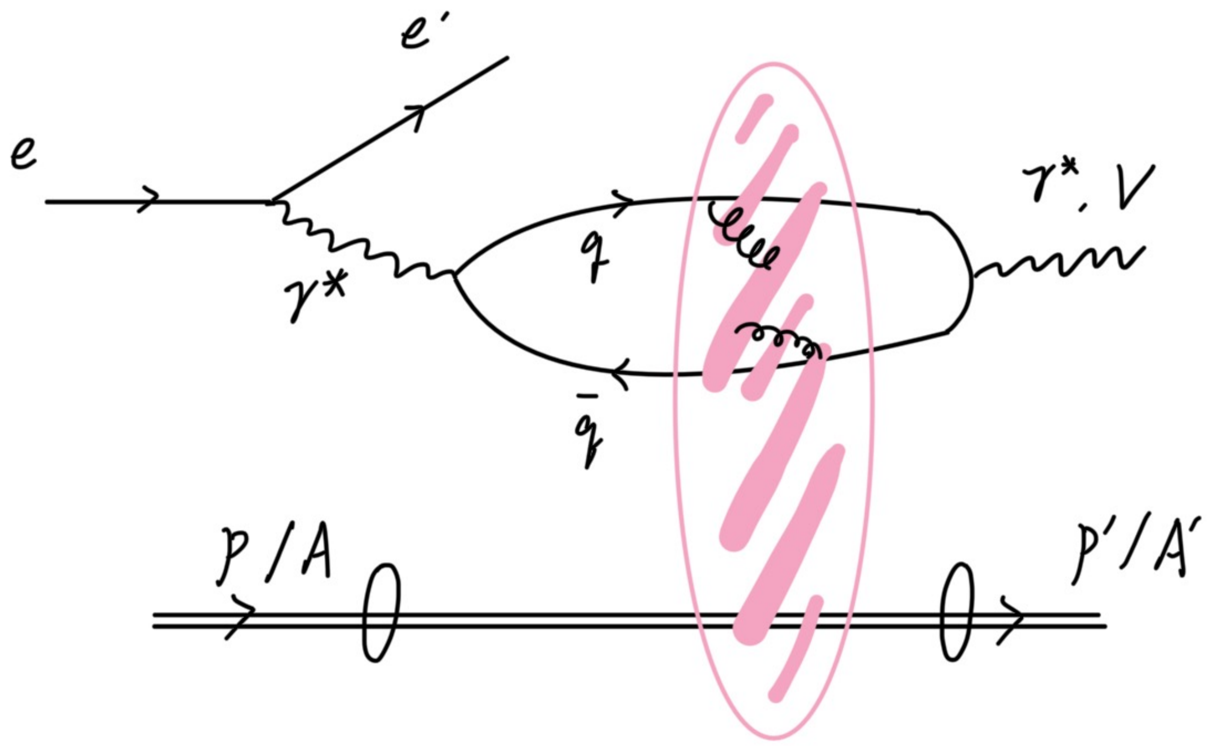
What is a jet?

An energetic QCD state that evolves and interacts.



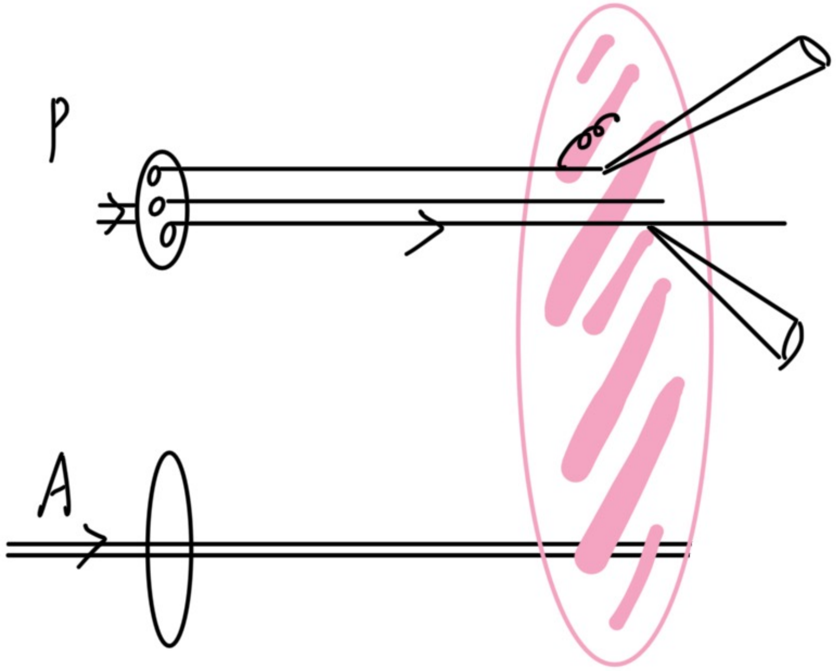
Quark jet scattering off a color field in:

Deep inelastic scattering e+p/A



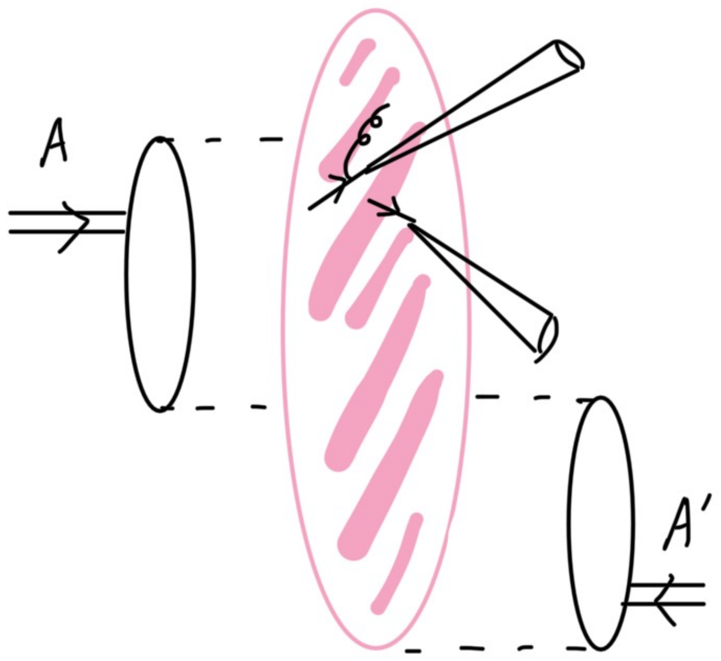
Quark jet scattering off a color field in:

Proton nucleus scattering p+A



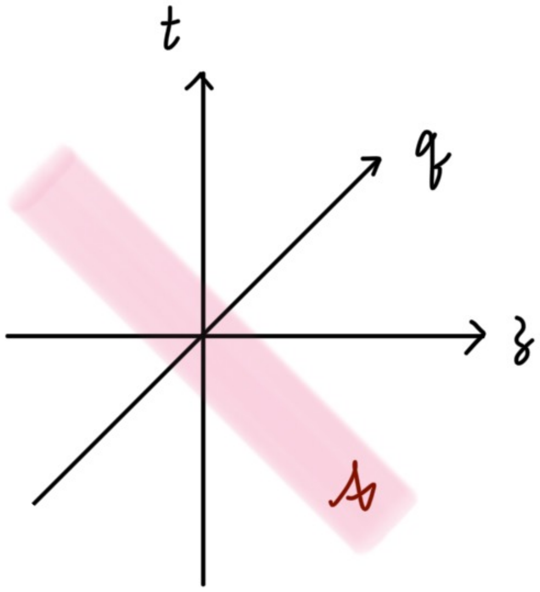
Quark jet scattering off a color field in:

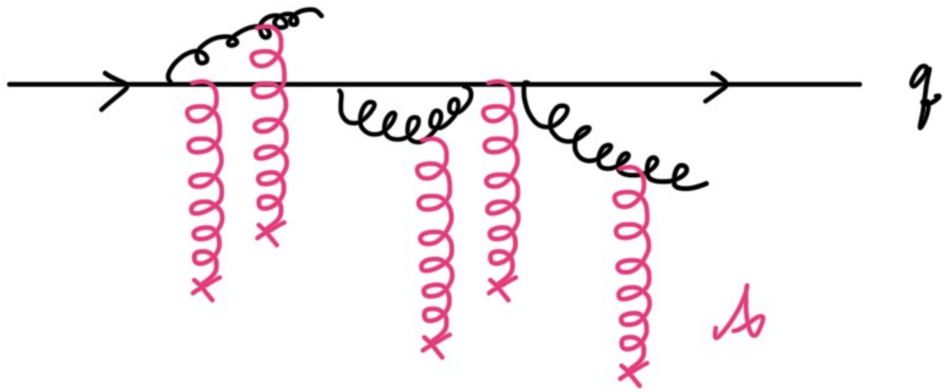
Heavy ion collisions A+A



Quark jet scattering off a color field

- The fundamental process



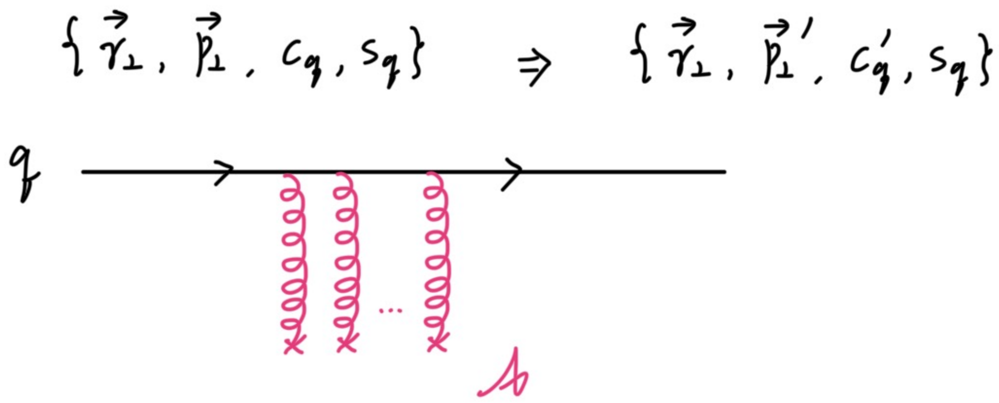


At high energy, the target has many gluons: $\mathcal{A}_\mu \gg 1/g$
⇒ described as a classical gluon field (Color Glass Condensate, MV model)

What has been established and approximated?

○ Eikonal limit

Quark is infinitely energetic: $p^+ \equiv p^0 + p^z = \infty$



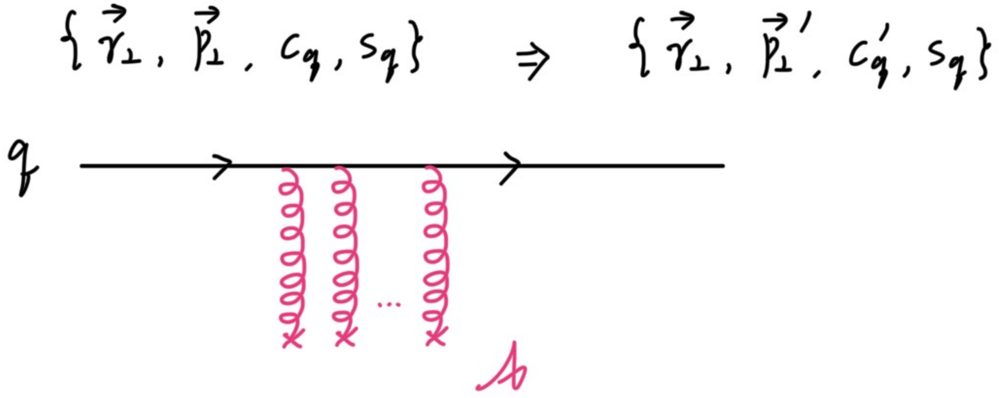
⇒ Wilson line: eikonal scattering amplitude, resummation of $\mathcal{A}_\mu$ in the path-ordered exponential

What has been established and approximated?

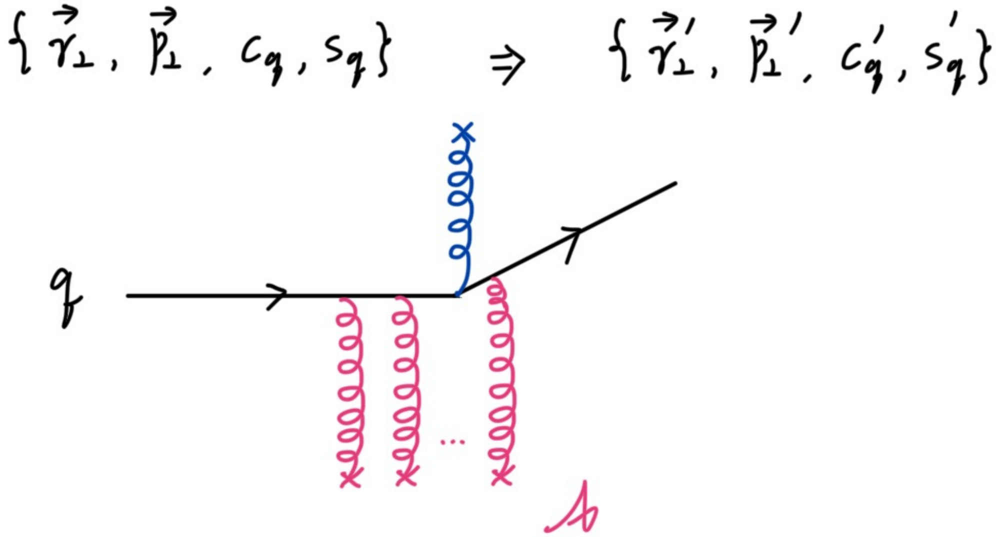
○ **Eikonal limit**

Quark is infinitely energetic:

$$p^+ \equiv p^0 + p^z = \infty$$



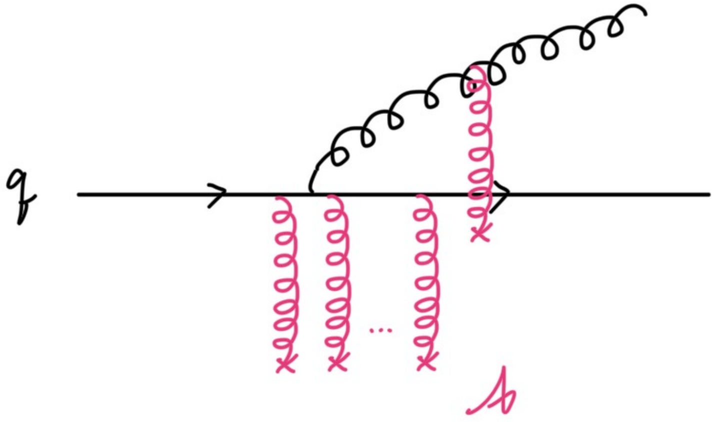
⇒ Sub-eikonal effects

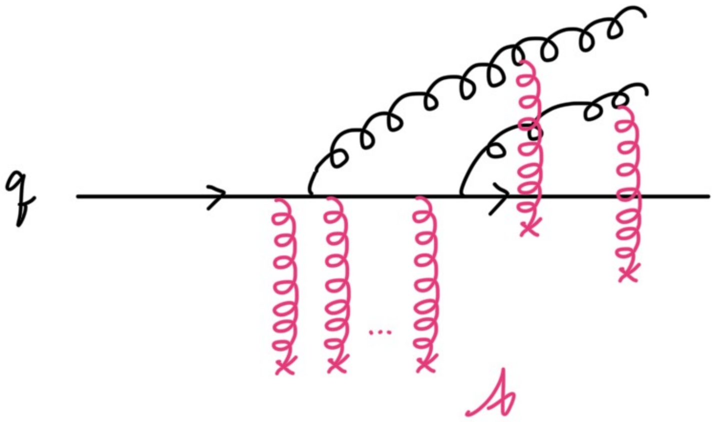


What has been established and approximated?

- **Perturbative approach**

  Expansion in powers of the coupling: one gluon emission at NLO, and two gluons at NNLO

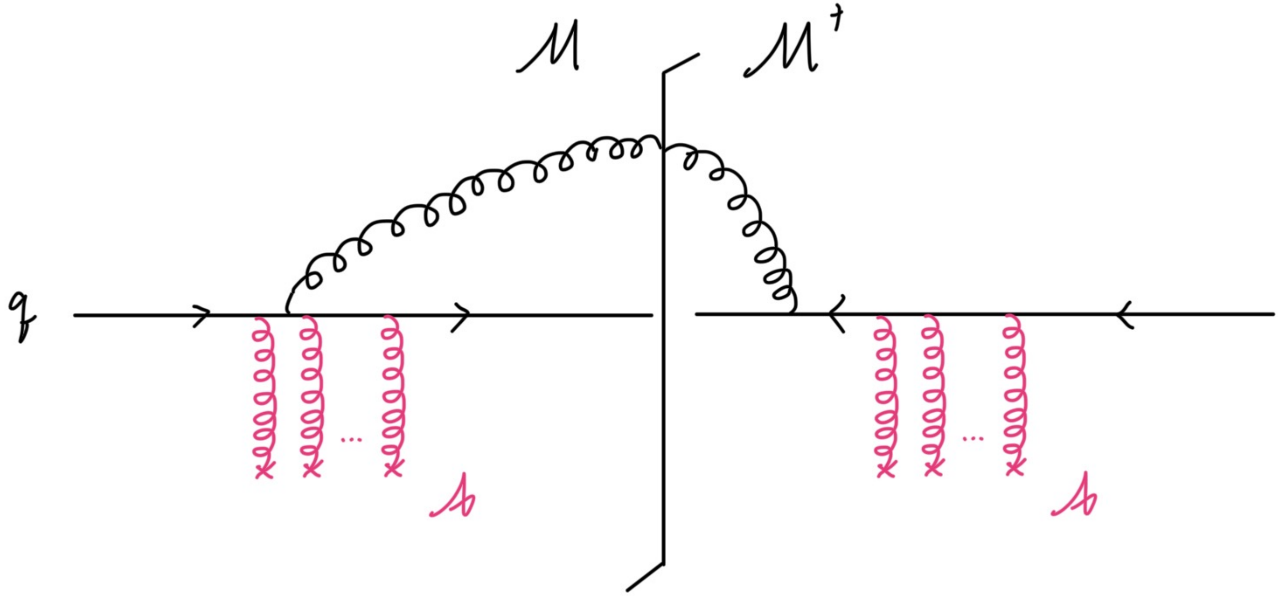




What has been established and approximated?

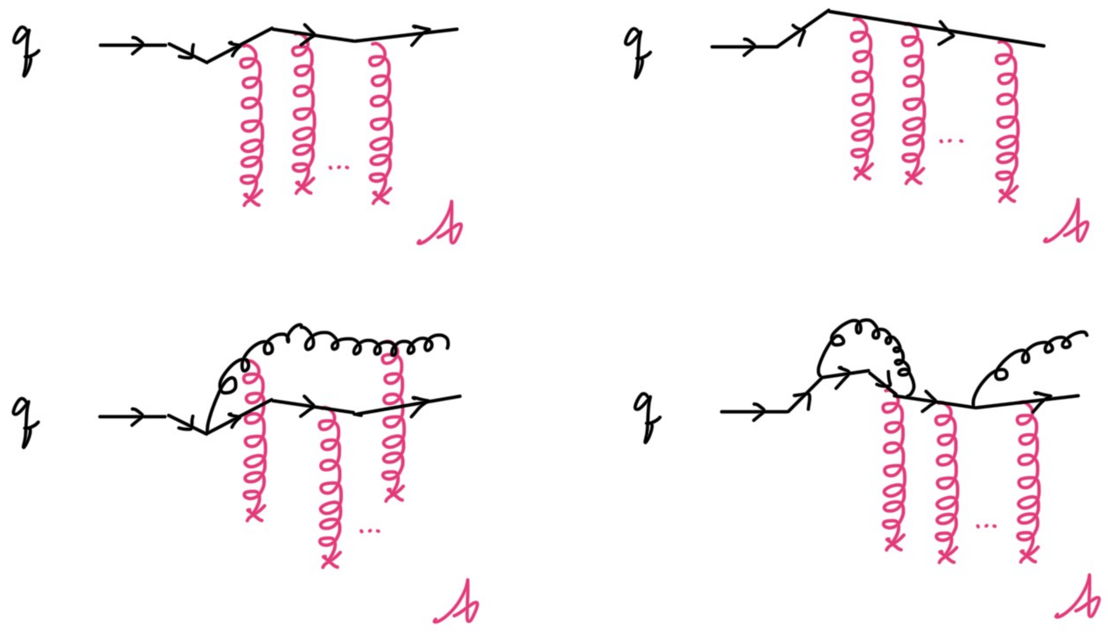
- **Perturbative approach**

  Calculation is on the probability level



What are the differences here?

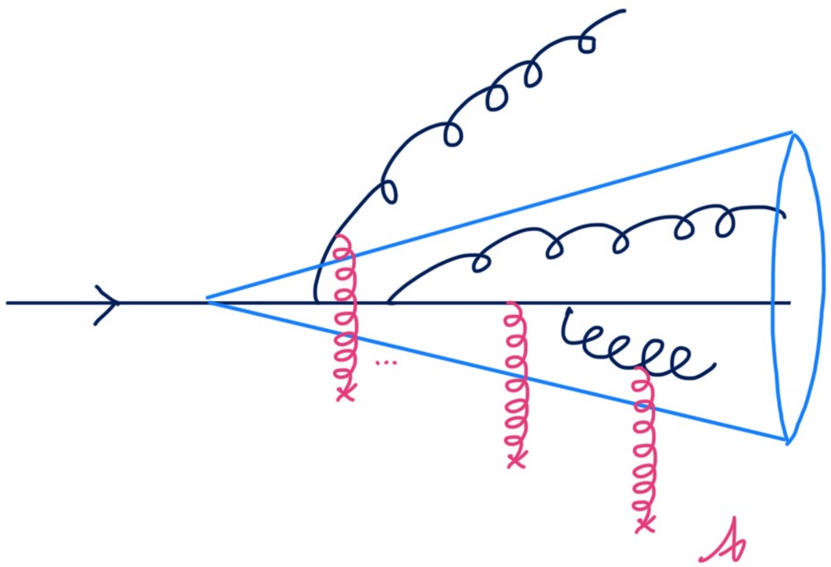
- **Non-perturbative approach**
  - ⇒ beyond eikonal
- **Amplitude level computation**
  - ⇒ jet is tracked as an evolving quantum state
- **Real-time simulation**
  - ⇒ accessibility to intermediate state



What are the differences here?

- **Dressed quark states**

  ⇒ distinguish jet intrinsic and external gluons

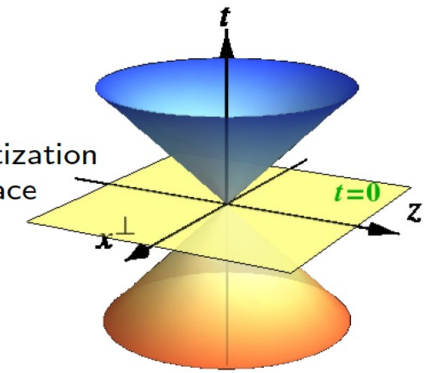
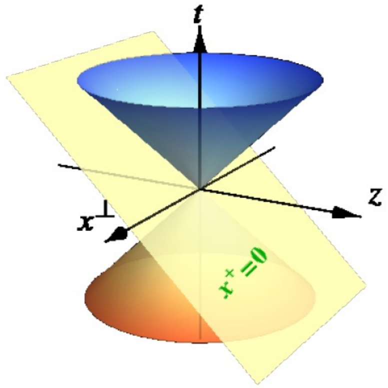
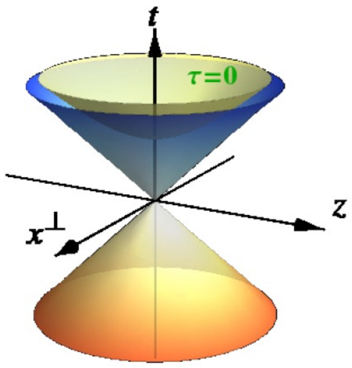


Outline

- Methodology
  - The light-front Hamiltonian approach: BLFQ & tBLFQ
- Application to jet physics
  1. Dressed quark
  2. In-medium dressed quark evolution
- Summary and outlooks

Light-front Hamiltonian approach: BLFQ & tBLFQ[1]

➢ Light-front dynamics

	instant form	front form	point form
time variable	$t = x^0$	$x^+ \triangleq x^0 + x^3$	$\tau \triangleq \sqrt{t^2 - \vec{x}^2 - a^2}$
quantization surface			
Hamiltonian	$H = P^0$	$P^- \triangleq P^0 - P^3$	P^μ
kinematical	$\vec{P}, \vec{J}$	$\vec{P}^\perp, P^+, \vec{E}^\perp, E^+, J^-$	$\vec{J}, \vec{K}$
dynamical	$\vec{K}, P^0$	$\vec{F}^\perp, P^-$	$\vec{P}, P^0$
dispersion relation	$p^0 = \sqrt{\vec{p}^2 + m^2}$	$p^- = (\vec{p}_\perp^2 + m^2)/p^+$	$p^\mu = m v^\mu \; (v^2 = 1)$

1. J. P. Vary, H. Honkanen, Jun Li, P. Maris, S. J. Brodsky, A. Harindranath, G. F. de Teramond, P. Sternberg, E. G. Ng, C. Yang., Phys. Rev. C81, 035205 (2010); X. Zhao, A. Ilderton, P. Maris, and J. P. Vary, Phys. Rev. D88, 065014 (2013).

Light-front Hamiltonian approach: BLFQ & tBLFQ[1]

➤ Hamiltonian formalism

- Bound states: eigenstates of the light-front Hamiltonain

$$P^-|\phi\rangle = P^-_\phi|\phi\rangle$$

$$\Updownarrow$$

$$\underbrace{(P^-P^+ - \vec{P}_\perp^2)}_{H_{\mathrm{LC}}}|\phi\rangle = M^2|\phi\rangle$$

- Time-dependent process: the state obeys the time-evolution equation

$$\frac{1}{2}P^-(x^+)|\psi(x^+)\rangle = i\frac{\partial}{\partial x^+}|\psi(x^+)\rangle$$

Light-front Hamiltonian approach: BLFQ & tBLFQ[1]

➢ Basis representation

- Optimal basis encodes certain symmetries of the system, and it is the key to computational efficiency

$$|\psi; x^+\rangle = \sum_\beta c_\beta(x^+)|\beta\rangle$$

Operators

$$\begin{pmatrix} \langle 1|U|1\rangle & \langle 1|U|2\rangle & \dots & \langle 1|U|n\rangle \\ \langle 2|U|1\rangle & \langle 2|U|2\rangle & \dots & \langle 2|U|n\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle n|U|1\rangle & \langle n|U|2\rangle & \dots & \langle n|U|n\rangle \end{pmatrix}$$

State

$$\begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix}$$

Light-front Hamiltonian approach: BLFQ & tBLFQ[1]

➤ Computational method

- **Basis Light-Front Quantization (BLFQ)**: the bound state is solved by diagonalizing the Hamiltonain matrix

$$H_{\mathrm{LC}} \to \begin{pmatrix} M_1^2 & & & \\ & M_2^2 & & \\ & & \ddots & \\ & & & M_2^2 \end{pmatrix}$$

Eignestates → LF wavefunctions
Eigenvalues → M^2

Light-front Hamiltonian approach: BLFQ & tBLFQ[1]

➢ Computational method

- **Basis Light-Front Quantization (BLFQ)**
- **time-dependent BLFQ (tBLFQ)**: the evolving state is solved by sequential matrix multiplications of the evolution operators

$$\begin{pmatrix} c_1(x^+) \\ c_2(x^+) \\ \vdots \\ c_n(x^+) \end{pmatrix} = \Big[\quad U_n \quad\Big] \cdots \Big[\quad U_2 \quad\Big]\Big[\quad U_1 \quad\Big] \begin{pmatrix} c_1(0) \\ c_2(0) \\ \vdots \\ c_n(0) \end{pmatrix}$$

$$U_k = \mathcal{T}_+ \exp\left[-\frac{\mathrm{i}}{2}\int_{x_{k-1}^+}^{x_k^+} dz^+ P^-(z^+)\right], \qquad x_n^+ = x^+$$

Light-front Hamiltonian approach: BLFQ & tBLFQ[1]

➢ Computational method

- Basis Light-Front Quantization (BLFQ)
- time-dependent BLFQ (tBLFQ)

✓ *First-principles*

✓ *Non-perturbative*

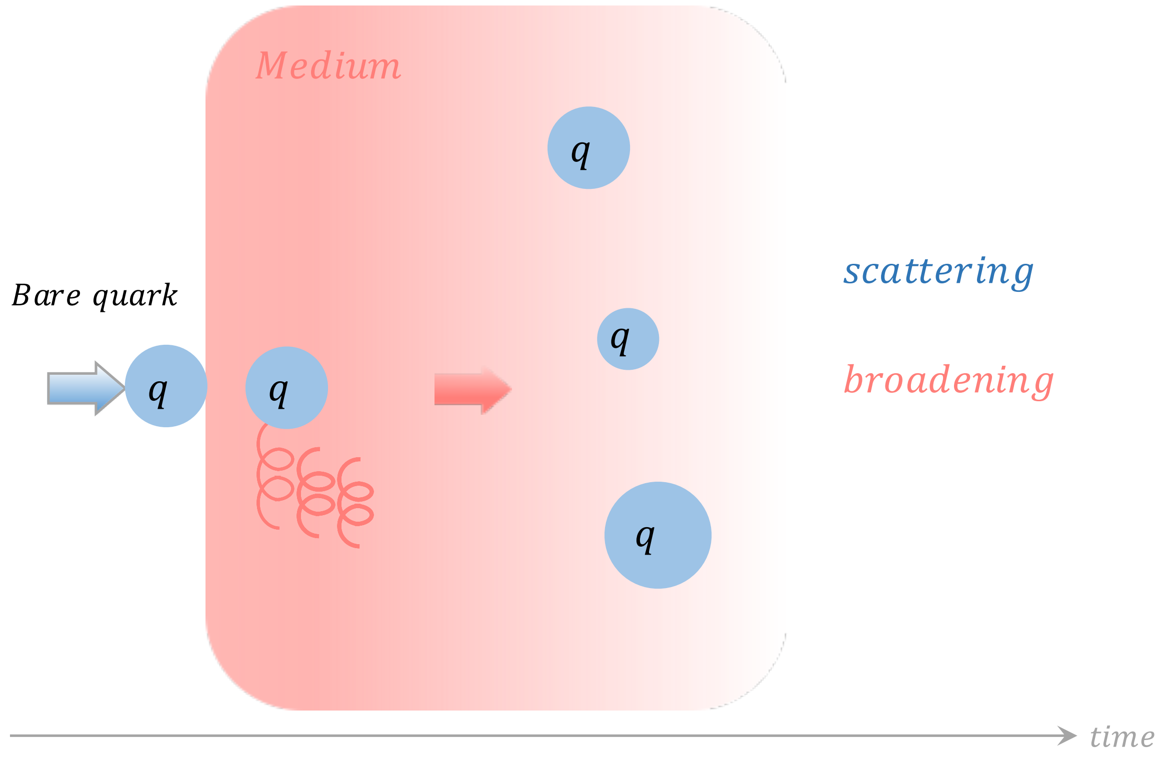
✓ *Fully quantum*

Outline

- ❑ Methodology
  - The light-front Hamiltonian approach: BLFQ & tBLFQ
- ❑ Application to jet physics
  1. Dressed quark
  2. In-medium dressed quark evolution
- ❑ Summary and outlooks

Applications of tBLFQ to Jet evolution

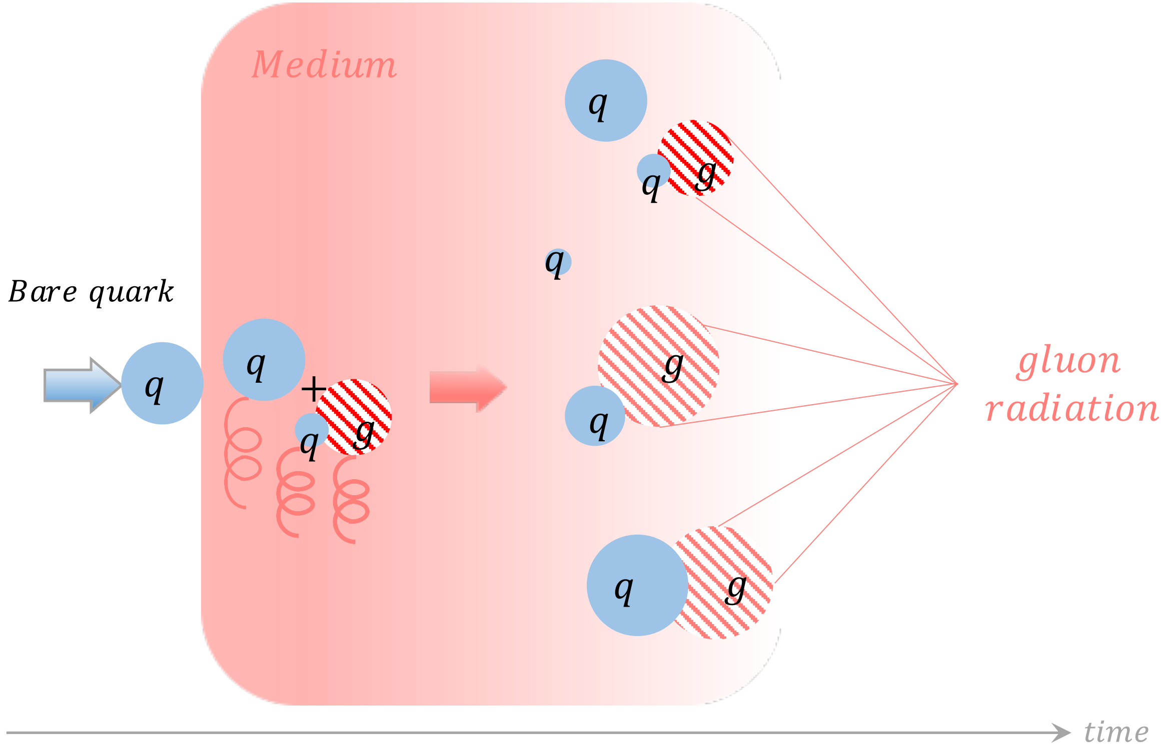
I. $|q\rangle$: quark jet scattering off a color field[1]



1. Phys.Rev.D 101(2020)7, 076016, ML, X. Zhao, P. Maris, G. Chen, Y. Li, K. Tuchin and J. P. Vary

Applications of tBLFQ to Jet evolution

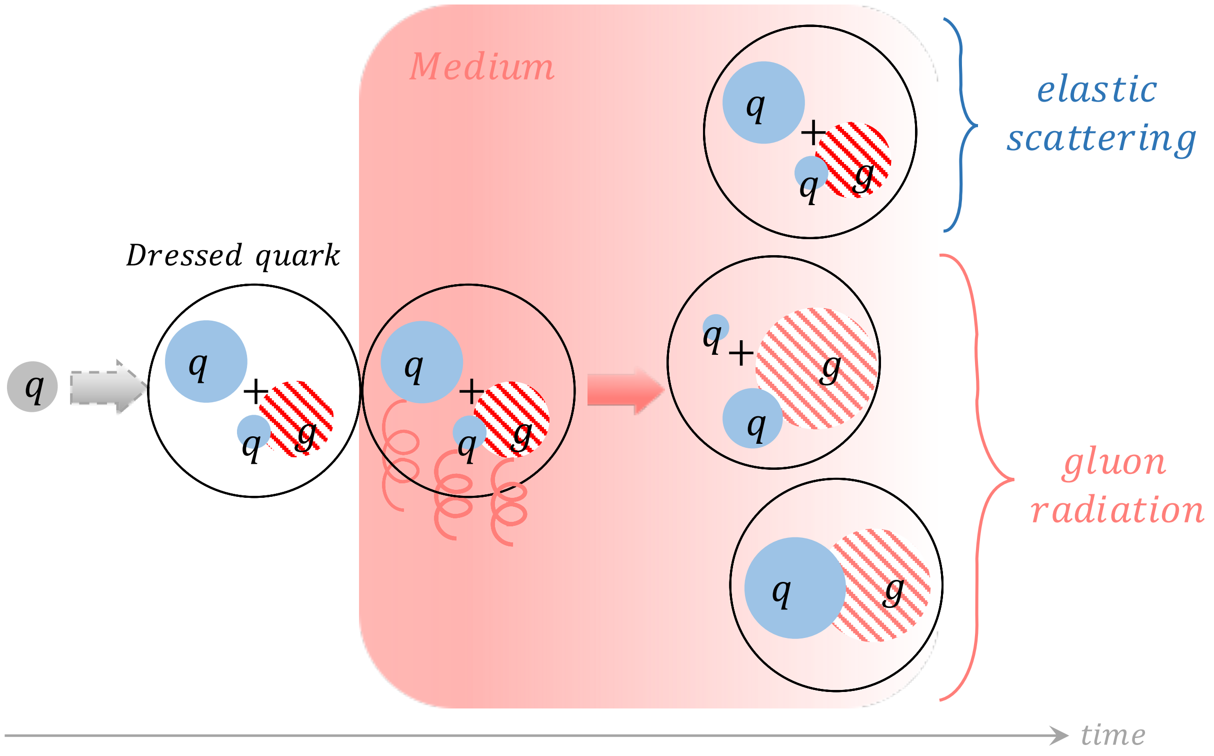
II. $|q\rangle + |qg\rangle$: quark jet scattering and gluon emission[1]



1. Phys.Rev.D 104 (2021) 5, 056014, ML, T. Lappi and X. Zhao; Phys.Rev.D 108 (2023) 3, 3, ML, T. Lappi, X. Zhao and C. A. Salgado

Applications of tBLFQ to Jet evolution

III. $|q\rangle + |qg\rangle$: dressed quark scattering and gluon emission



Applications of tBLFQ to Jet evolution

- **Basis representation:** discrete momentum states

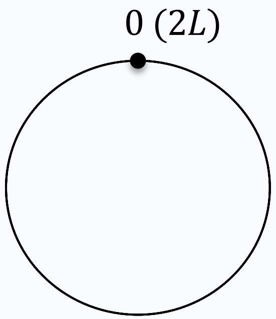
$$P_{\mathrm{KE}}^{-}|\beta\rangle = P_{\beta}^{-}|\beta\rangle,\ \beta_l = \{k_l^x, k_l^y, k_l^+, \lambda_l, c_l\}, (l = q, g)$$

$$|q\rangle: |\beta_q\rangle; \quad |qg\rangle: |\beta_{qg}\rangle = |\beta_q\rangle \otimes |\beta_g\rangle$$

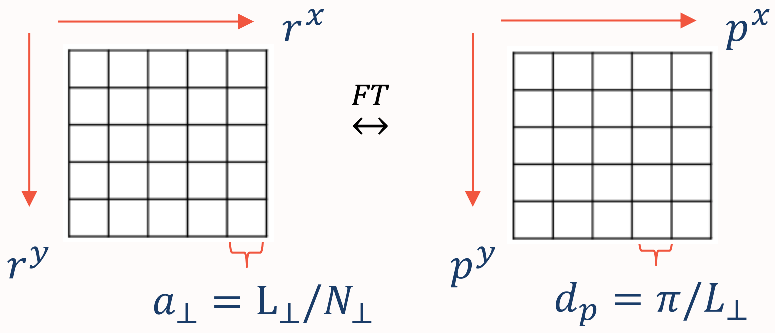
- The longitudinal space
  - $x^- = [0, 2L]$
  - $p_l^+ = \frac{2\pi}{L} k_l^+$

    $k_q^+ = \frac{1}{2}, \frac{3}{2}, \ldots, K + \frac{1}{2}$

    $k_g^+ = 1, 2, \ldots, K$



- The transverse coordinate space
  - $r_l^\perp = [-N_\perp, \ldots, N_\perp - 1]\, L_\perp / N_\perp$
  - $p_l^\perp = \frac{2\pi}{2L_\perp} k_l^\perp, k_l^\perp = -N_\perp, \ldots, N_\perp - 1$



Basis size: $N_{tot} = (2N_\perp)^2 \times 2 \times 3 + K \times (2N_\perp)^4 \times 4 \times 24$

Dressed quark

- **QCD eigenstates in** $|q\rangle + |qg\rangle$
  - The dressed quark state is described as the eigenstate of the light-front QCD Hamiltonian with the quark quantum numbers:

$$\mathcal{L}_{QCD} = -\frac{1}{4}\mathrm{F}_{\mathrm{a}}^{\mu\nu}F_{\mu\nu}^{a} + \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi \quad \rightarrow \quad \mathrm{P}_{QCD}^{-} = \mathrm{P}_{KE}^{-} + V_{qg}$$

$$P_{QCD}^{-}|\phi\rangle = P_{\phi}^{-}|\phi\rangle$$

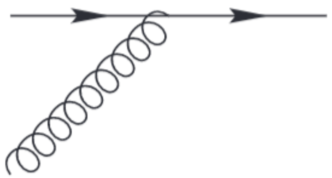
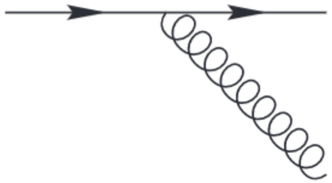
$$\Updownarrow$$

$$\underbrace{(P_{QCD}^{-}P^{+} - \vec{P}_{\perp}^{2})}_{H_{\mathrm{LC}}}|\phi\rangle = M^{2}|\phi\rangle$$

Dressed quark

- **QCD eigenstates in $|q\rangle + |qg\rangle$**
  - Sector-dependent mass renormalization

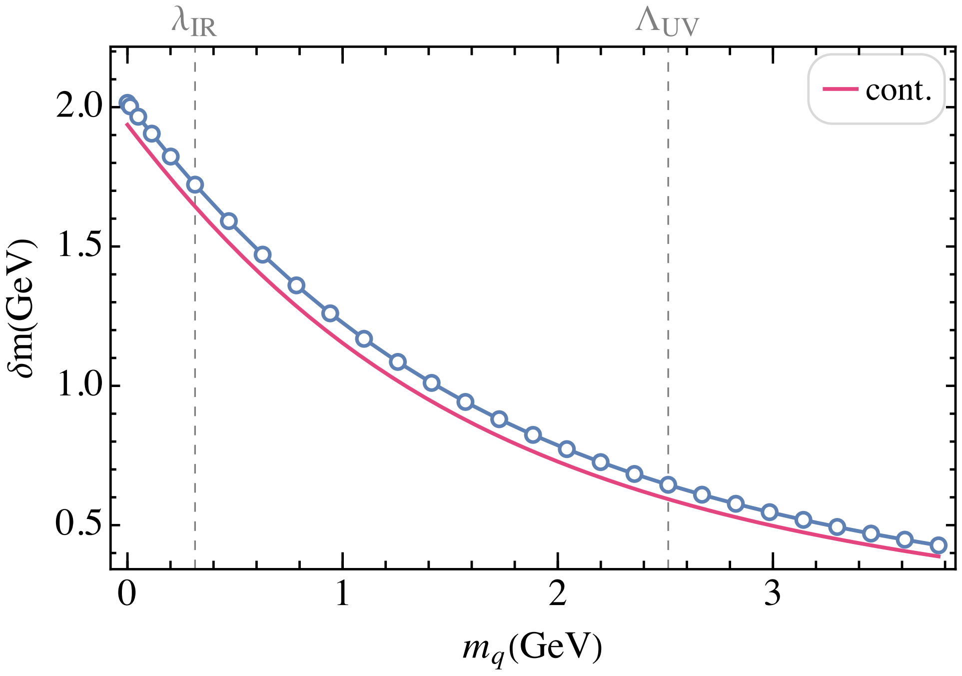
$$H_{\mathrm{LC}}(\delta m)|\phi\rangle = m_q^2|\phi\rangle$$

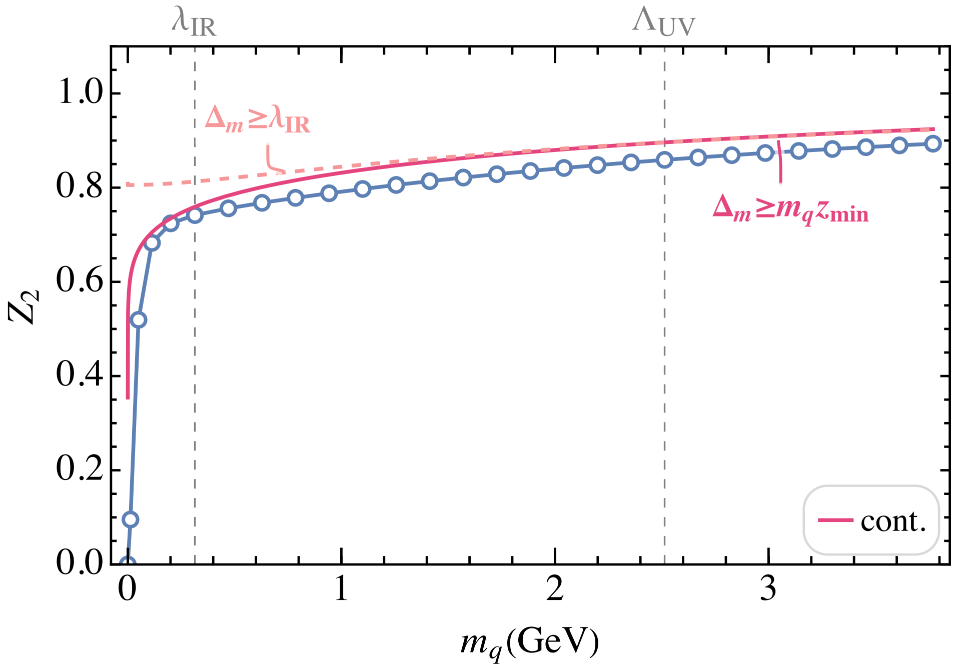
Fock sector	$\lvert q\rangle$	$\lvert qg\rangle$
$\langle q\rvert$	$\mathrm{P}^-_{KE}(m_Q = m_q + \delta m)$	
$\langle qg\rvert$		$\mathrm{P}^-_{KE}(m_q)$

Dressed quark

- **QCD eigenstates in $|q\rangle + |qg\rangle$**
  - Sector-dependent mass renormalization

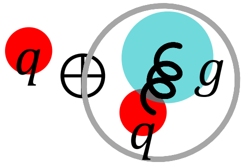
$$H_{\mathrm{LC}}(\delta m)|\phi\rangle = m_q^2|\phi\rangle$$



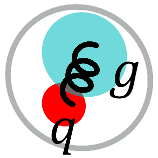


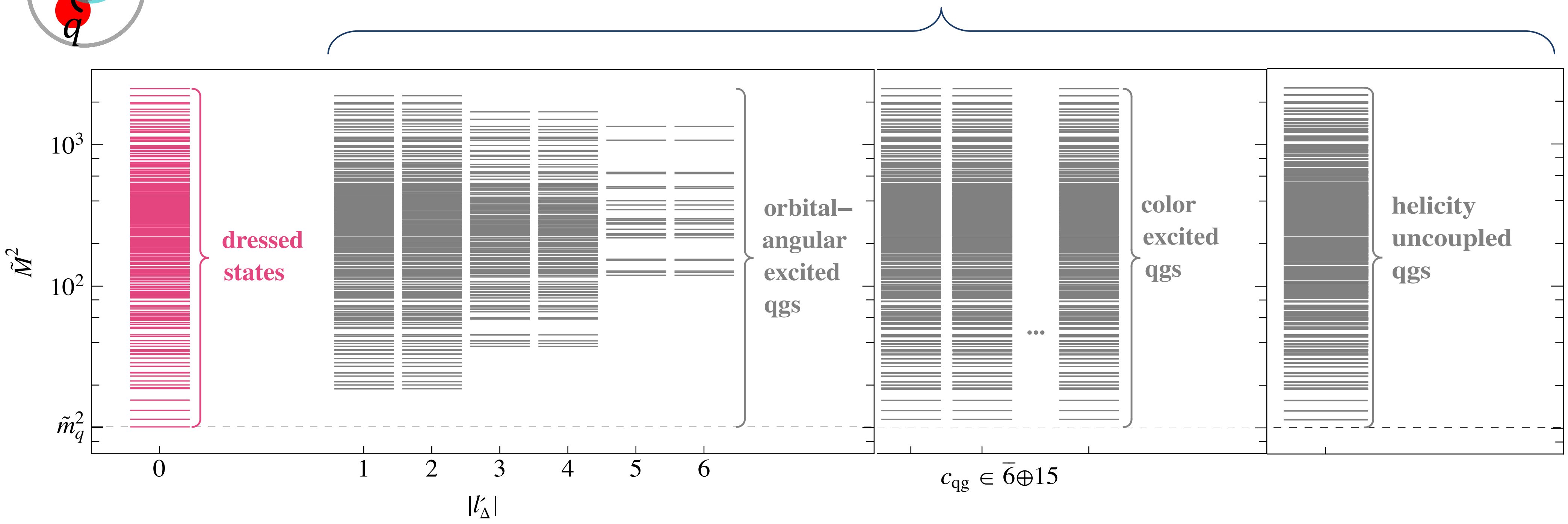
Dressed quark

- **QCD eigenstates in $|q\rangle + |qg\rangle$**
  - the dressed states



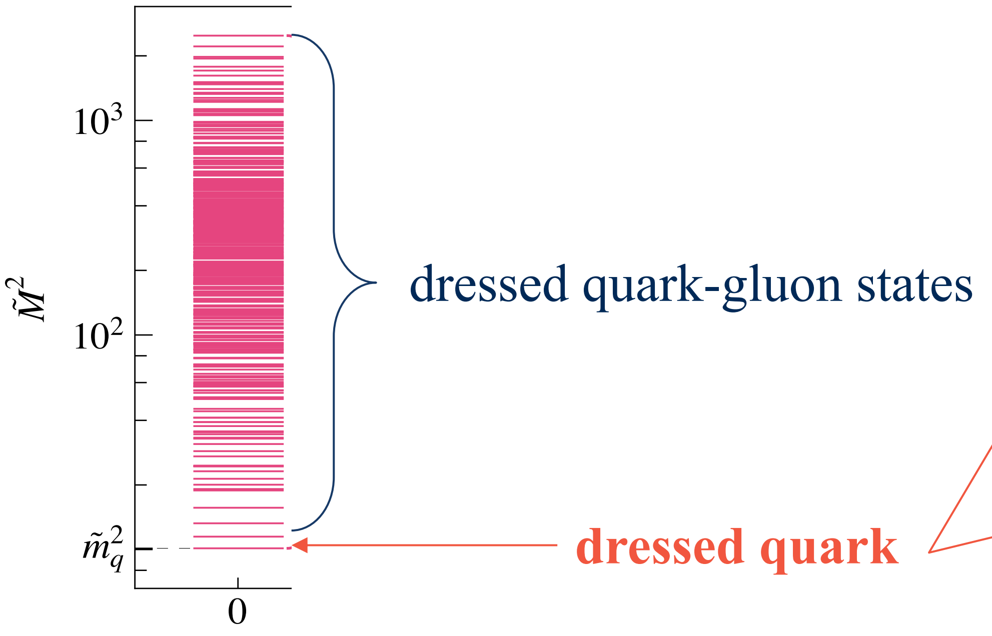
  - quark-gluon states

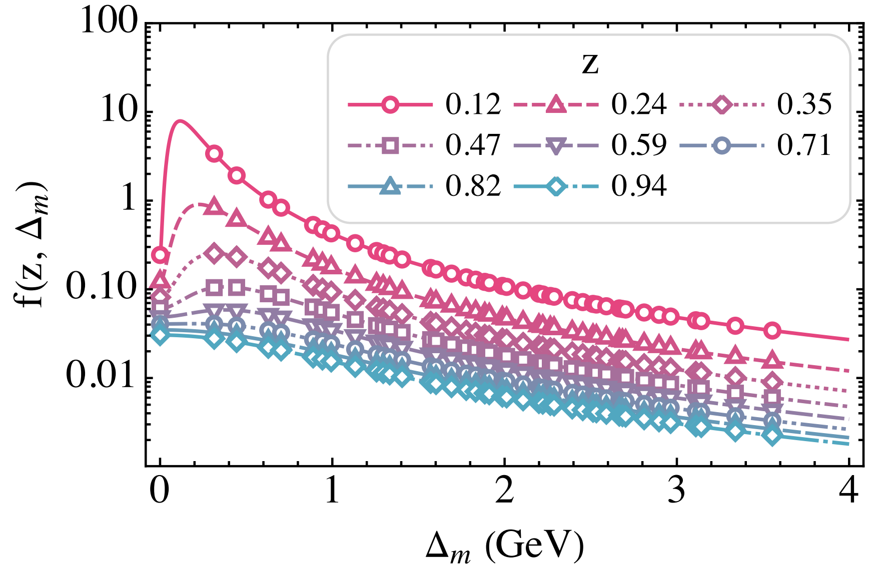




Dressed quark

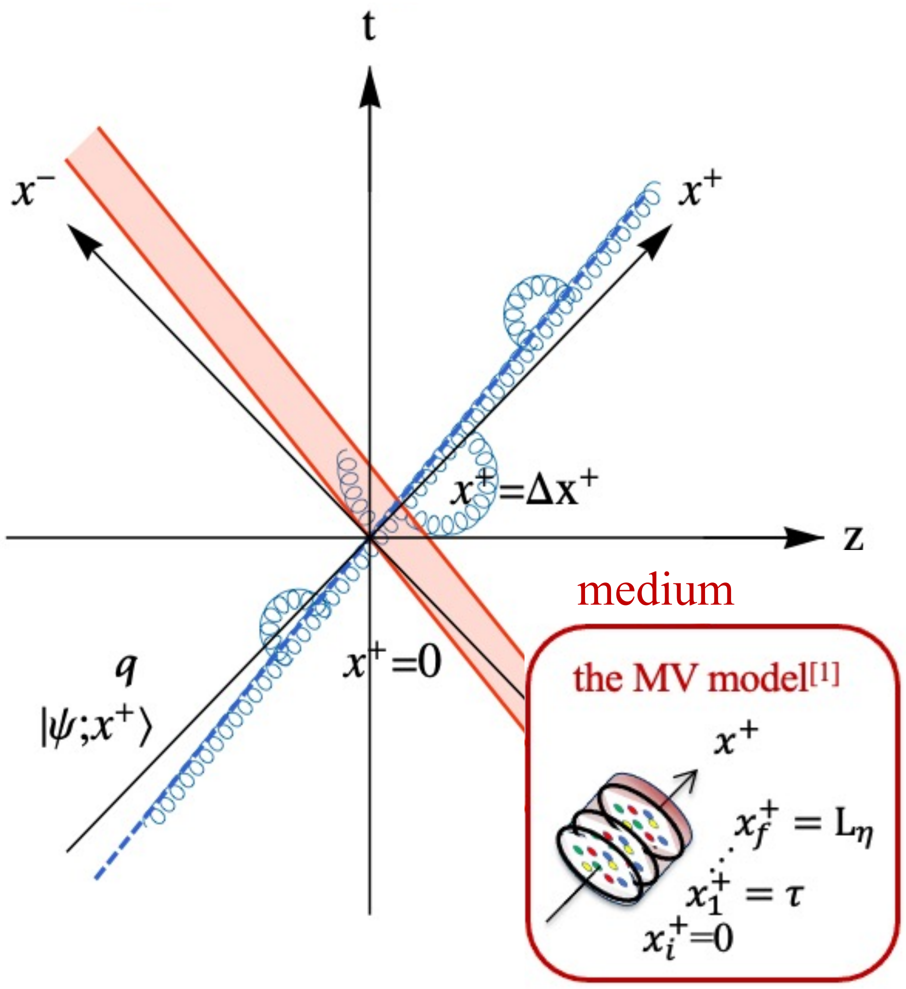
- **QCD eigenstates in $|q\rangle + |qg\rangle$**
  - the dressed states





In-medium quark jet evolution

- **The physics process**



- The medium, $\mathcal{A}(x^+, \vec{x}_\perp)$, is a classical gluon field[1]
  - Color charges

$$< \rho_a(x)\rho_b(y) > = g^2\tilde{\mu}^2\delta_{ab}\delta^{(3)}(x-y)$$

  - The color field

$$(m_g^2 - \nabla_\perp^2)\mathcal{A}_a^-(x^+, \vec{x}_\perp) = \rho_a\,(x^+, \vec{x}_\perp)$$

where m_g is a chosen infrared regulator.

  - Saturation scale: $Q_s^2 = C_F\,(g^2\tilde{\mu})^2 L_\eta/(2\pi)$

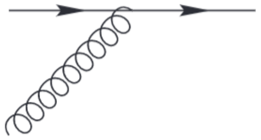
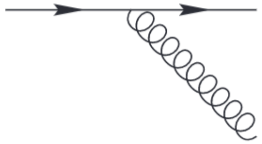
[1]L. D. McLerran and R. Venugopalan, Phys. Rev. D49, 2233 (1994); L. D. McLerran and R. Venugopalan, Phys. Rev. D49, 3352 (1994); L. D. McLerran and R. Venugopalan, Phys. Rev. D50, 2225 (1994).

In-medium quark jet evolution

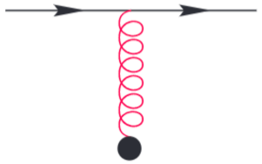
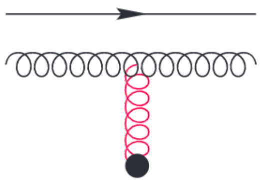
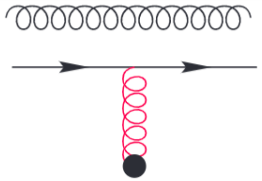
- **The evolution Hamiltonian**

$$D_\mu = \partial_\mu + ig(A_\mu + \mathcal{A}_\mu)$$

$$\mathcal{L}_{QCD} = -\frac{1}{4}\mathrm{F}_{\mathrm{a}}^{\mu\nu} F^{a}_{\mu\nu} + \bar{\psi}\left(i\gamma^{\mu} D_\mu - m\right)\psi \quad \rightarrow \quad \mathrm{P}^-(x^+) = \mathrm{P}^-_{QCD} + V_{\mathcal{A}}(x^+)$$

Fock sector	$\vert q\rangle$	$\vert qg\rangle$
$\langle q\vert$	$\mathrm{P}^-_{KE}(m_Q = m_q + \delta m)$	
$\langle qg\vert$		$\mathrm{P}^-_{KE}(m_q)$

+

Fock sector	$\vert q\rangle$	$\vert qg\rangle$
$\langle q\vert$		
$\langle qg\vert$		 

In-medium quark jet evolution

- **Solve the time-evolution equation**

$$\frac{1}{2} V_I(x^+)|\psi; x^+\rangle_{\mathrm{I}} = i\frac{\partial}{\partial x^+}|\psi; x^+\rangle_{\mathrm{I}}$$

  - P_{KE}^{-} as a phase factor:

$$|\psi; x^+\rangle_{\mathrm{I}} = \mathrm{e}^{\frac{\mathrm{i}}{2}\mathrm{P}_{KE}^{-} x^+}|\psi; x^+\rangle, \qquad \mathrm{V_I(x^+)} = \mathrm{e}^{\frac{\mathrm{i}}{2}\mathrm{P}_{KE}^{-} x^+} V(x^+) e^{-\frac{\mathrm{i}}{2}\mathrm{P}_{KE}^{-} x^+}$$

  - Time evolution as a product of many small timesteps

$$|\psi; x^+\rangle_{\mathrm{I}} = \lim_{n\to\infty} \prod_{k=1}^{n} \underbrace{\mathcal{T}_+ \exp\{-\frac{\mathrm{i}}{2}\int_{x_{k-1}^+}^{x_k^+} dz^+ V_I(z^+)\}}\, |\psi; 0\rangle_{\mathrm{I}}$$

$$\mathcal{T}_+ \exp\{-\frac{\mathrm{i}}{2}\int_{x_{k-1}^+}^{x_k^+} dz^+ V_{\mathcal{A},I}(z^+)\} \times \mathcal{T}_+ \exp\{-\frac{\mathrm{i}}{2}\int_{x_{k-1}^+}^{x_k^+} dz^+ V_{qg,I}(z^+)\}$$

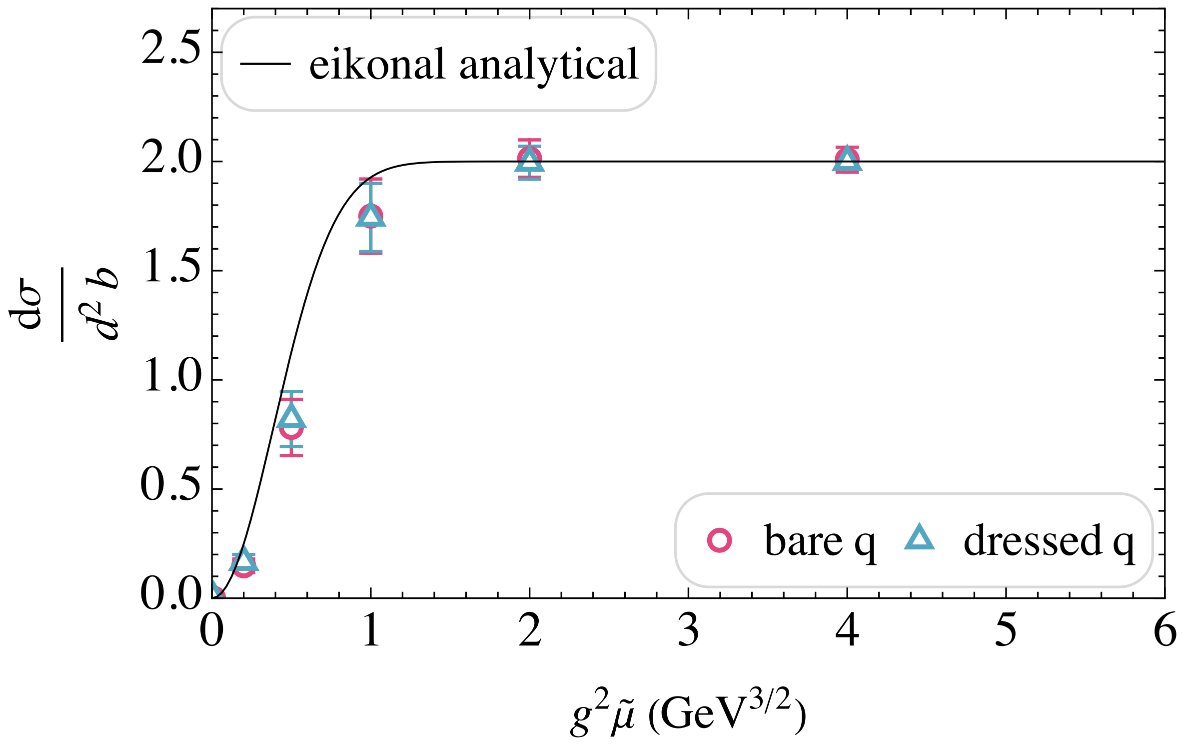
matrix exponential in coordinate space + Fast Fourier Transform, $\sim O(N_{tot} \log N_{tot})$

4th-order Runge-Kutta method, $\sim O(N_{tot})$

Results: cross section

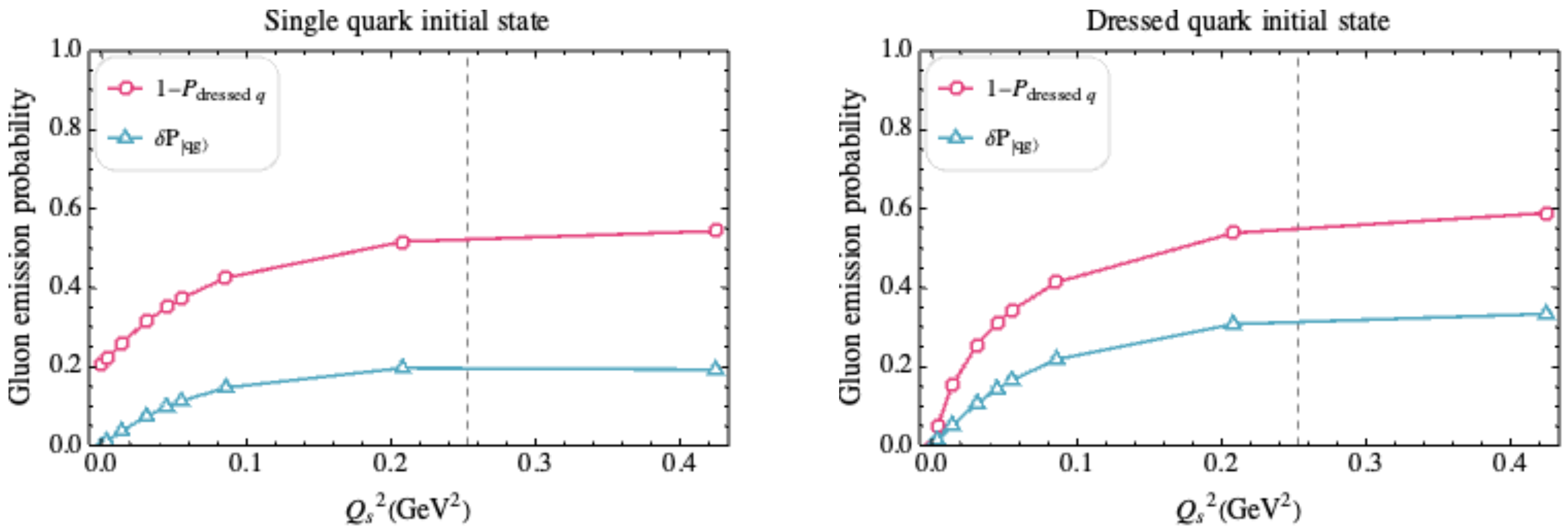
- **Bare/dressed quark scattering off the medium**

$$\frac{d\sigma}{d^2b} = \sum_{\phi_{out}} |M(\phi_{out};\psi_{in})|^2 = \sum_{\phi_{out}} |\langle\phi_{out}|\psi_{out}\rangle - \langle\phi_{out}|\psi_{in}\rangle|^2$$



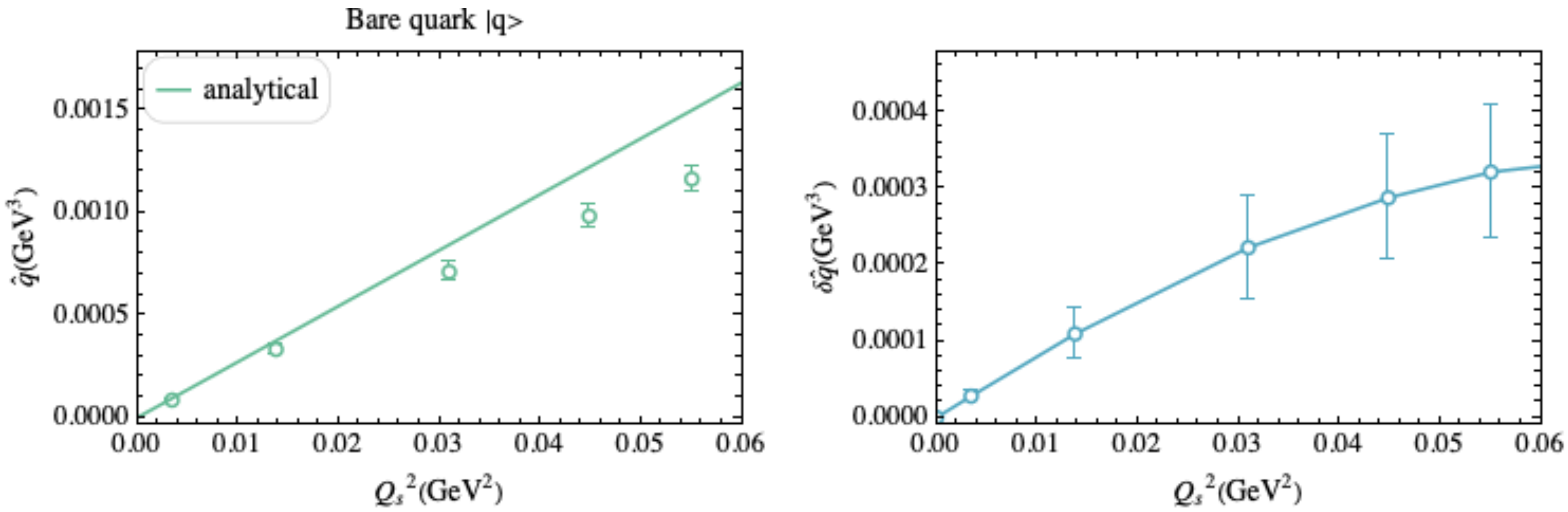
Results: gluon emission

- **Medium-induced gluon emission**
  - Total gluon: $\delta P_{|qg\rangle}(Q_s) = P_{|qg\rangle}(Q_s) - P_{|qg\rangle}(Q_s = 0)$
  - External gluon: $1 - P_{dressed\ q}$



Results: momentum broadening

- **“NLO” Fock sector contribution**
  - Quenching parameter $\hat{q} \equiv \Delta\langle P_\perp^2\rangle/\Delta x^+$
  - Non-eikonal effect: $\delta\hat{q} = \hat{q}_{\text{dressed q}} - \hat{q}_{bare\ q}$



Summary and outlooks

- We applied a light-front Hamiltonian approach, BLFQ and tBLFQ, to study in-medium quark jet evolution:
  1. we obtained the dressed quark states and the excited states
  2. we analyzed gluon emission from non-perturbative perspectives
  3. we extracted non-eikonal effect of momentum broadening

- Further applications
  1. Sub-jet structures
  2. Quantum simulation of QCD jets
  3. Jet evolution in Glasma

Thank you!