INSTYTUT FIZYKI JĄDROWEJ
IM. HENRYKA NIEWODNICZAŃSKIEGO
POLSKIEJ AKADEMII NAUK

Light Front Wave Functions and $\gamma^*\gamma^* - \mathit{Quarkonium}$ transition form factors

Izabela Babiarz

izabela.babiarz@ifj.edu.pl

Department of the Theory of Strong Interactions and Many Body Systems

May 8^{th} 2024 LFQCD Seminar

Content

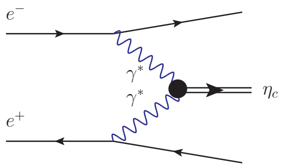
Motivation and useful application: $\gamma^*\gamma^* \to \mathcal{Q}$

The first observation of quarkonium was announced independently in November 1974 by two groups:

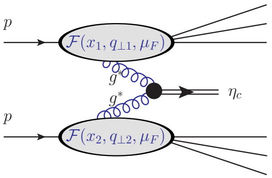
- the Brookhaven National Laboratory's (BNL) led by Samuel Chao Chung Ting, in the reaction:

$$p + Be \to e^+ e^- x$$

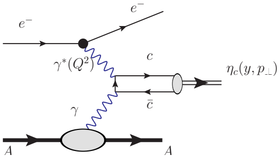
  at 30-GeV in 1974. They observed a clear sharp peak at mass 3.1 GeV. ($J \to e^+ e^-$).
  Phys.Rev.Lett. 33,1404 (1974)
- Stanford Linear Accelerator Center (SLAC) led by Burton Richter $e^+ e^- \to \psi(3105)$
  Phys.Rev.Lett. 33,1406 (1974)
- Since one can relate the transition form factor to radiative decay width, can we learn about the internal structure of mesons (exotic mesons) or the meson formation process?
- Can one combine the perturbative and nonperturbative nature of quarkonium?



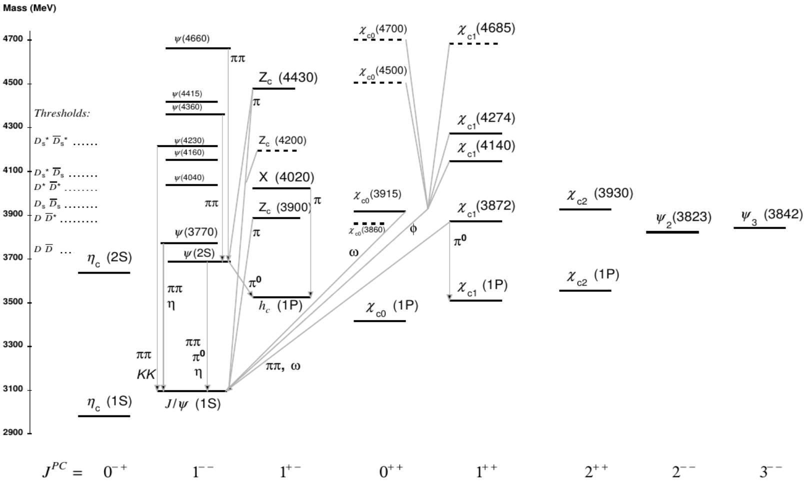
- Can one explain meson transverse distribution in proton-proton collisions?



- Can one consider electron-ion reactions as supplementary information of photon-photon fusion?



Spectrum of charmonium system and beyond



R.L. Workman *et al.* (Particle Data Group), Prog.Theor.Exp.Phys. 2022, 083C01 (2022)

$J^{PC} = 0^{-+}$ - pseudoscalar ; 0^{++} - scalar ; 1^{--} - vector ; 1^{++} - axial vector

Wave Function in the rest frame of $q\bar{q}$

the Schrödinger equation with central potential V(r) in the rest frame of the $q\bar{q}$ bound state:

$$\left(\frac{\vec{p}^2}{2\mu} + V(r)\right)|E, nlm\rangle = E|E, nlm\rangle\,, \quad \psi_{nlm}(\vec{r}) = R_{nl}(r)Y_{lm}(\theta, \phi)$$

after angular and radial decomposition, we are interested in the solution of

J. Cepila et al., Eur. Phys. J. C 79.6 (2019)

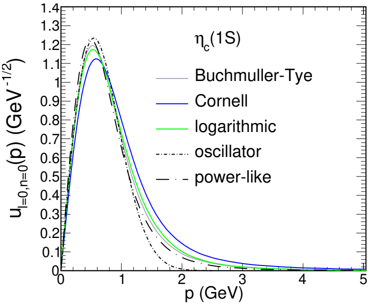
$$\frac{\partial^2 u_{nl}(r)}{\partial r^2} = \left(m_c V(r) + \frac{l(l+1)}{r^2} - mE\right)u_{nl}(r)$$

the Fourier Bessel transformation → the wave function in the momentum space:

$$u_{nl}(p) = i^l\sqrt{\frac{2}{\pi}}\int_0^\infty dr\, pr\, j_l(pr)\, u_{nl}(r)$$

Models of heavy $q\bar{q}$ interacting potential:

- Harmonic oscillator,
- Cornell potential,
- Logarithmic potential,
- Effective power law,
- Buchmüller-Tye



Light-Front Wave Functions from rest-frame: $J = 0$

$$\psi_{NR}(\vec{p}, \lambda_1, \lambda_2) = \sum_{m_l,\, m_s} \underbrace{Y_{l\, m_l}(\hat{p}) \langle \frac{1}{2}\frac{1}{2}, \lambda_1 \lambda_2 | S, m_s \rangle \langle L, S, m_l m_s | J J_z \rangle}_{\text{spin-orbit}} \underbrace{\varphi(|\vec{p}|)}_{\text{radial}}$$

$$= \underbrace{\frac{1}{\sqrt{2}} \xi_Q^{\tau\dagger}\, \hat{\mathcal{O}} i\sigma_2\, \xi_{\bar{Q}}^{\bar{\tau}*}}_{\text{spin-orbit}} \underbrace{\frac{u_{nl}(p)}{p}}_{\text{radial}} \frac{1}{\sqrt{4\pi}};$$

$$\vec{p} = (\vec{p}_\perp, p_z) = \left(\vec{k}_\perp, \frac{1}{2}(2z-1) M_{Q\bar{Q}} \right).$$

$$\hat{\mathcal{O}} = \begin{cases} \mathbb{I} & S = 0, L = 0. \\ \frac{\vec{\sigma}\cdot\vec{p}}{p}, & S = 1, L = 1. \end{cases}$$

Spherical harmonic for S-wave (l =0):

$$Y_{00} = \sqrt{\frac{1}{4\pi}}$$

Spherical harmonic for P-wave (l =1):

$$Y_{10} = \sqrt{\frac{3}{4\pi}} \cos(\theta)\,,$$

$$Y_{11} = -\sqrt{\frac{3}{8\pi}} \sin(\theta) e^{i\phi}\,,$$

$$Y_{1-1} = \sqrt{\frac{3}{8\pi}} \sin(\theta) e^{-i\phi}\,.$$

Clebsch-Gordan coefficients

$$\langle L = 1, S = 1; m_l, m_s | J = 0, J_z = 0 \rangle\,,$$

$$\langle 1, 1; +1, -1 | 00 \rangle = \sqrt{\frac{1}{3}}\,,$$

$$\langle 1, 1; -1, +1 | 00 \rangle = \sqrt{\frac{1}{3}}\,,$$

$$\langle 1, 1; 0, 0 | 00 \rangle = -\sqrt{\frac{1}{3}}\,,$$

Light-Front Wave Functions from rest-frame: $J = 0$ - Melosh-transf.

Melosh-transformation of spin-orbit part: $\xi_Q = R(z, \vec{k}_\perp)\chi_Q, \quad \xi^*_{\bar{Q}} = R^*(1-z, -\vec{k}_\perp)\chi^*_{\bar{Q}}$,

$$R(z, \vec{k}_\perp) = \frac{m_Q + zM - i\vec{\sigma}\cdot(\vec{n}\times\vec{k}_\perp)}{\sqrt{(m_Q + zM)^2 + \vec{k}_\perp^2}}$$

$\hat{\mathcal{O}}' = R^\dagger(z, \vec{k}_\perp)\mathcal{O}\, i\sigma_2 R^*(1-z, -\vec{k}_\perp)(i\sigma_2)^{-1}$ from Pauli matrices properties: $i\sigma_2\vec{\sigma}^*(i\sigma_2)^{-1} = -\vec{\sigma}$

$$\hat{\mathcal{O}}' = R^\dagger(z, \vec{k}_\perp)\hat{\mathcal{O}}\, R(1-z, -\vec{k}_\perp)\,.$$

Pseudoscalar (S-wave)

$$\begin{pmatrix} \Psi_{++}(z,\vec{k}_\perp) & \Psi_{+-}(z,\vec{k}_\perp) \\ \Psi_{-+}(z,\vec{k}_\perp) & \Psi_{--}(z,\vec{k}_\perp) \end{pmatrix} = \frac{\psi_S(z,\vec{k}_\perp)}{\sqrt{z(1-z)}} \begin{pmatrix} -k_x + ik_y & m_Q \\ -m_Q & -k_x - ik_y \end{pmatrix}$$

$$\psi_S(z,\vec{k}_\perp) = \frac{\pi}{\sqrt{2M}}\frac{u_{n0}(p)}{p}$$

Scalar (P-wave)

$$\begin{pmatrix} \Psi_{++}(z,\vec{k}_\perp) & \Psi_{+-}(z,\vec{k}_\perp) \\ \Psi_{-+}(z,\vec{k}_\perp) & \Psi_{--}(z,\vec{k}_\perp) \end{pmatrix} = \frac{-\psi_P(z,\vec{k}_\perp)}{\sqrt{z(1-z)}} \begin{pmatrix} k_x - ik_y & m_Q(1-2z) \\ m_Q(1-2z) & -k_x - ik_y \end{pmatrix}$$

$$\psi_P(z,\vec{k}_\perp) = \frac{\pi\sqrt{M}}{\sqrt{2}\sqrt{M^2 - 4m_Q^2}}\frac{u_{n1}(p)}{p}$$

Normalisation

$$1 = \int_0^1 \frac{\mathrm{d}z}{z(1-z)}\int\frac{\mathrm{d}^2\vec{k}_\perp}{16\pi^3}\sum_{\lambda\bar{\lambda}}|\Psi_{\lambda\bar{\lambda}}(z,\vec{k}_\perp)|^2 = \int_0^1\frac{\mathrm{d}z}{z(1-z)}\int\frac{\mathrm{d}^2\vec{k}_\perp}{16\pi^3}2M_{c\bar{c}}\psi(z,\vec{k}_\perp)$$

Light-Front Wave Functions from rest-frame: $J = 1$

Axial meson, $J = 1 \to$ three possible polarizations states $\lambda_A : \pm 1, 0$

$$\psi_{NR}(\vec{p}, \lambda_1, \lambda_2, \lambda_A) = \sum_{m_l,\, m_s} \underbrace{Y_{1\, m_l}(\hat{p}) \langle \frac{1}{2} \frac{1}{2}, \lambda_1 \lambda_2 | 1, m_s \rangle \langle 1, 1, m_l m_s | 1 \lambda_A \rangle}_{\text{spin}-\text{orbit}} \underbrace{\varphi(|\vec{p}|)}_{\text{radial}}$$

$$= \frac{1}{2} \sqrt{\frac{3}{4\pi}} \xi_Q^{\lambda_1 \dagger} \left(\vec{\sigma} \cdot \frac{\vec{p} \times \vec{E}(\lambda_A)}{p} \right) i\sigma_2 \, \xi_{\bar{Q}}^{\lambda_2 *} \, \frac{u(p)}{|\vec{p}|}$$

Meson polarizations:

$$\vec{E}(\pm) = (\vec{E}_\perp(\pm), 0) \,, \qquad \vec{E}(0) = \vec{n} \,, \qquad \vec{E}_\perp(\lambda_A) = -\frac{1}{\sqrt{2}} \Big(\lambda_A e_x + i e_y \Big) \,.$$

The LFWF then is obtained as follows

$$\Psi^{(\lambda_A)}_{\lambda \bar{\lambda}}(z, \vec{k}_\perp) = \chi_Q^{\lambda \dagger} \, \mathcal{O}'_{\lambda_A} \, i\sigma_2 \, \chi_{\bar{Q}}^{\bar{\lambda} *} \, \psi(z, \vec{k}_\perp) \sqrt{2(M_{Q\bar{Q}}^2 - 4m_Q^2)} \,,$$

where we pull out a square-root factor to simplify formulas further on. The spin-orbital part is encoded in the 2×2-matrix,

$$\mathcal{O}'_{\lambda_A} = \sqrt{\frac{3}{2}} \, R^\dagger(z, \vec{k}_\perp) \Big(\vec{\sigma} \cdot \frac{\vec{p} \times \vec{E}(\lambda_A)}{\sqrt{2} p} \Big) R(1 - z, -\vec{k}_\perp) \,.$$

Axial Meson wave function, $\lambda_A = \pm 1$

The "radial" part:

$$\psi_A(z, \vec{k}_\perp) = \frac{\pi\sqrt{M_{Q\bar{Q}}}}{2\sqrt{2}}\ \frac{u(p)}{p^2}\,.$$

Invariant Mass of $Q\bar{Q}$ system $M_{Q\bar{Q}} = \sqrt{\frac{\vec{k}_\perp^{\mathbf{2}} + m_Q^{\mathbf{2}}}{z(1-z)}}$

Transverse component of the wave function ($\lambda_A = \pm 1$)

$$\Psi^{(\lambda_A)}_{\lambda\bar{\lambda}}(z, \vec{k}_\perp) = \frac{\psi_A(z, \vec{k}_\perp)}{\sqrt{z(1-z)}}\sqrt{\frac{3}{2}}$$
$$\times \begin{pmatrix} m_Q(1-2z)\sqrt{2}i[\vec{e}_\perp(-), \vec{E}_\perp(\lambda_A)] & -(1-2z)(\vec{E}_\perp(\lambda_A)\vec{k}_\perp) + i[\vec{E}_\perp(\lambda_A), \vec{k}_\perp] \\ (1-2z)(\vec{E}_\perp(\lambda_A)\vec{k}_\perp) + i[\vec{E}_\perp(\lambda_A), \vec{k}_\perp] & m_Q(1-2z)\sqrt{2}i[\vec{e}_\perp(+), \vec{E}_\perp(\lambda_A)] \end{pmatrix},$$

Here we can point out several combinations, which could appear in the amplitude

$$\Psi^{*(\lambda_A)}_{+-}(z, \vec{k}_\perp) - \Psi^{*(\lambda_A)}_{-+}(z, \vec{k}_\perp) = \sqrt{\frac{3}{2}}\frac{\psi_A(z, \vec{k}_\perp)}{\sqrt{z(1-z)}}\,2(2z-1)(\vec{E}^*_\perp(\lambda_A)\vec{k}_\perp)\,,$$

$$\Psi^{*(\lambda_A)}_{+-}(z, \vec{k}_\perp) + \Psi^{*(\lambda_A)}_{-+}(z, \vec{k}_\perp) = \sqrt{\frac{3}{2}}\frac{\psi_A(z, \vec{k}_\perp)}{\sqrt{z(1-z)}}\,(-2i)[\vec{E}^*_\perp(\lambda_A), \vec{k}_\perp]\,,$$

$$\sqrt{2}(\vec{e}_\perp(-)\vec{q}_{\perp}1)\Psi^{*(\lambda_A)}_{++}(z, \vec{k}_\perp) + \sqrt{2}(\vec{e}_\perp(+)\vec{q}_{\perp}1)\Psi^{*(\lambda_A)}_{--}(z, \vec{k}_\perp) = \sqrt{\frac{3}{2}}\frac{\psi_A(z, \vec{k}_\perp)}{\sqrt{z(1-z)}}\,2m_Q(2z-1)i[\vec{q}_{\perp}1, \vec{E}^*_\perp(\lambda_A)]\,.$$

Longitudinal component of the wave function ($\lambda_A = 0$)

$$\Psi^{(0)}_{\lambda\bar{\lambda}}(z, \vec{k}_\perp) = \frac{\psi_A(z, \vec{k}_\perp)}{\sqrt{z(1-z)}} \sqrt{\frac{3}{2}} \frac{1}{M_{Q\bar{Q}}} \begin{pmatrix} i2m_Q\sqrt{2}[\vec{e}_\perp(-), \vec{k}_\perp] & -2\vec{k}_\perp^2 \\ 2\vec{k}_\perp^2 & i2m_Q\sqrt{2}[\vec{e}_\perp(+), \vec{k}_\perp] \end{pmatrix} .$$

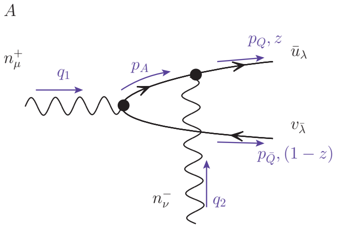
$$\Psi^{*(\lambda_A)}_{+-}(z, \vec{k}_\perp) - \Psi^{*(\lambda_A)}_{-+}(z, \vec{k}_\perp) = \sqrt{\frac{3}{2}} \frac{\psi_A(z, \vec{k}_\perp)}{\sqrt{z(1-z)}} \frac{1}{M_{Q\bar{Q}}} (-4\vec{k}_\perp^{\mathbf{2}}) ,$$

$$\Psi^{*(\lambda_A)}_{+-}(z, \vec{k}_\perp) + \Psi^{*(\lambda_A)}_{-+}(z, \vec{k}_\perp) = 0 ,$$

$$\sqrt{2}(\vec{e}_\perp(-)\vec{q}_{\mathbf{1}\perp})\Psi^{*(\lambda_A)}_{++}(z, \vec{k}_\perp) + \sqrt{2}(\vec{e}_\perp(+)\vec{q}_{\mathbf{1}\perp})\Psi^{*(\lambda_A)}_{--}(z, \vec{k}_\perp) = \sqrt{\frac{3}{2}} \frac{\psi_A(z, \vec{k}_\perp)}{\sqrt{z(1-z)}} \frac{2m_Q}{M_{Q\bar{Q}}} (-2i)[\vec{q}_{\mathbf{1}\perp}, \vec{k}_\perp] .$$

Helicity amplitude $\gamma^*\gamma^* \to q\bar{q}$

$$n^{+\mu} n^{-\nu} \mathcal{M}_{\mu\nu} = \frac{4\pi\alpha_{em} e_Q^2 \,\mathrm{Tr}\mathbb{1}_{\mathrm{color}}}{\sqrt{N_c}} \int \frac{dz d^2\vec{k}_\perp}{z(1-z)16\pi^3} \sum_{\lambda\bar{\lambda}} \Psi^*_{\lambda\bar{\lambda}} n^{+\mu} n^{-\nu} \mathcal{A}^{\lambda\bar{\lambda}}_{\mu\nu}$$



$$n^+_\mu n^-_\nu \mathcal{A}^{\lambda\bar{\lambda}}_{\mu\nu}\left(\gamma^*(q_1)\gamma^*(q_2) \to Q_\lambda(z, \vec{p}_{\perp Q})\, \bar{Q}_{\bar{\lambda}}(1-z, \vec{p}_{\perp\bar{Q}})\right)$$

$$= \bar{u}_\lambda(p_Q)\, \hat{n}^+ \frac{\hat{p}_A + m_Q}{p_A^2 - m_Q^2}\, \hat{n}^-\, v_{\bar{\lambda}}(p_{\bar{Q}}) + \bar{u}_\lambda(p_Q)\hat{n}^- \frac{\hat{p}_B + m_Q}{p_B^2 - m_Q^2}\, \hat{n}^+\, v_{\bar{\lambda}}(p_{\bar{Q}}),$$

We work in light front frame, thus $p^\mu = (p^+, p^-, p_\perp)$, and $p^+ = (p^0 + p^3)/\sqrt{2}$, $p^- = (p^0 - p^3)/\sqrt{2}$

$p_\mu \gamma^\mu = \hat{p}\,, \quad \hat{n}^+ = n^+_\mu \gamma^\mu = \gamma^-\,, \quad \hat{n}^- = n^-_\mu \gamma^\mu = \gamma^+$

Brodsky-Lepage spinors for particle: $\bar{u}_\lambda(p_Q)$, and antiparticle: $v_{\bar{\lambda}}(p_{\bar{Q}})$

Helicity amplitude $\gamma^*\gamma^* \to q\bar{q}$

Let us have a look at its light-cone decomposition

$$p_{A\mu} = p_{A+} n^+_\mu + p_{A-} n^-_\mu + p^\perp_{A\mu} = p_{A+} n^+_\mu + \frac{p_A^2 + \vec{p}^{\,2}_{A\perp}}{2p_{A+}} n^-_\mu + p^\perp_{A\mu} \,.$$

add and subtract $\propto m_Q^2/(2p_{A+})$ in the minus-component

$$p_{A\mu} = \underbrace{p_{A+} n^+_\mu + \frac{m_Q^2 + \vec{p}^{\,2}_{A\perp}}{2p_{A+}} n^-_\mu + p^\perp_{A\mu}}_{\mathrm{on-shell}} + \frac{p_A^2 - m_Q^2}{2p_{A+}} n^-_\mu = \underbrace{p^{\mathrm{os}}_{A\mu}}_{(p^{\mathrm{os}}_A)^2 = m_Q^2} + \frac{p_A^2 - m_Q^2}{2p_{A+}} n^-_\mu \,,$$

in case of on-shell momentum, we can write

$$\hat{p}^{\mathrm{os}}_A + m_Q = \sum_\sigma u_\sigma(p^{\mathrm{os}}_A)\bar{u}_\sigma(p^{\mathrm{os}}_A) \,,$$

so that the quark propagator becomes

$$\frac{\hat{p}_A + m_Q}{p_A^2 - m_Q^2} = \frac{\sum_\sigma u_\sigma(p^{\mathrm{os}}_A)\bar{u}_\sigma(p^{\mathrm{os}}_A)}{p_A^2 - m_Q^2} + \frac{1}{2p_{A+}} \hat{n}^- \,.$$

Now, in diagram A, the quark propagator is between the matrices $\hat{n}^-$ and $\hat{n}^+$:

$$\hat{n}^- \frac{\hat{p}_A + m_Q}{p_A^2 - m_Q^2} \hat{n}^+ = \hat{n}^- \frac{\sum_\sigma u_\sigma(p^{\mathrm{os}}_A)\bar{u}_\sigma(p^{\mathrm{os}}_A)}{p_A^2 - m_Q^2} \hat{n}^+ + \frac{1}{2p_{A+}} \underbrace{\hat{n}^- \hat{n}^- \hat{n}^+}_{=0},$$

Helicity amplitude $\gamma^*\gamma^* \to q\bar{q}$

$$\int \frac{dz d^2\vec{k}_\perp}{z(1-z)16\pi^3} \sum_{\lambda\bar{\lambda}} \Psi^*_{\lambda\bar{\lambda}} n^{+\mu} n^{-\nu} \mathcal{A}^{\lambda\bar{\lambda}}_{\mu\nu} = (-2) \int \frac{dz\, d^2\vec{k}_\perp}{\sqrt{z(1-z)}16\pi^3}$$

$$\times \Big\{ -m_Q \Big[\frac{1}{\vec{l}_A{}^2+\varepsilon^2} - \frac{1}{\vec{l}_B{}^2+\varepsilon^2} \Big]$$

$$\times \Big(\sqrt{2}(\vec{e}_\perp(-)\vec{q}_{1\perp})\Psi^*_{++}(z,\vec{k}_\perp) + \sqrt{2}(\vec{e}_\perp(+)\vec{q}_{1\perp})\Psi^*_{--}(z,\vec{k}_\perp) \Big)$$

$$+\Big(2z(1-z)\vec{q}_{1\perp}{}^2 + (1-2z)(\vec{k}_\perp \cdot \vec{q}_{1\perp}) \Big) \Big[\frac{1}{\vec{l}_A{}^2+\varepsilon^2} - \frac{1}{\vec{l}_B{}^2+\varepsilon^2} \Big]$$

$$\times \Big(\Psi^*_{+-}(z,\vec{k}_\perp) + \Psi^*_{-+}(z,\vec{k}_\perp) \Big)$$

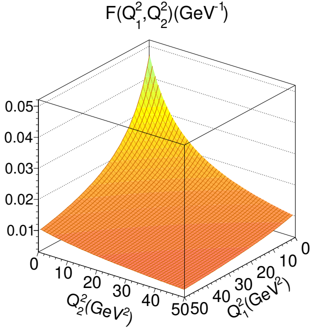
$$-(1-2z)(\vec{q}_{1\perp} \cdot \vec{q}_{2\perp}) \Big[\frac{1-z}{\vec{l}_A{}^2+\varepsilon^2} + \frac{z}{\vec{l}_B{}^2+\varepsilon^2} \Big] \Big(\Psi^*_{+-}(z,\vec{k}_\perp) + \Psi^*_{-+}(z,\vec{k}_\perp) \Big)$$

$$+i[\vec{k}_\perp, \vec{q}_{1\perp}] \Big[\frac{1}{\vec{l}_A{}^2+\varepsilon^2} - \frac{1}{\vec{l}_B{}^2+\varepsilon^2} \Big] \Big(\Psi^*_{+-}(z,\vec{k}_\perp) - \Psi^*_{-+}(z,\vec{k}_\perp) \Big)$$

$$+i[\vec{q}_{1\perp}, \vec{q}_{2\perp}] \Big[\frac{1-z}{\vec{l}_A{}^2+\varepsilon^2} + \frac{z}{\vec{l}_B{}^2+\varepsilon^2} \Big] \Big(\Psi^*_{+-}(z,\vec{k}_\perp) - \Psi^*_{-+}(z,\vec{k}_\perp) \Big) \Big\} .$$

with $\varepsilon^2 = z(1-z)\vec{q}_{1\perp}{}^2 + m_c^2$, $\vec{l}_A = \vec{p}_{\bar{Q}} - (1-z)\vec{q}_{1\perp} = -\vec{k} + (1-z)\vec{q}_{2\perp}$,
$\vec{l}_B = \vec{p}_{Q\perp} - z\vec{q}_{1\perp} = \vec{k}_\perp + z\vec{q}_{2\perp}$

$\gamma^*\gamma^* \to \eta_c(1S)$ Transition Form Factor



The definition of the transition form factor $\gamma^*\gamma^* \to \mathcal{Q}$, where $\mathcal{Q}$ is pseudoscalar meson

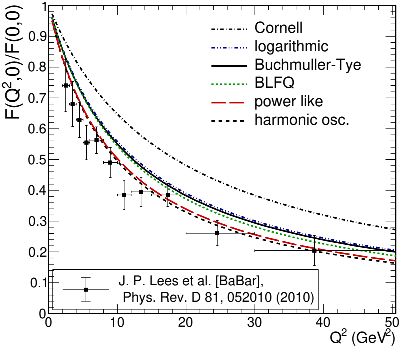
$$M_{\mu\nu}(\gamma^*(q_1)\gamma^*(q_2) \to \mathcal{Q}) = -i4\pi\alpha_{em}\varepsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta F(Q_1^2, Q_2^2)$$

$$n^{+\mu}n^{-\nu}M_{\mu,\nu}(\gamma^*(q_1)\gamma^*(q_2) \to \eta_c) = -i4\pi\alpha_{em}(q_1^x q_2^y - q_1^y q_2^x)F(Q_1^2, Q_2^2)$$

$$F(Q_1^2, Q_2^2) = e_c^2\sqrt{N_c}4m_c \cdot \int \frac{dzd^2\vec{k}_\perp \psi_S(z, \vec{k}_\perp)}{z(1-z)16\pi^3}\left\{\frac{(1-z)}{(\vec{k}_\perp - (1-z)\vec{q}_{2\perp})^2 + \varepsilon^2} + \frac{z}{(\vec{k}_\perp + z\vec{q}_{2\perp})^2 + \varepsilon^2}\right\},$$

$$Q_i^2 = \vec{q}_{i\perp}\,, \varepsilon^2 = z(1-z)\vec{q}_{1\perp}{}^2 + m_c^2$$

Normalized transition form factor: $\gamma^*\gamma \to \eta_c$



In the Melosh spin-rotation formulation $\tilde{\psi}_{\uparrow\downarrow}(z,k_\perp), \tilde{\psi}_{\uparrow\uparrow}(z,k_\perp)$, are related to the same radial wave function $\psi(z,k_\perp)$ as:

$$\tilde{\psi}_{\uparrow\downarrow}(z,k_\perp) \to \frac{m_c}{\sqrt{z(1-z)}}\psi(z,k_\perp)\,,$$

$$\tilde{\psi}_{\uparrow\uparrow}(z,k_\perp) \to \frac{-|\vec{k}_\perp|}{\sqrt{z(1-z)}}\psi(z,k_\perp)\,,$$

$$F(Q^2,0) = e_c^2\sqrt{N_c}\,4\int\frac{dzd^2\vec{k}_\perp}{\sqrt{z(1-z)}16\pi^3}\left\{\frac{1}{\vec{k}_\perp{}^2+\varepsilon^2}\tilde{\psi}_{\uparrow\downarrow}(z,k_\perp) + \frac{\vec{k}_\perp{}^2}{[\vec{k}_\perp{}^2+\varepsilon^2]^2}\left(\tilde{\psi}_{\uparrow\downarrow}(z,k_\perp)+\frac{m_c}{k_\perp}\tilde{\psi}_{\uparrow\uparrow}(z,k_\perp)\right)\right\},$$

F_{TT} and F_{LL} $\gamma^*\gamma^* \to \chi_{c0}$ Transition Form Factors

$$\mathcal{M}_{\mu\nu}(\gamma^*(q_1)\gamma^*(q_2) \to \chi_{Q0}(P)) = 4\pi\alpha_{\rm em}\Big(- \delta^{\perp}_{\mu\nu}(q_1, q_2)\, F_{TT}(q_1^2, q_2^2) + e^L_\mu(q_1) e^L_\nu(q_2)\, F_{LL}(q_1^2, q_2^2)\Big)\,,$$

here the projector on transverse polarization states is:

$$-\delta^{\perp}_{\mu\nu}(q_1, q_2) = -g_{\mu\nu} + \frac{1}{X}\Big((q_1\cdot q_2)(q_{1\mu}q_{2\nu} + q_{1\nu}q_{2\mu}) - q_1^2 q_{2\mu}q_{2\nu} - q_2^2 q_{1\mu}q_{1\nu}\Big),$$

and $X = (q_1 \cdot q_2)^2 - q_1^2 q_2^2$. The longitudinal polarization states of virtual photons read:

$$e^L_\mu(q_1) = \sqrt{\frac{-q_1^2}{X}}\Big(q_{2\mu} - \frac{q_1\cdot q_2}{q_1^2} q_{1\mu}\Big)\,, \quad e^L_\nu(q_2) = \sqrt{\frac{-q_2^2}{X}}\Big(q_{1\nu} - \frac{q_1\cdot q_2}{q_2^2} q_{2\nu}\Big)\,.$$

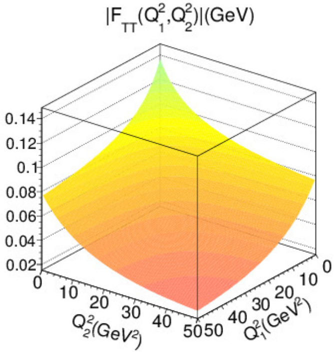
$$\int \frac{dz d^2\vec{k}_\perp}{z(1-z)16\pi^3} \sum_{\lambda\bar\lambda} \Psi^*_{\lambda\bar\lambda} n^{+\mu} n^{-\nu} \mathcal{A}^{\lambda\bar\lambda}_{\mu\nu} = |\vec{q}_{1\perp}||\vec{q}_{2\perp}| F_1 + (\vec{q}_{1\perp}\cdot\vec{q}_{2\perp}) F_2\,.$$

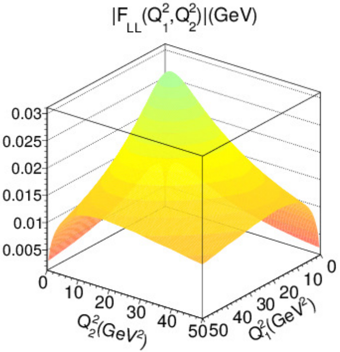
These form factors F_1 and F_2 have the integral form written as

$$\begin{aligned}
F_1(\vec{q}_{1\perp}{}^2, \vec{q}_{2\perp}{}^2) &= |\vec{q}_{1\perp}||\vec{q}_{2\perp}| \frac{4m_Q}{\vec{q}_{2\perp}{}^2} \int \frac{dzd^2\vec{k}_\perp}{z(1-z)16\pi^3} \psi(z, \vec{k}_\perp) \\
&\times \; 2z(1-z)(2z-1)\left[\frac{1}{\vec{l}_A{}^2+\varepsilon^2} - \frac{1}{\vec{l}_B{}^2+\varepsilon^2}\right], \\
F_2(\vec{q}_{1\perp}{}^2, \vec{q}_{2\perp}{}^2) &= 4m_Q \int \frac{dzd^2\vec{k}_\perp}{z(1-z)16\pi^3} \psi(z, \vec{k}_\perp)\left[\frac{1-z}{\vec{l}_A{}^2+\varepsilon^2} + \frac{z}{\vec{l}_B{}^2+\varepsilon^2}\right] \\
&+ \; \frac{4m_Q}{\vec{q}_{2\perp}{}^2} \int \frac{dzd^2\vec{k}_\perp}{z(1-z)16\pi^3} \psi(z, \vec{k}_\perp) 4z(1-z)\left[\frac{\vec{q}_{2\perp}\cdot\vec{l}_A}{\vec{l}_A{}^2+\varepsilon^2} - \frac{\vec{q}_{2\perp}\cdot\vec{l}_B}{\vec{l}_B{}^2+\varepsilon^2}\right].
\end{aligned}$$

$$\begin{aligned}
F_{TT} &= e_c^2\sqrt{N_c}[-|\vec{q}_{1\perp}||\vec{q}_{2\perp}|F_1 + (q_1\cdot q_2)F_2] \\
F_{LL} &= e_c^2\sqrt{N_c}[-|\vec{q}_{1\perp}||\vec{q}_{2\perp}|F_2 + (q_1\cdot q_2)F_1]
\end{aligned}$$

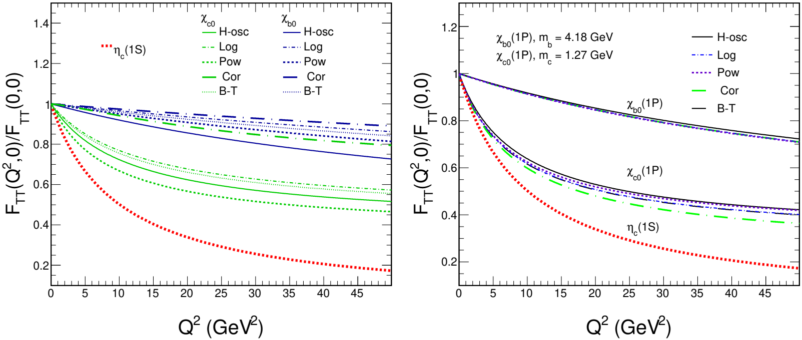
Dependence on two photons virtualities





The relative sign of the form factor for two transverse photons and the form factor for two longitudinal photons is opposite.

Normalized Transition Form Factor at the on-shell photon



The normalized form factor at the on-shell point, for one real photon.

$\eta_c(1S)$ and χ_{c0} decay width

$$\Gamma(\eta_c \to \gamma\gamma) = \frac{\pi\alpha_{\rm em}^2 M_{\eta_c}^3}{4}|F_{\eta_c}(0,0)|^2\,, \quad \Gamma(\chi_{c0} \to \gamma\gamma) = \frac{\pi\alpha_{\rm em}^2}{M_{\chi_{c0}}}|F_{\chi_{c0}}(0,0)|^2$$

$$\begin{aligned} F_{\eta_c}(0,0) &= e_c^2\sqrt{N_c}\,4m_c \cdot \int \frac{dz d^2\vec{k}_\perp}{z(1-z)16\pi^3}\frac{\psi_S(z,\vec{k}_\perp)}{\vec{k}_\perp^2 + m_c^2} \\ F_{\chi_{c0}}(0,0) &= e_c^2\sqrt{N_c}\,\frac{M_{\chi_{c0}}^2}{2}\,4m_c \cdot \int \frac{dz d^2\vec{k}_\perp}{z(1-z)16\pi^3}\frac{\psi_P(z,\vec{k}_\perp)}{\vec{k}_\perp^2 + m_c^2} \end{aligned}$$

potential type	m_c [GeV]	$\|F_{\chi_{c0}}(0,0)\|$ [GeV]	$\Gamma(\chi_{c0} \to \gamma\gamma)$ [keV]	$\|F_{\eta_c}(0,0)\|$ [GeV^{-1}]	$\Gamma(\eta_c \to \gamma\gamma)$ [keV]
harmonic oscillator	1.4	0.18	1.56	0.051	2.89
logarithmic	1.5	0.14	0.91	0.052	2.95
powerlike	1.334	0.16	1.32	0.059	3.87
Cornell	1.84	0.10	0.44	0.039	1.69
Buchmülller-Tye	1.48	0.14	0.96	0.052	2.95

$\gamma^*\gamma^*$-transition form factors for $J^{PC} = 1^{++}$ axial mesons

$$\frac{1}{4\pi\alpha_{\rm em}}\mathcal{M}_{\mu\nu\rho} = i\left(q_1 - q_2 + \frac{Q_1^2 - Q_2^2}{(q_1+q_2)^2}(q_1+q_2)\right)_\rho \tilde{G}_{\mu\nu}\,\frac{M}{2X}F_{\rm TT}(Q_1^2, Q_2^2)$$
$$+ \; ie^L_\mu(q_1)\tilde{G}_{\nu\rho}\,\frac{1}{\sqrt{X}}F_{\rm LT}(Q_1^2, Q_2^2) + ie^L_\nu(q_2)\tilde{G}_{\mu\rho}\,\frac{1}{\sqrt{X}}F_{\rm TL}(Q_1^2, Q_2^2)\,.$$

- Above we introduced

$$\tilde{G}_{\mu\nu} = \varepsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta\,,\; X = (q_1\cdot q_2)^2 - q_1^2 q_2^2$$

  and the polarization vectors of longitudinal photons

$$e^L_\mu(q_1) = \sqrt{\frac{-q_1^2}{X}}\left(q_{2\mu} - \frac{q_1\cdot q_2}{q_1^2}q_{1\mu}\right)\,, \qquad e^L_\nu(q_2) = \sqrt{\frac{-q_2^2}{X}}\left(q_{1\nu} - \frac{q_1\cdot q_2}{q_2^2}q_{2\nu}\right)\,.$$

- $F_{\rm TT}(0,0) = 0$, there is **no decay to two photons** (Landau-Yang).
- $F_{\rm LT}(Q^2, 0) \propto Q$ (absence of kinematical singularities).

$$f_{\rm LT}(Q^2) = \frac{F_{\rm LT}(Q^2,0)}{Q}$$

- $f_{\rm LT}(0)$ gives rise to so-called "reduced width" $\tilde{\Gamma}$.

- We found several interesting properties

$$Q_1 F_{\rm LT} = \frac{e_f^2\sqrt{N_c}}{2}\Big\{(\nu - Q_1^2)\underbrace{(\Phi_1+\Phi_2)}_{antisymetric} - (\nu + Q_1^2)\underbrace{(\Phi_1-\Phi_2)}_{symetric}\Big\},$$

$$Q_2 F_{\rm TL} = \frac{e_f^2\sqrt{N_c}}{2}\Big\{(\nu - Q_2^2)(\Phi_1+\Phi_2) + (\nu + Q_2^2)(\Phi_1-\Phi_2)\Big\},$$

and

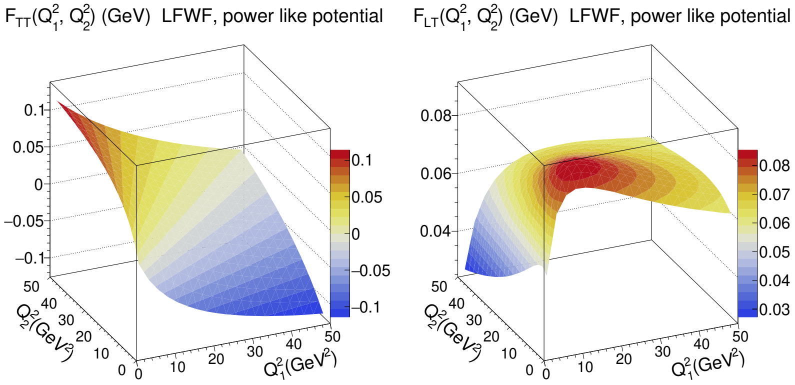
$$F_{\rm TT} = -\frac{1}{M_{\chi_c}}\Big\{Q_1 F_{\rm LT} + Q_2 F_{\rm TL}\Big\}.$$

$$\Phi_1(Q_1^2,Q_2^2) = -4\sqrt{\frac{3}{2}}\int \frac{dzd^2\vec{k}_\perp}{z(1-z)16\pi^3}\psi_A(z,\vec{k}_\perp)(1-2z)\Big\{(\vec{k}_\perp^2 + m_Q^2)\Big(\frac{1}{\vec{l}_A^2+\varepsilon^2} - \frac{1}{\vec{l}_B^2+\varepsilon^2}\Big) - (\vec{q}_{2\perp}\cdot\vec{k}_\perp)\Big(\frac{1-z}{\vec{l}_A^2+\varepsilon^2} + \frac{z}{\vec{l}_B^2+\varepsilon^2}\Big)\Big\},$$

$$\Phi_2(Q_1^2,Q_2^2) = -8\sqrt{\frac{3}{2}}\frac{Q_1^2}{Q_2^2}\int \frac{dzd^2\vec{k}_\perp}{z(1-z)16\pi^3}\psi_A(z,\vec{k}_\perp)z(1-z)(\vec{q}_{2\perp}\cdot\vec{k}_\perp)\Big(\frac{1}{\vec{l}_A^2+\varepsilon^2} - \frac{1}{\vec{l}_B^2+\varepsilon^2}\Big)$$

$\nu \equiv q_1 \cdot q_2 = \frac{1}{2}(M^2 + Q_1^2 + Q_2^2)$

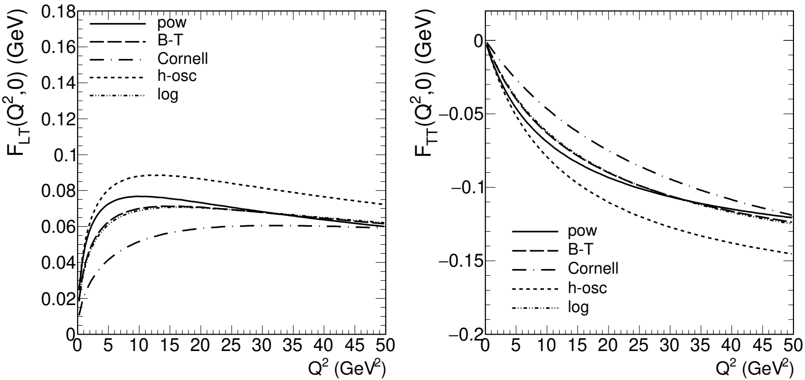
Dependence on two virtualities



- $F_{TT}(Q_1^2, Q_2^2)$ and $F_{LT}(Q_1^2, Q_2^2)$ by it self are not symmetric under exchange of $Q_1^2 \leftrightarrow Q_2^2$, but they combine in symmetric and antisymmetric functions.

$$F_S(Q_1^2, Q_2^2) \equiv Q_2 F_{\mathrm{TL}}(Q_1^2, Q_2^2) - Q_1 F_{\mathrm{LT}}(Q_1^2, Q_2^2)\,,$$
$$F_A(Q_1^2, Q_2^2) \equiv Q_2 F_{\mathrm{TL}}(Q_1^2, Q_2^2) + Q_1 F_{\mathrm{LT}}(Q_1^2, Q_2^2)\,.$$

$\gamma^*\gamma \to \chi_{c1}$ transition form factors



There is clearly no decay $\chi_{c1} \to \gamma\gamma$ (see the right panel).

Reduced width of $\chi_{c1}(1P)$

While the 1^{++} meson does not decay into two real photons, it is common practice to introduce reduced width $\tilde{\Gamma}_{\gamma^*\gamma^*}$
We follow the convention of Ref. H. Aihara (TPC/Two Gamma Collaboration) Phys.Rev.D 38 (1988) 1

$$\sigma_{ij} = \frac{32\pi}{N_i N_j}(2J+1)\frac{W^2}{2\sqrt{X}}\frac{\Gamma\Gamma^*_{ij}(Q^2)}{(W^2-M^2)^2+M^2\Gamma^2} = \frac{32\pi}{N_i N_j}(2J+1)\frac{W^2}{2M\sqrt{X}}\,\mathrm{BW}(W^2,M^2)\,\Gamma^*_{ij}(Q^2)\,.$$

$$\Gamma^{\mathrm{LT}}_{\gamma^*\gamma^*}(Q_1^2,Q_2^2,M^2) = \frac{\pi\alpha_{\mathrm{em}}^2}{3M}\;F^2_{\mathrm{LT}}(Q_1^2,Q_2^2)\,.$$

$$\tilde{\Gamma}(A) = \lim_{Q^2\to 0}\frac{M^2}{Q^2}\Gamma^{\mathrm{LT}}_{\gamma^*\gamma^*}(Q^2,0,M^2) = \frac{\pi\alpha_{\mathrm{em}}^2 M}{3}\,f^2_{\mathrm{LT}}\,,\quad \text{with } f_{\mathrm{LT}} = \lim_{Q^2\to 0}\frac{F_{\mathrm{LT}}(Q^2,0)}{Q}\,,$$

potential model	m_c (GeV)	$\tilde{\Gamma}(\chi_{c1})$ (keV)
power-law	1.33	0.50
Buchmüller-Tye	1.48	0.30
Cornell	1.84	0.09
harmonic oscillator	1.4	0.53
logarithmic	1.5	0.27

- Considerably larger values of $\tilde{\Gamma}(\chi_{c1})$ are quoted in the literature. For example Danilkin & Vanderhaeghen (2017) report a value of $\tilde{\Gamma}(\chi_{c1}) \approx 1.6\,\mathrm{keV}$ from a sum rule analysis. Li et al. (2022) obtain $\tilde{\Gamma}(\chi_{c1}) \approx 3\,\mathrm{keV}$ from a LFWF approach.
- A measurement of the reduced width would therefore be very valuable.

Reduced $\gamma_L^*\gamma$ width for $\chi_{c1}(3872)$

The reduced width of the $\chi_{c1}(2P)$ state for several models of the charmonium wave functions

$c\bar{c}$ potential	m_c (GeV)	$f_{\rm LT}(0)$	$\tilde{\Gamma}_{\gamma\gamma}$ (keV)
harmonic oscillator	1.4	0.041	0.36
power-law	1.334	0.033	0.24
Buchmüller-Tye	1.48	0.029	0.18
logarithmic	1.5	0.025	0.14
Cornell	1.84	0.018	0.07
BLFQ	1.6	0.044	0.42

- First evidence for the production of $\chi_{c1}(3872)$ in single-tag e^+e^- collisions was reported by Belle Phys. Rev. Lett. **126** (2021) no.12, 122001 From three measured events, they provided a range for its reduced width, $0.02\,\mathrm{keV} < \tilde{\Gamma}_{\gamma\gamma} < 0.5\,\mathrm{keV}$. Recent update by Achasov et al. Phys. Rev. D **106** (2022) no.9, 093012 using a corrected value for the branching ratio $\mathrm{Br}(\chi_{c1}(3872) \to \pi^+\pi^- J/\psi)$ and reads

$$0.024\,\mathrm{keV} < \tilde{\Gamma}_{\gamma\gamma}(\chi_{c1}(3872)) < 0.615\,\mathrm{keV}$$

- all our results, including the BLFQ approach, lie **well within the experimentally allowed range**. Therefore, $\gamma\gamma$ data do not exclude the $c\bar{c}$ option, although there is certainly some room for a contribution from an additional meson-meson component.

$\gamma^*\gamma \to \chi_{c2}$ Transition Form Factor

$$\frac{1}{4\pi\alpha_{em}}\mathcal{M}_{\mu\nu\alpha\beta} = \delta^{\perp}_{\mu\nu}(q_2-q_1)_\alpha(q_2-q_1)_\beta\, F_{\rm TT,0}(Q^2) + \delta^{\perp}_{\mu\alpha}\delta^{\perp}_{\nu\beta}\, F_{\rm TT,2}(Q^2) + \Big(q_{1\mu} - \frac{q_1^2}{q_1\cdot q_2}q_{2\mu}\Big)\delta^{\perp}_{\nu\alpha}(q_2-q_1)_\beta\, F_{\rm LT}(Q^2)\,,$$

$$F_{\rm TT,0}(Q^2) = \sqrt{6N_c}e_f^2\frac{M^2}{M^2+Q^2}\int\frac{dz\,k_\perp dk_\perp}{\sqrt{z(1-z)}8\pi^2}\,\frac{1}{[k_\perp^2+\varepsilon^2]^2}\Big[m_f k_\perp\tilde\psi^{0}_{\uparrow\uparrow}(z,k_\perp) - \frac{\varepsilon^2}{2}\Big((2z-1)\Big(\tilde\psi^{0}_{\uparrow\downarrow}(z,k_\perp)+\tilde\psi^{0}_{\downarrow\uparrow}(z,k_\perp)\Big) + \Big(\tilde\psi^{0}_{\uparrow\downarrow}(z,k_\perp)-\tilde\psi^{0}_{\downarrow\uparrow}(z,k_\perp)\Big)\Big)\Big]\,,$$

$$F_{\rm TT,2}(Q^2) = -2\sqrt{N_c}e_f^2(M^2+Q^2)\int\frac{dz\,k_\perp dk_\perp}{\sqrt{z(1-z)}8\pi^2}\,\frac{1}{[k_\perp^2+\varepsilon^2]^2}\Big[m_f k_\perp\tilde\psi^{+2}_{\uparrow\uparrow}(z,k_\perp) + \frac{k_\perp^2}{2}\Big((2z-1)\Big(\tilde\psi^{+2}_{\uparrow\downarrow}(z,k_\perp)+\tilde\psi^{+2}_{\downarrow\uparrow}(z,k_\perp)\Big) + \Big(\tilde\psi^{+2}_{\uparrow\downarrow}(z,k_\perp)-\tilde\psi^{+2}_{\downarrow\uparrow}(z,k_\perp)\Big)\Big)\Big]\,,$$

$$F_{\rm LT}(Q^2) = 4\sqrt{N_c}e_f^2\,M\int\frac{dz k_\perp dk_\perp}{\sqrt{z(1-z)}8\pi^2}\,\frac{z(1-z)k_\perp}{[k_\perp^2+\varepsilon^2]^2}\Big(\tilde\psi^{+1}_{\uparrow\downarrow}(z,k_\perp)+\tilde\psi^{+1}_{\downarrow\uparrow}(z,k_\perp)\Big)\,.$$

The single-tag cross-section

$$\frac{d\sigma}{dQ^2} = 2\int dW \frac{dL}{dWdQ^2}\left(\sigma_{\rm TT}(W^2,Q^2) + \epsilon_0 \sigma_{\rm LT}(W^2,Q^2)\right).$$

The factor two appears because each of the lepton can emit the off-shell photon. In the narrow-width approximation, we therefore have

$$\frac{d\sigma}{dQ^2} = 4\pi^2 \frac{(2J+1)}{M^2}\left(1+\frac{Q^2}{M^2}\right)^{-1} \frac{2\,dL}{dWdQ^2}\Big|_{W=M} \Gamma_{\gamma^*\gamma}(Q^2),$$

Off-shell widths are convention-dependent, and to compare to the experimental data from Ref.Phys.Rev.D 97 (2018) 5, 052003, we note that the Belle collaboration writes

$$\frac{d\sigma}{dQ^2} = 4\pi^2 \frac{(2J+1)}{M^2}\left(1+\frac{Q^2}{M^2}\right) \frac{2\,dL}{dWdQ^2}\Big|_{W=M} \Gamma^{\rm Belle}_{\gamma^*\gamma}(Q^2),$$

which means, that

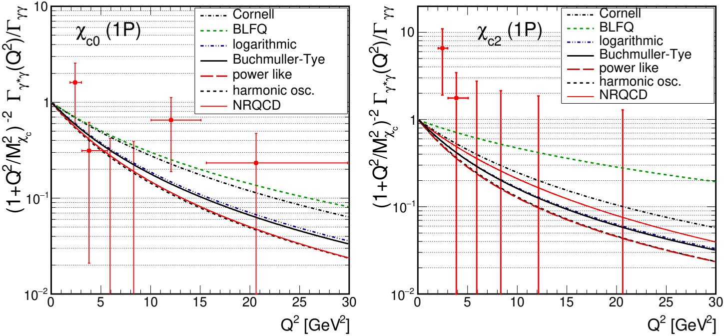
$$\Gamma^{\rm Belle}_{\gamma^*\gamma}(Q^2) = \left(1+\frac{Q^2}{M^2}\right)^{-2} \Gamma_{\gamma^*\gamma}(Q^2).$$

$$\sigma_{ij} = \frac{32\pi}{N_i N_j}(2J+1)\frac{W^2}{2M\sqrt{X}}\,{\rm BW}(W^2,M^2)\,\Gamma^*_{ij}(Q^2).$$

$$\sigma_{\rm TT} \quad = \quad \frac{(4\pi\alpha_{\rm em})^2}{4\sqrt{X}}\left\{F^2_{{\rm TT},2}(Q^2) + \frac{2}{3}\left(1+\frac{Q^2}{M^2}\right)^4 M^4\, F^2_{{\rm TT},0}(Q^2)\right\}{\rm BW}(W^2,M^2).$$

$$\sigma_{\rm LT} = \frac{Q^2\sqrt{X}}{W^2}(4\pi\alpha_{\rm em})^2\, F^2_{\rm LT}(Q^2)\, BW(W^2,M^2),$$

$\gamma^*\gamma$ cross-section and off-shell width



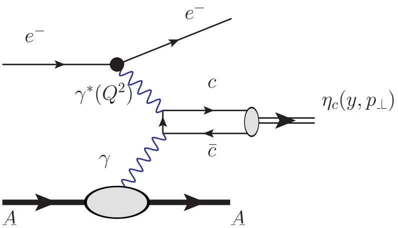
Off-shell decay width $\Gamma^*(Q^2)$ for χ_{c0} (on the l.h.s.) and χ_{c2} (on the r.h.s.) compared to the Belle data Phys.Rev.D 97 (2018

$$\Gamma_{\gamma^*\gamma}(Q^2) = \Gamma^*_{\mathrm{TT}}(Q^2) + \epsilon_0 2\Gamma^*_{\mathrm{LT}}(Q^2)$$

$$\Gamma^*(Q^2) = \frac{(4\pi\alpha_{\mathrm{em}})^2}{16\pi M} F^2_{\mathrm{TT}}(Q^2)\,. \qquad \Gamma^*_{\mathrm{TT}}(Q^2) = (4\pi\alpha_{\mathrm{em}})^2 \left\{ \frac{F^2_{\mathrm{TT2}}(Q^2)}{80\pi M} + \frac{M^3 F^2_{\mathrm{TT0}}(Q^2)}{120\pi}\left(1+\frac{Q^2}{M^2}\right)^4 \right\}.$$

$$\Gamma^*_{\mathrm{LT}}(Q^2) = (4\pi\alpha_{\mathrm{em}})^2 \frac{1}{160\pi}\left(1+\frac{Q^2}{M^2}\right)^2 M Q^2 F^2_{\mathrm{LT}}(Q^2)\,.$$

Electron-ion collisions



the nuclear radius: $R_A = r_0 A^{1/3}$, with $r_0 = 1.1\,\mathrm{fm}$

$$\sigma(eA \to e\eta_c A) = \int d\omega_e dQ^2 \frac{d^2 N_e}{d\omega_e dQ^2} \times \sigma(\gamma^* A \to \eta_c A)$$

$$\sigma(\gamma^* A \to \eta_c A) = \int d\omega_A \frac{dN_A}{d\omega_A} \times \sigma_{\mathrm{TT}}(\gamma^*\gamma \to \eta_c; W_{\gamma\gamma}, Q^2, 0)$$

$$W_{\gamma\gamma} = \sqrt{4\omega_e\omega_A - p_\perp^2}$$

$$\omega_e = \frac{\sqrt{M^2 + p_\perp^2}}{2} e^{+y}$$

$$\omega_A = \frac{\sqrt{M^2 + p_\perp^2}}{2} e^{-y}$$

$$p_\perp^2 = \left(1 - \frac{\omega_e}{E_e}\right) Q^2$$

$$\frac{dN_A}{d\omega_A} = \frac{2Z^2\alpha_{em}}{\pi\omega_A}\left[\xi K_0(\xi)K_1(\xi) - \frac{\xi^2}{2}(K_1^2(\xi) - K_0^2(\xi))\right]$$

$\xi = R_A\omega_A/\gamma_L$, K_0 and K_1 -modified Bessel functions

i.e.: *Ann.Rev.Nucl.Part.Sci.*55, 271(2005)

$$\frac{d^2 N_e}{d\omega_e dQ^2} = \frac{\alpha_{em}}{\pi\omega_e Q^2}\left[\left(1 - \frac{\omega_e}{E_e}\right)\left(1 - \frac{Q^2_{min}}{Q^2}\right) + \frac{\omega_e^2}{2E_e^2}\right]$$

$Q^2_{min} = m_e^2\omega_e^2/[E_e(E_e - \omega_e)]$ and $Q^2_{max} = 4E_e(E_e - \omega_e)$

$$\sigma_{\rm TT}(W_{\gamma\gamma}, Q_1^2, Q_2^2) = \frac{1}{4\sqrt{X}} \frac{M_{\eta_c}\Gamma_{\rm tot}}{(W_{\gamma\gamma}^2 - M_{\eta_c}^2)^2 + M_{\eta_c}^2\Gamma_{\rm tot}^2} \mathcal{M}^*(++)\mathcal{M}(++)$$

The helicity amplitude $\mathcal{M}(\lambda_1, \lambda_2) = e_\mu^1(\lambda_1)e_\nu^2(\lambda_2)\mathcal{M}^{\mu\nu}$

$$\mathcal{M}_{\mu\nu}(\gamma^*(q_1)\gamma^*(q_2) \to \eta_c) = 4\pi\alpha_{\rm em}\,(-i)\varepsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta\, F(Q_1^2, Q_2^2)\,.$$

$X = (q_1 \cdot q_2)^2 - q_1^2 q_2^2$, in the limit $Q_2^2 \to 0$, $\sqrt{X} = q_1 \cdot q_2 = (M_{\eta_c}^2 + Q^2)/2$

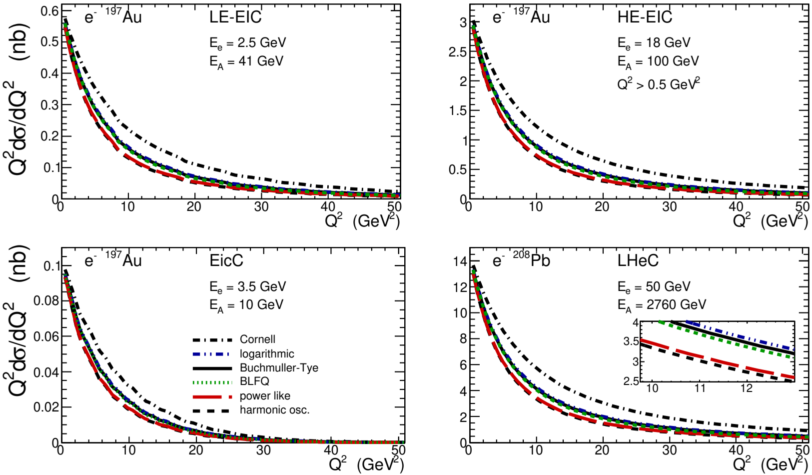
$$\sigma_{\rm TT}(W_{\gamma\gamma}, Q^2, 0) = 2\pi^2\alpha_{\rm em}^2 \frac{M_{\eta_c}\Gamma_{\rm tot}}{(W_{\gamma\gamma}^2 - M_{\eta_c}^2)^2 + M_{\eta_c}^2\Gamma_{\rm tot}^2}\,(M_{\eta_c}^2 + Q^2)\,F^2(Q^2, 0)\,.$$

We can take advantage of the relation $\Gamma(\gamma\gamma \to \eta_c) = \frac{\pi}{4}\alpha_{em}^2 M_{\eta_c}^3 |F(0,0)|^2$

$$\begin{aligned}
\sigma_{\rm TT}(W_{\gamma\gamma}, Q^2, 0) &= 8\pi \frac{\Gamma_{\gamma\gamma}\Gamma_{\rm tot}}{(W_{\gamma\gamma}^2 - M_{\eta_c}^2)^2 + M_{\eta_c}^2\Gamma_{\rm tot}^2}\left(1 + \frac{Q^2}{M_{\eta_c}^2}\right)\left(\frac{F(Q^2,0)}{F(0,0)}\right)^2 \\
&\approx 8\pi^2\,\delta(W_{\gamma\gamma}^2 - M_{\eta_c}^2)\,\frac{\Gamma_{\gamma\gamma}}{M_{\eta_c}}\left(1 + \frac{Q^2}{M_{\eta_c}^2}\right)\left(\frac{F(Q^2,0)}{F(0,0)}\right)^2
\end{aligned}$$

Phys.Lett.B 843(2023)138046

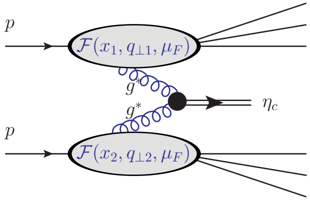
Differential distribution in photon virtuality

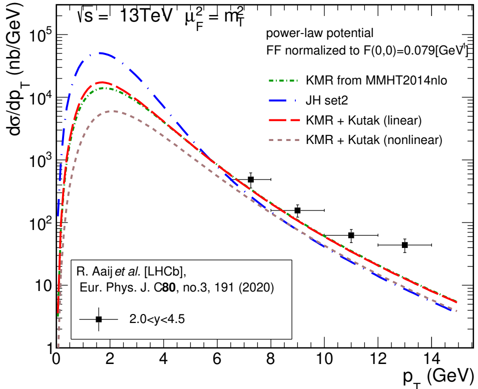


$Q^2 > 0.5\,\mathrm{GeV}^2$

Phys.Lett.B 843(2023)138046

Hadroproduction of $\eta_c(1S, 2S)$ via gluon-gluon fusion





I. Babiarz, R. Pasechnik, W. Schäfer, A. Szczurek JHEP 02, 037 (2020)

$$\mathcal{M}^{ab}_{\mu\nu}(g^* g^* \to \eta_c) = (-i) 4\pi\alpha_s\, \varepsilon_{\mu\nu\alpha\beta}\, q_1^\alpha q_2^\beta \times \frac{\mathrm{Tr}[t^a t^b]}{\sqrt{N_c}} \frac{F(Q_1^2, Q_2^2)}{e_c^2 \sqrt{N_c}}$$

$$\frac{d\sigma}{dy d^2\vec{p}_\perp} = \int \frac{d^2\vec{q}_{1\perp}}{\pi \vec{q}_{1\perp}^2} \mathcal{F}(x_1, \vec{q}_{1\perp}{}^2, \mu_F) \int \frac{d^2\vec{q}_{2\perp}}{\pi \vec{q}_{2\perp}{}^2} \mathcal{F}(x_2, \vec{q}_{2\perp}{}^2, \mu_F)\, \delta^{(2)}(\vec{q}_{1\perp} + \vec{q}_{2\perp} - \vec{p}_\perp) \times \frac{\pi}{(x_1 x_2 s)^2} \overline{|\mathcal{M}(g^* g^* \to \eta_c)|^2}$$

Summary

- Helicty amplitude technique was introduced with the light-front wave function of $Q\bar{Q}$ bound state in several spin configurations
- The properties of the photon-photon transition form factor were presented. The comparison to BaBar(2010) for $\gamma^*\gamma^* \to \eta_c$ shows that a relatively good description gives wave function power-like and the harmonic oscillator. The form factor strongly depends on the quark mass.
- The relation at the on-shell point of the form factor and decay width or reduced decay width were shown.
  The radiative decay width for $\Gamma(\eta_c \to \gamma\gamma)$: $(1.69 - 2.95)$ keV, $\Gamma(\chi_{c0} \to \gamma\gamma)$: $(0.44 - 1.56)$ keV
  The reduced width for $\tilde{\Gamma}(\chi_{c1}(1P) \to \gamma^*\gamma^*)$: in the range: $(0.09 - 0.53)$ keV, $\tilde{\Gamma}(\chi_{c1}(2P) \to \gamma^*\gamma^*)$: $(0.07 - 0.36)$ keV, estimates by Achasov for X(3872): $(0.024 - 0.625)$ keV.
- For χ_{c2} and χ_{c0} off-shell widths from single tag cross-section were compared to Belle data (2017).
- We proposed two investigate photon-photon fusion mechanisms in electron-Ion future colliders. Differential distributions for photon virtuality have been shown for LE-EIC, HE-EIC, EicC and LHeC. The estimated cross-section is in the range: $(0.1 - 60)$ nb.
- We applied the transition form factor for η_c to a proton-proton collision including proper color factors in $k_\perp$-factorization approach. From comparison to LHCb data (2020) for 13TeV, we see that other production mechanisms must be included for two points around 11 GeV and 13 GeV of the meson transverse momentum.

For more details visit references

- I. Babiarz, Victor P. Goncalves, R. Pasechnik, W. Schäfer Cracow,A. Szczurek, Phys.Rev.D 100 (2019) 5, 054018
- I. Babiarz, R. Pasechnik, W. Schäfer, A. Szczurek, JHEP 02 (2020) 037
- I. Babiarz, R. Pasechnik, W. Schäfer, A. Szczurek, JHEP 06 (2020) 101
- I. Babiarz, R. Pasechnik, W. Schäfer, A. Szczurek, JHEP 09 (2022) 170
- I. Babiarz, R. Pasechnik, W. Schäfer, A. Szczurek, Phys.Rev.D 107 (2023) 7, L071503
- I. Babiarz, Victor P. Goncalves, W. Schäfer, A. Szczurek, Phys.Lett.B 843 (2023) 138046
- I. Babiarz, R. Pasechnik, W. Schäfer, A. Szczurek, e-Print:2402.13910 [hep-ph]