Transverse structure of hadrons beyond leading twist with basis light-front quantization

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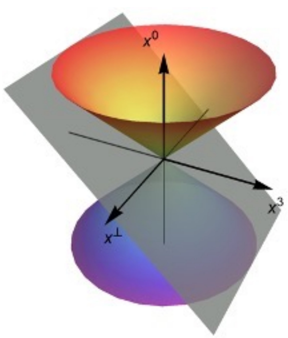

Institute of Modern Physics, Chinese Academy of Sciences

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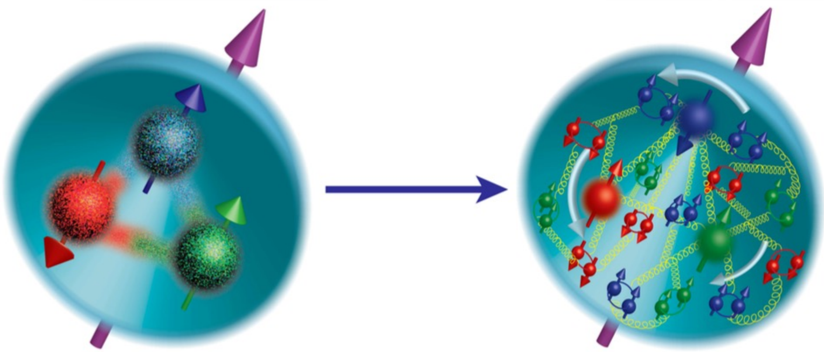
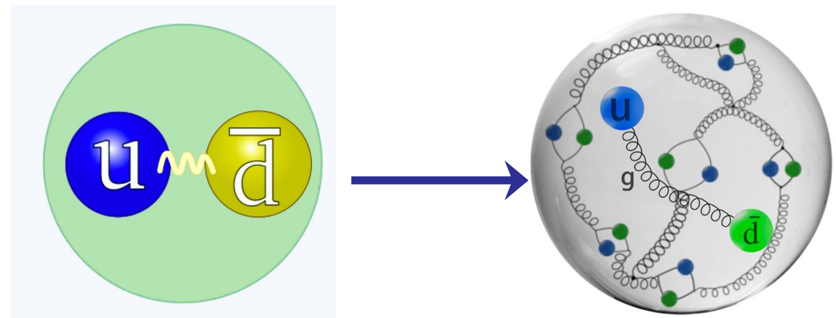
Contents

- Hadron structure，PDFs & TMD-PDFs
- Basis light front quantization
- Numerical results

Hadron structures

- Proton is made of
  - 2 up quarks + 1 down quark ➡ **valence quarks**
  - + any number of quark-antiquark pairs ➡ **sea quarks**
  - + any number of gluons





$$|\text{proton}\rangle = \psi_{uud}|uud\rangle + \psi_{uudg}|uudg\rangle + \psi_{uudq\bar{q}}|uudq\bar{q}\rangle + \cdots$$

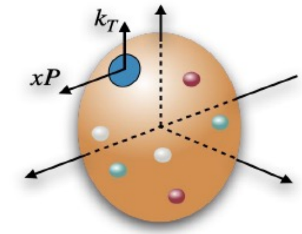
$$|\text{meson}\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \cdots$$

Fock sector expansion

- Pion is made of
  - 1 quark + 1 antiquark ➡ **valence quarks**
  - + any number of quark-antiquark pairs ➡ **sea quarks**
  - + any number of gluons

✓ Many body system
✓ Rich structure of hadrons

Overview on TMDs (spin 1/2)



Quark correlator

$$\Phi_q^{[\Gamma]}\left(P,S;x=\frac{k^+}{P^+},\vec{k}_\perp\right)=\frac{1}{2}\int\frac{\mathrm{d}z^-\mathrm{d}z^\perp}{2(2\pi)^3}e^{ik\cdot z}\left\langle P,S\right|\bar{\Psi}_q(0)\Gamma\mathcal{W}(0^\perp,z^\perp)\Psi_q(z)\left|P,S\right\rangle\Big|_{z^+=0},$$

Parameterization:

8 twist-2 TMDs:

6 T-even terms
2 T-odd terms

$$\Phi^{[\gamma^+]}=f_1-\frac{\epsilon_\perp^{ij}k_\perp^i S_\perp^j}{M}f_{1T}^\perp,$$

$$\Phi^{[\gamma^+\gamma^5]}=\Lambda g_{1L}+\frac{k_\perp\cdot\boldsymbol{S}_\perp}{M}g_{1T},$$

$$\Phi^{[i\sigma^{j+}\gamma^5]}=S_\perp^j h_1+\Lambda\frac{k_\perp^j}{M}h_{1L}^\perp+S_\perp^i\frac{2k_\perp^i k_\perp^j-(k_\perp)^2\delta^{ij}}{2M^2}h_{1T}^\perp+\frac{\epsilon_\perp^{ji}k_\perp^i}{M}h_1^\perp,$$

16 twist-3 TMDs:

8 T-even terms
8 T-odd terms

$$\Phi^{[1]}=\frac{M}{P^+}\left[e-\frac{\epsilon_T^{\rho\sigma}k_{\perp\rho}S_{T\sigma}}{M}e_T^\perp\right],$$

$$\Phi^{[i\gamma_5]}=\frac{M}{P^+}\left[S_L e_L-\frac{k_\perp\cdot S_T}{M}e_T\right],$$

$$\Phi^{[\gamma^\alpha]}=\frac{M}{P^+}\left[-\epsilon_T^{\alpha\rho}S_{T\rho}f_T-S_L\frac{\epsilon_T^{\alpha\rho}k_{\perp\rho}}{M}f_L^\perp-\frac{k_\perp^\alpha k_\perp^\rho-\frac{1}{2}k_\perp^2 g_T^{\alpha\rho}}{M^2}\epsilon_{T\rho\sigma}S_T^\sigma f_T^\perp+\frac{k_\perp^\alpha}{M}f^\perp\right],$$

$$\Phi^{[\gamma^\alpha\gamma_5]}=\frac{M}{P^+}\left[S_T^\alpha g_T+S_L\frac{k_\perp^\alpha}{M}g_L^\perp-\frac{k_\perp^\alpha k_\perp^\rho-\frac{1}{2}k_\perp^2 g_T^{\alpha\rho}}{M^2}S_{T\rho}g_T^\perp-\frac{\epsilon_T^{\alpha\rho}k_{\perp\rho}}{M}g^\perp\right],$$

$$\Phi^{[i\sigma^{\alpha\beta}\gamma_5]}=\frac{M}{P^+}\left[\frac{S_T^\alpha k_\perp^\beta-k_\perp^\alpha S_T^\beta}{M}h_T^\perp-\epsilon_T^{\alpha\beta}h\right],$$

$$\Phi^{[i\sigma^{+-}\gamma_5]}=\frac{M}{P^+}\left[S_L h_L-\frac{k_\perp\cdot S_T}{M}h_T\right].$$

Jaffe-Ji notation:

f, e → unpolarized quarks
g → longitudinally polarized quarks
h → transversely polarized quarks

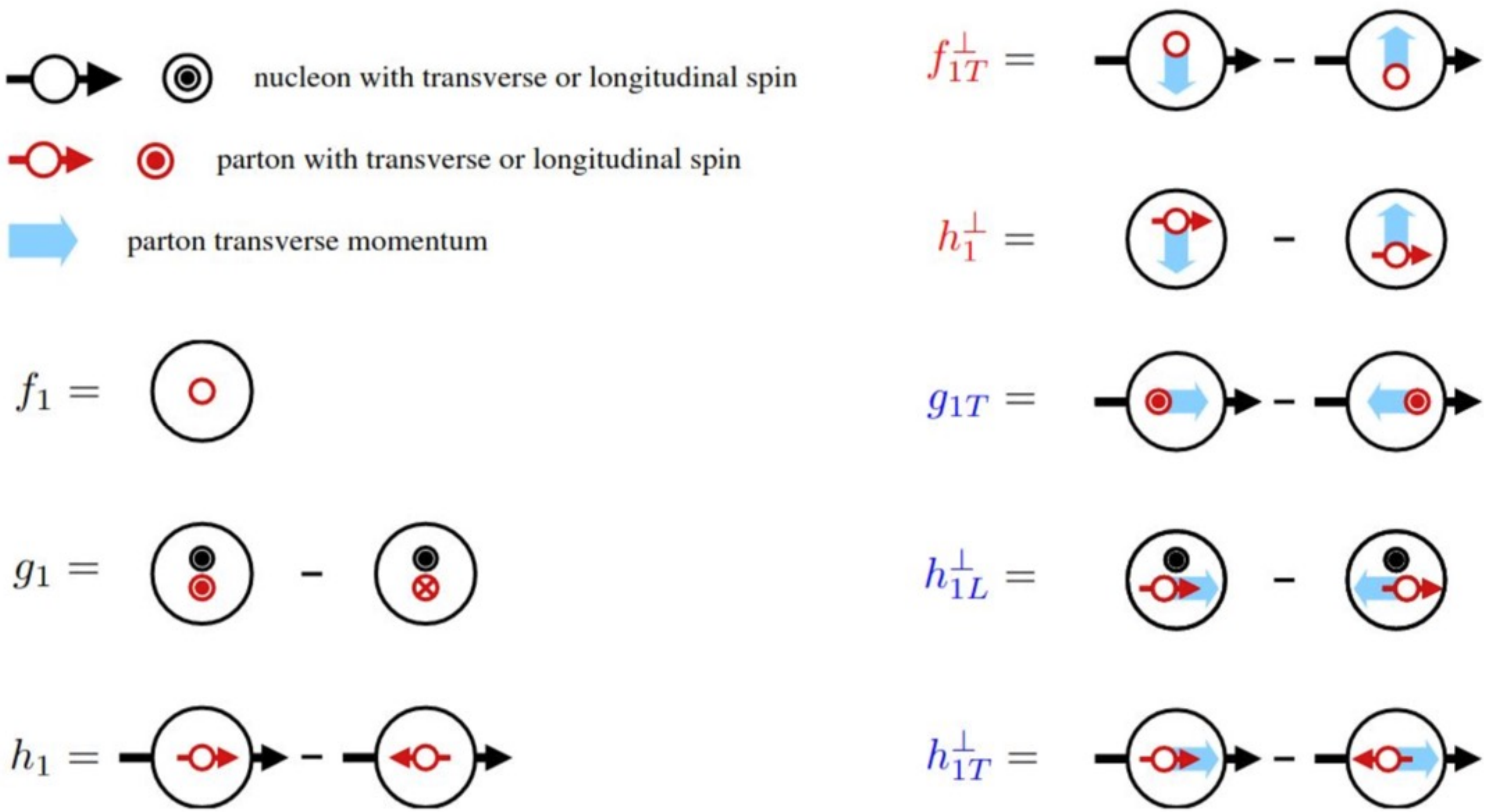
1 → the leading twist
L → longitudinally polarized hadron
T → transversely polarized hadron
$\perp$→ existing $k_\perp$ with a non-contracted index

[Meißner, et. al. JHEP08 (2009) 056]

Twist-2 TMDs

f → unpolarized quarks
g → longitudinally polarized quarks
h → transversely polarized quarks

1 → the leading twist
L → longitudinally polarized hadron
T → transversely polarized hadron
⊥→ existing $k_{\perp}$ with a non-contracted index

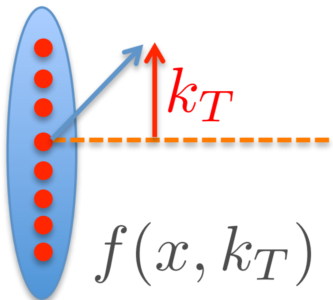


Hadron structure: encoded in TMDs

TMD-PDFs

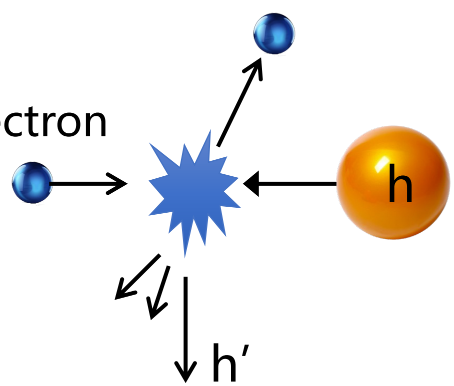
Parton Distribution Functions (PDFs): 1D longitudinal

Transverse-momentum-dependent PDFs: 1D longitudinal + 2D transverse

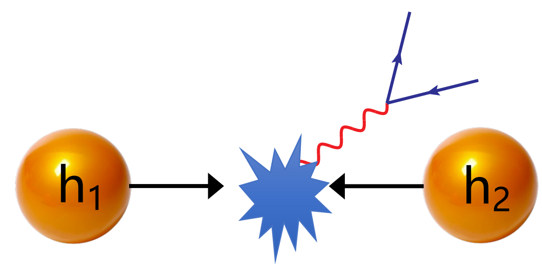


Longitudinal + transverse motion

Probability for finding a parton in a hadron with momentum fraction x and transverse momentum k_T



Semi-inclusive Deep Inelastic Scattering (SIDIS)



Drell-Yan process

SIDIS and Drell-Yan processes: extracting TMDs

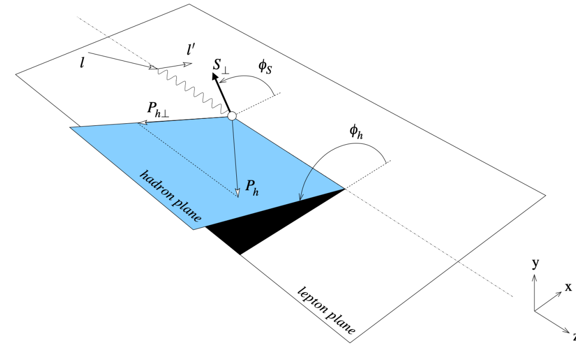
Semi-inclusive DIS

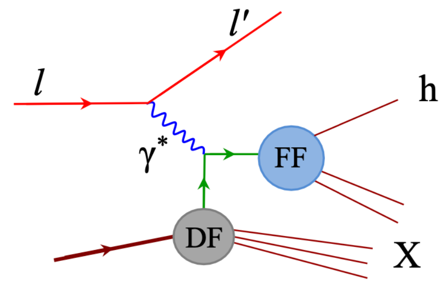
$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\varphi_h\,d\psi} = \left[\frac{\alpha}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right]\left(F_{UU,T}+\varepsilon F_{UU,L}\right)\times$$

$$\left\{\begin{aligned}
&1+\cos\phi_h\left(\sqrt{2\varepsilon(1+\varepsilon)}A_{UU}^{\cos\phi_h}\right)+\cos 2\phi_h\left(\varepsilon A_{UU}^{\cos 2\phi_h}\right)\\
&+\lambda\sin\phi_h\left(\sqrt{2\varepsilon(1-\varepsilon)}A_{LU}^{\sin\phi_h}\right)\\
&+S_L\left[\sin\phi_h\left(\sqrt{2\varepsilon(1+\varepsilon)}A_{UL}^{\sin\phi_h}\right)+\sin 2\phi_h\left(\varepsilon A_{UL}^{\sin 2\phi_h}\right)\right]\\
&+S_L\lambda\left[\sqrt{1-\varepsilon^2}A_{LL}+\cos\phi_h\left(\sqrt{2\varepsilon(1-\varepsilon)}A_{LL}^{\cos\phi_h}\right)\right]\\
&+S_T\left[\begin{aligned}&\sin(\phi_h-\phi_S)\left(A_{UT}^{\sin(\phi_h-\phi_S)}\right)\\
&+\sin(\phi_h+\phi_S)\left(\varepsilon A_{UT}^{\sin(\phi_h+\phi_S)}\right)\\
&+\sin(3\phi_h-\phi_S)\left(\varepsilon A_{UT}^{\sin(3\phi_h-\phi_S)}\right)\\
&+\sin\phi_S\left(\sqrt{2\varepsilon(1+\varepsilon)}A_{UT}^{\sin\phi_S}\right)\\
&+\sin(2\phi_h-\phi_S)\left(\sqrt{2\varepsilon(1+\varepsilon)}A_{UT}^{\sin(2\phi_h-\phi_S)}\right)\end{aligned}\right]\\
&+S_T\lambda\left[\begin{aligned}&\cos(\phi_h-\phi_S)\left(\sqrt{(1-\varepsilon^2)}A_{LT}^{\cos(\phi_h-\phi_S)}\right)\\
&+\cos\phi_S\left(\sqrt{2\varepsilon(1-\varepsilon)}A_{LT}^{\cos\phi_S}\right)\\
&+\cos(2\phi_h-\phi_S)\left(\sqrt{2\varepsilon(1-\varepsilon)}A_{LT}^{\cos(2\phi_h-\phi_S)}\right)\end{aligned}\right]
\end{aligned}\right\}$$

SSA

DSA





Twist-2

Twist-3

$$A_{UT}^{\sin(\phi_h-\phi_S)} \propto f_{1T}^{\perp}\otimes D_1$$

$$A_{UT}^{\sin(\phi_h+\phi_S)} \propto h_1\otimes H_1^{\perp}$$

$$A_{UT}^{\sin(3\phi_h-\phi_S)} \propto h_{1T}^{\perp}\otimes H_1^{\perp}$$

$$A_{UT}^{\sin(\phi_S)} \propto \frac{M}{Q}\left(f_T\otimes D_1 + h_1\otimes H_1^{\perp}+\cdots\right)$$

$$A_{UT}^{\sin(2\phi_h-\phi_S)} \propto \frac{M}{Q}\left(h_T\otimes H_1^{\perp} + h_T^{\perp}H_1^{\perp}+\cdots\right)$$

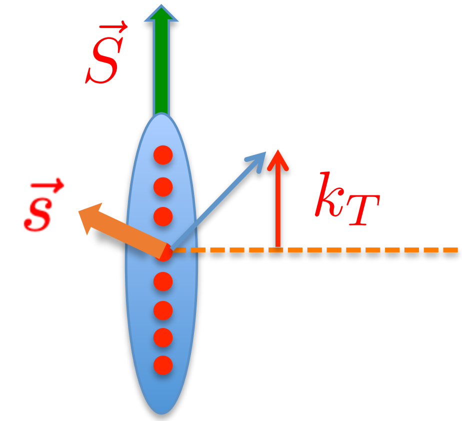
$$A_{LT}^{\cos(\phi_h-\phi_S)} \propto g_{1T}\otimes D_1$$

$$A_{LT}^{\cos(\phi_S)} \propto \frac{M}{Q}\left(g_T\otimes D_1 + e_T\otimes H_1^{\perp}+\cdots\right)$$

$$A_{LT}^{\cos(2\phi_h-\phi_S)} \propto \frac{M}{Q}\left(e_T\otimes H_1^{\perp} + e_T^{\perp}\otimes H_1^{\perp}+\cdots\right)$$

[Bacchetta, et al, JHEP 02 (2007) 093]

Quark TMD-PDF for hadrons



Quark correlator

$\mathcal{W} \to 1$

$$\Phi_q^{[\Gamma]}\left(P,S;x=\frac{k^+}{P^+},k_\perp\right)=\frac{1}{2}\int\frac{\mathrm{d}z^-\mathrm{d}z^\perp}{2(2\pi)^3}e^{ik\cdot z}\langle P,S|\,\bar{\Psi}_q(0)\Gamma\mathcal{W}(0^\perp,z^\perp)\Psi_q(z)\,|P,S\rangle\Big|_{z^+=0},$$

Twist-2: $\Phi^{[\gamma^+]}(x,k_\perp)=f_1(x,k_\perp),$

……

(averaging hadron spin S)

Twist-3: $\Phi^{[1]}(x,k_\perp)=\dfrac{M}{P^+}e(x,k_\perp),$

……

Hadron states:

$$|\text{proton}\rangle=\psi_{uud}|uud\rangle+\psi_{uudg}|uudg\rangle+\cdots$$

$$|\text{meson}\rangle=\psi_{q\bar{q}}|q\bar{q}\rangle+\psi_{q\bar{q}g}|q\bar{q}g\rangle+\cdots$$

Light-front wave functions

Overlap form of TMDs:

Proton: $f_1\sim\langle P,S|\bar{\Psi}\gamma^+\Psi|P,S\rangle\sim\int[D]\psi^*_{uud}\psi_{uud}+\int[D]\psi^*_{uudg}\psi_{uudg}$

Meson: $f_1\sim\langle P|\bar{\Psi}\gamma^+\Psi|P\rangle\sim\int[D]\psi^*_{q\bar{q}}\psi_{q\bar{q}}+\int[D]\psi^*_{q\bar{q}g}\psi_{q\bar{q}g}$

Equation of motion relation

Using the unintegrated quark correlator

$$\Phi_{\alpha\beta}(x, k_\perp) = \int \frac{\mathrm{d}^4 z}{(2\pi)^4} e^{ik\cdot z} \langle P | \bar{\psi}_\beta(0) \psi_\alpha(z) | P \rangle ,$$

Inserting the equation of motion

$$0 = \int \frac{\mathrm{d}^4 z}{(2\pi)^4} e^{ik\cdot z} \langle P | \bar{\psi}(0) \Gamma (i \not{D}(z) - m) \psi(z) | P \rangle ,$$

Through some algebra,

$$0 = \int \frac{\mathrm{d}z^- \mathrm{d}^2 z_\perp}{2(2\pi)^3} e^{ik\cdot z} \langle P | \bar{\psi}(0) [k^+ \mathbb{1} - m\ \gamma^+ + g A^j(0) i \sigma^{j+}] \psi(z) | P \rangle |_{z^+=0} .$$

$$e(x, k_\perp) = \frac{m}{M} \frac{f_1(x, k_\perp)}{x} + \tilde{e}(x, k_\perp) ,$$

twist-3 | twist-2 | genuine twist-3

involve qqg interactions

$$|\text{proton}\rangle = \psi_{uud} |uud\rangle + \psi_{uudg} |uudg\rangle + \cdots$$

twist-2 TMD: $f_1(x, k_\perp) \sim \langle P | \bar{\psi} \gamma^+ \psi | P \rangle \sim \int [\mathrm{D}] \psi^*_{uud} \psi_{uud} + \int [\mathrm{D}] \psi^*_{uudg} \psi_{uudg}$

genuine twist-3 TMD: $\tilde{e}(x, k_\perp) \sim \langle P | \bar{\psi} \sigma^{j+} A_j \psi | P \rangle \sim \int [\mathrm{D}] \left[\psi^*_{uud} \psi_{uudg} + \text{h.c.} \right]$

Equation of motion relation [Bacchetta, JHEP 02 (2007) 093]

Quark-quark-gluon correlator

$$\Phi_A^{\alpha[\Gamma]}(x,k_\perp) = g\int \frac{d\xi^- d^2\xi_\perp}{2(2\pi)^3} e^{ik\cdot\xi}\langle P,S|\bar\psi(0)\Gamma A^\alpha(\xi)\psi(\xi)|P,S\rangle|_{\xi^+=0}$$

16 terms Genuine twist-3 TMDs:

8 T-even terms

8 T-odd terms

$$\frac{1}{2Mx}\mathrm{Tr}\left[\tilde\Phi_{A\alpha}\sigma^{\alpha+}\right] = \tilde h + i\tilde e + \frac{\epsilon_T^{\rho\sigma}k_{\perp\rho}S_{T\sigma}}{M}\left(\tilde h_T^\perp - i\tilde e_T^\perp\right),$$
$$\frac{1}{2Mx}\mathrm{Tr}\left[\tilde\Phi_{A\alpha}i\sigma^{\alpha+}\gamma_5\right] = S_L\left(\tilde h_L + i\tilde e_L\right) - \frac{k_\perp\cdot S_T}{M}\left(\tilde h_T + i\tilde e_T\right),$$
$$\frac{1}{2Mx}\mathrm{Tr}\left[\tilde\Phi_{A\rho}\left(g_T^{\alpha\rho} + i\epsilon_T^{\alpha\rho}\gamma_5\right)\gamma^+\right] = \frac{k_\perp^\alpha}{M}\left(\tilde f^\perp - i\tilde g^\perp\right) - \epsilon_T^{\alpha\rho}S_{T\rho}\left(\tilde f_T + i\tilde g_T\right)$$
$$- S_L\frac{\epsilon_T^{\alpha\rho}k_{\perp\rho}}{M}\left(\tilde f_L^\perp + i\tilde g_L^\perp\right) - \frac{k_\perp^\alpha k_\perp^\rho - \frac{1}{2}k_\perp^2 g_T^{\alpha\rho}}{M^2}\epsilon_{T\rho\sigma}S_T^\sigma\left(\tilde f_T^\perp + i\tilde g_T^\perp\right),$$

8 EOM relations:

$$e(x,k_\perp^2) = \frac{m}{M}\frac{f_1(x,k_\perp^2)}{x} + \tilde e(x,k_\perp^2),$$
$$f^\perp(x,k_\perp^2) = \frac{f_1(x,k_\perp^2)}{x} + \tilde f^\perp(x,k_\perp^2),$$
$$g_T'(x,k_\perp^2) = \frac{m}{M}\frac{h_{1T}(x,k_\perp^2)}{x} + \tilde g_T'(x,k_\perp^2),$$
$$g_L^\perp(x,k_\perp^2) = \frac{g_{1L}(x,k_\perp^2)}{x} + \frac{m}{M}\frac{h_{1L}^\perp(x,k_\perp^2)}{x} + \tilde g_L^\perp(x,k_\perp^2),$$
$$g_T^\perp(x,k_\perp^2) = \frac{g_{1T}(x,k_\perp^2)}{x} + \frac{m}{M}\frac{h_{1T}^\perp(x,k_\perp^2)}{x} + \tilde g_T^\perp(x,k_\perp^2),$$
$$h_T^\perp(x,k_\perp^2) = \frac{h_{1T}(x,k_\perp^2)}{x} + \tilde h_T^\perp(x,k_\perp^2),$$
$$h_L(x,k_\perp^2) = \frac{m}{M}\frac{g_{1L}(x,k_\perp^2)}{x} - \frac{k_\perp^2}{M^2}\frac{h_{1L}^\perp(x,k_\perp^2)}{x} + \tilde h_L(x,k_\perp^2),$$
$$h_T(x,k_\perp^2) = \frac{m}{M}\frac{g_{1T}(x,k_\perp^2)}{x} - \frac{h_{1T}(x,k_\perp^2)}{x} - \frac{k_\perp^2}{M^2}\frac{h_{1T}^\perp(x,k_\perp^2)}{x} + \tilde h_T(x,k_\perp^2),$$

Basis light front quantization (BLFQ)

Light-front Schrödinger equation

[Vary et al, 2008]

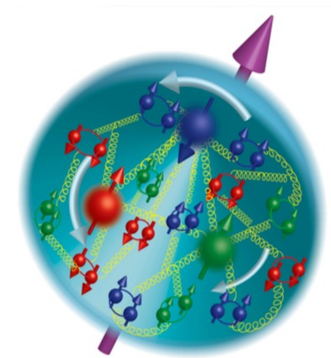
$$P^+P^-|\Psi\rangle = M^2|\Psi\rangle$$

Convert to

Eigenvalue problem

$$P^-|\beta\rangle = P^-_\beta|\beta\rangle$$

- P^-: light-front Hamiltonian
- $|\beta\rangle$: mass eigenstate
- P^-_β: eigenvalue for $|\beta\rangle$



- Fock sector expansion

$$|\text{proton}\rangle = \psi_{uud}|uud\rangle + \psi_{uudg}|uudg\rangle + \cdots$$

- Transverse: 2D harmonic oscillator basis: $\Phi^b_{n,m}(\vec{p}_\perp)$.
- Longitudinal: plane-wave basis, labeled by k.
- Basis truncation:

$$\sum_i (2n_i + |m_i| + 1) \leq N_{max},$$
$$\sum_i k_i = K_{\max}.$$

$N_{max}, K_{\max}$ are basis truncation parameters.

Proton with one dynamic gluon

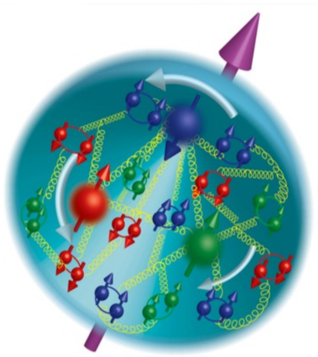
$$P^+P^-|\Psi\rangle = M^2|\Psi\rangle$$

$$|\text{proton}\rangle = \psi_{uud}|uud\rangle + \psi_{uudg}|uudg\rangle$$

[Siqi XU et al, arXiv: 2209.0858]

QCD Interaction:

$$P^-_{\text{QCD}} = \int \mathrm{d}x^- \mathrm{d}^2 x^\perp \Big\{ \frac{1}{2}\bar{\psi}\gamma^+ \frac{m_0^2 + (i\partial^\perp)^2}{i\partial^+}\psi - \frac{1}{2}A^i_a\left[m_g^2 + (i\partial^\perp)^2\right]A^i_a + g_s\bar{\psi}\gamma_\mu T^a A^\mu_a \psi + \frac{1}{2}g_s^2\bar{\psi}\gamma^+ T^a\psi \frac{1}{(i\partial^+)^2}\bar{\psi}\gamma^+ T^a \psi \Big\},$$



Confinement:

$$P^-_{\text{C}} P^+ = \frac{\kappa^4}{2}\sum_{i\neq j}\left\{ \{\vec{r}^{\,2}_{ij\perp} - \frac{\partial_{x_i}(x_i x_j \partial_{x_j})}{(m_i + m_j)^2}\right\}$$

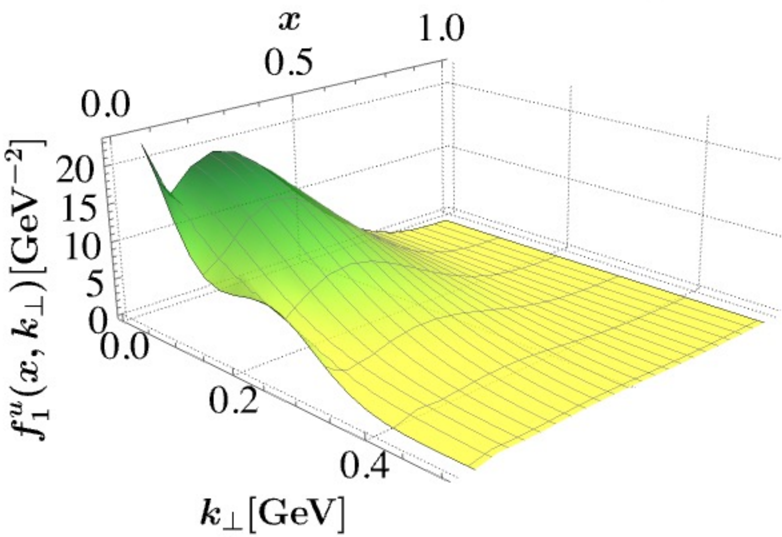
Parameters:

Truncation: Nmax=9, K=16.5
HO parameters: b=0.7GeV, b_{inst}=3GeV

m_u	m_d	m_g	κ	m_f	g
0.31GeV	0.25GeV	0.50GeV	0.54GeV	1.80GeV	2.40

Twist-2 TMDs of proton

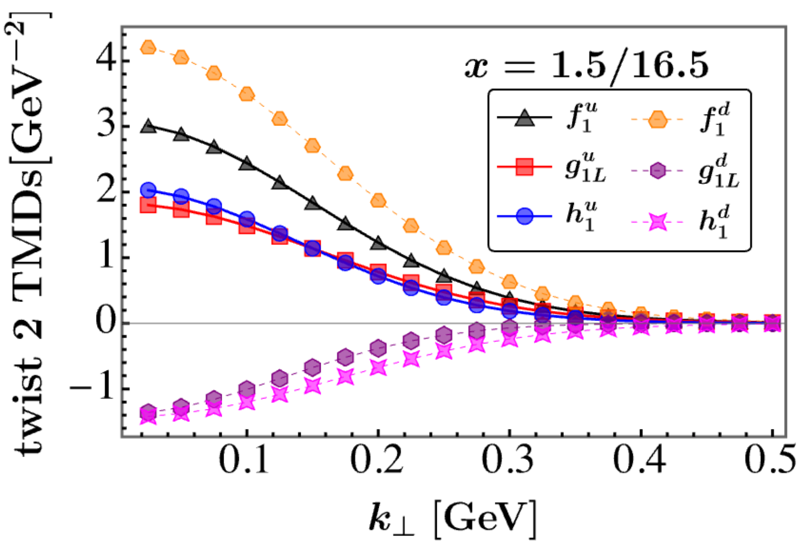
[Hongyao YU et al, in preparation]

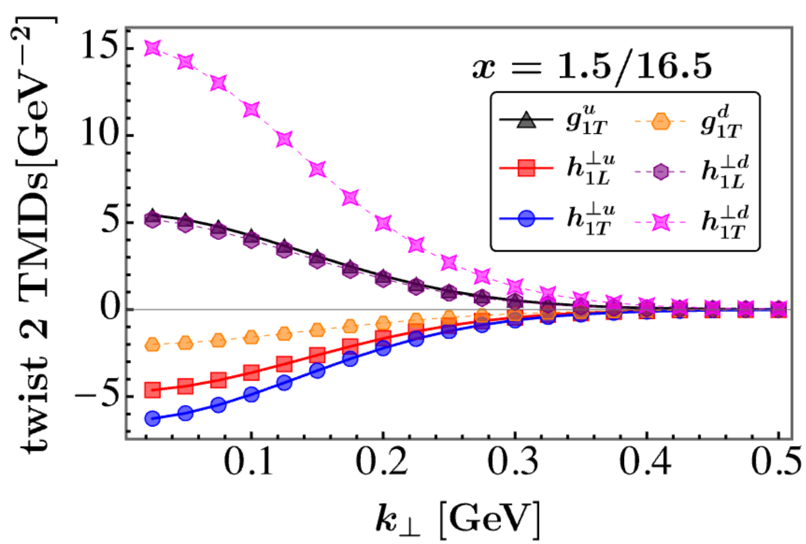


$$\Phi^{[\gamma^+]} = f_1 - \frac{\epsilon_\perp^{ij} k_\perp^i S_\perp^j}{M} f_{1T}^\perp,$$

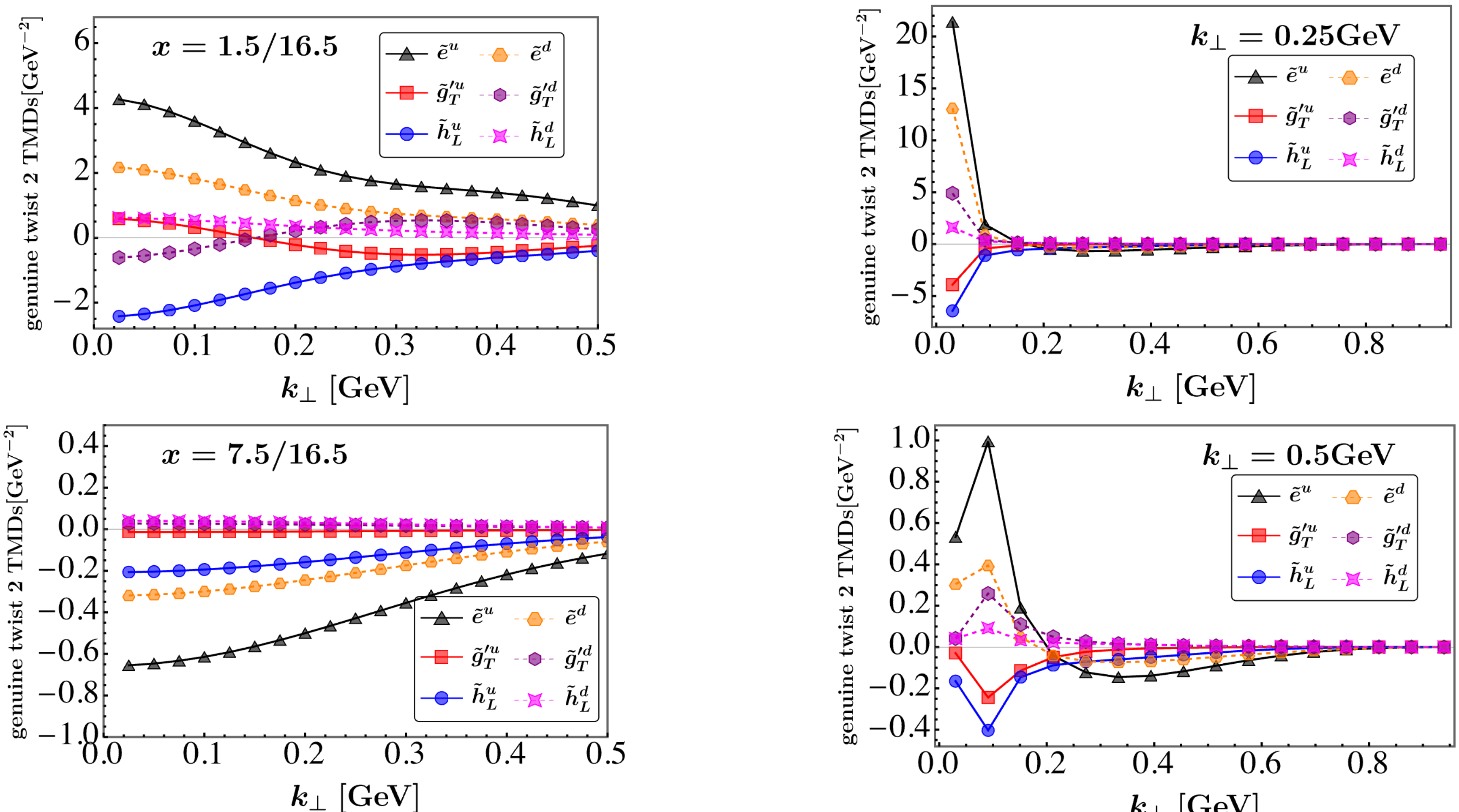
$$\Phi^{[\gamma^+\gamma^5]} = \Lambda g_{1L} + \frac{k_\perp \cdot \boldsymbol{S}_\perp}{M} g_{1T},$$

$$\Phi^{[i\sigma^{j+}\gamma^5]} = S_\perp^j h_1 + \Lambda \frac{k_\perp^j}{M} h_{1L}^\perp + S_\perp^i \frac{2k_\perp^i k_T^j - (k_\perp)^2 \delta^{ij}}{2M^2} h_{1T}^\perp + \frac{\epsilon_\perp^{ji} k_\perp^i}{M} h_1^\perp,$$





Genuine twist-3 TMDs of proton



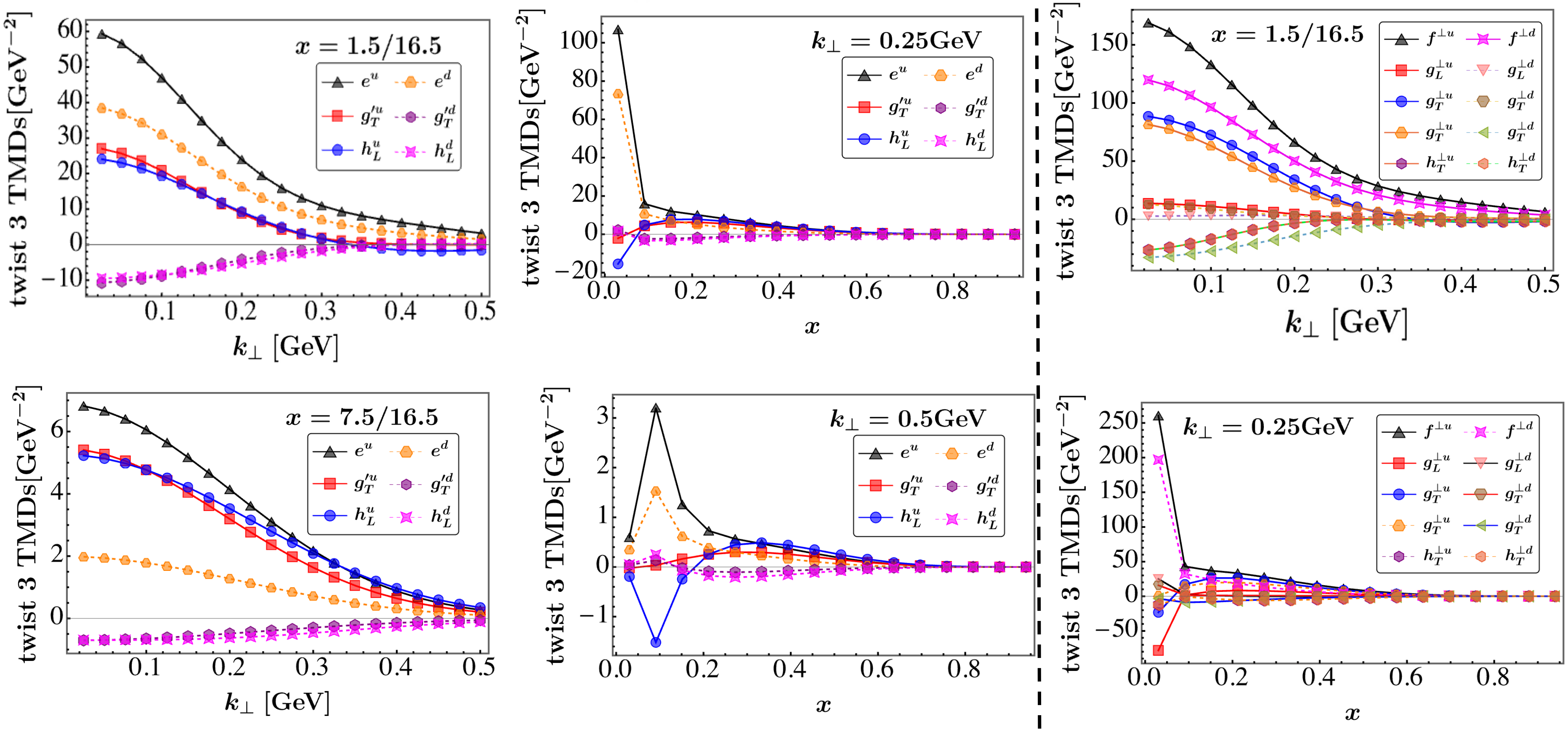
- more concentrating in small x and k_T
- same magnitude as twist-2 parts

$$e\left(x,k_\perp^2\right)=\frac{m}{M}\frac{f_1\left(x,k_\perp^2\right)}{x}+\tilde{e}\left(x,k_\perp^2\right)$$

[Zhimin ZHU et al, in preparation]

Twist-3 TMDs of proton

[Zhimin ZHU et al, in preparation]



- more concentrating in small x and k_T
- large then twist-2 TMDs

$$e\left(x,k_\perp^2\right) = \frac{m}{M}\frac{f_1\left(x,k_\perp^2\right)}{x} + \tilde{e}\left(x,k_\perp^2\right)$$

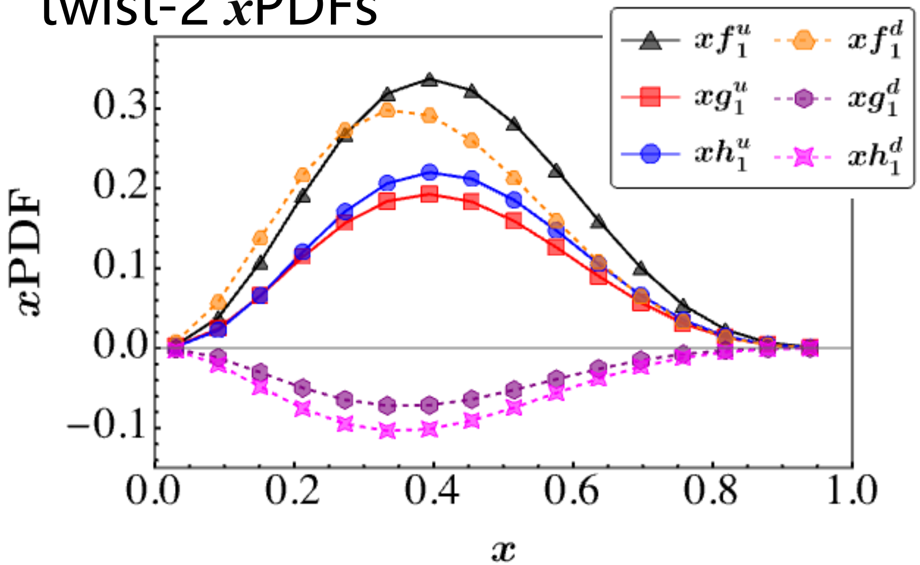
$$g_T'\left(x,k_\perp^2\right) = \frac{m}{M}\frac{h_{1T}\left(x,k_\perp^2\right)}{x} + \tilde{g}_T'\left(x,k_\perp^2\right)$$

$$h_L\left(x,k_\perp^2\right) = \frac{m}{M}\frac{g_{1L}\left(x,k_\perp^2\right)}{x} - \frac{k_\perp^2}{M^2}\frac{h_{1L}^\perp\left(x,k_\perp^2\right)}{x} + \tilde{h}_L\left(x,k_\perp^2\right)$$

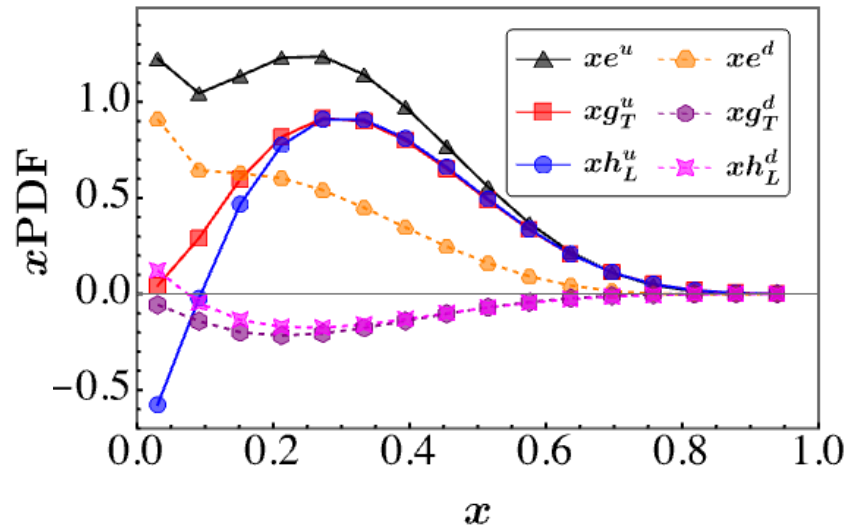
xPDFs of proton

[Zhimin ZHU et al, in preparation]

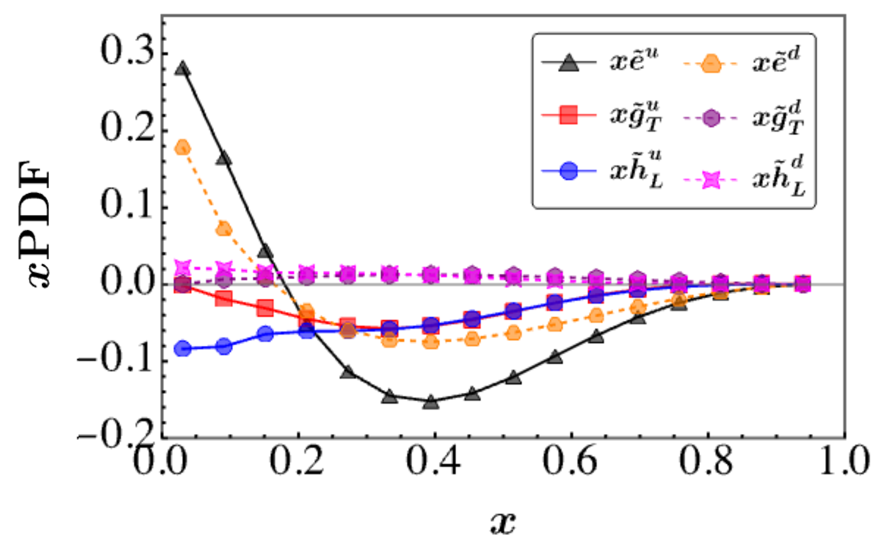
twist-2 xPDFs



twist-3 xPDFs



genuine twist-3 xPDFs

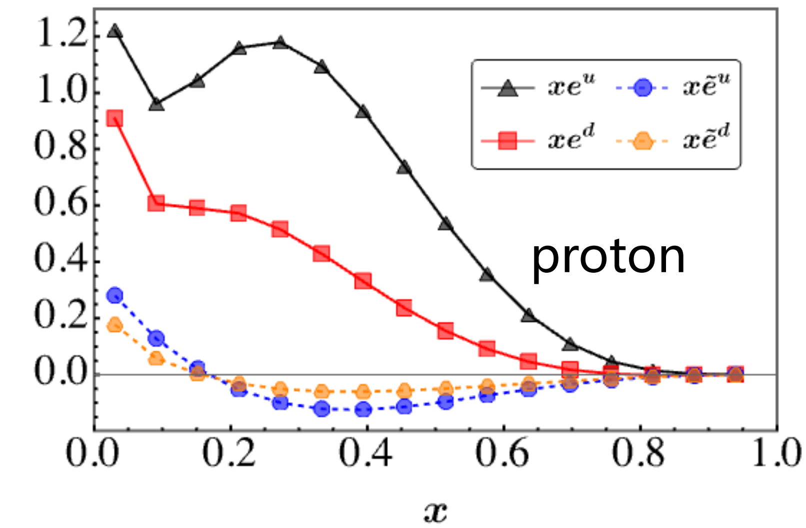


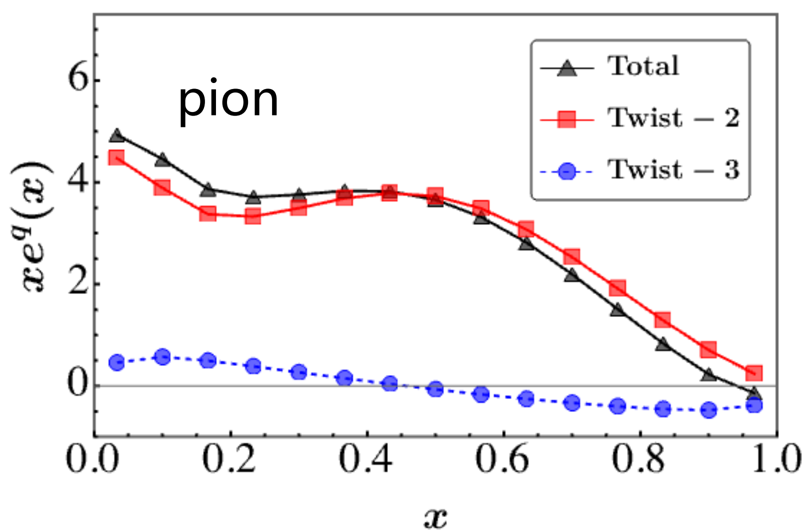
$$\int \frac{d^2 k_\perp}{(2\pi)^2} f(x, k_\perp) = f(x)$$

Twist-3 PDFs:

- more concentrating in small x
- lager then twist-2 parts

Twist-3 xPDFs of proton & pion





Sum rules

1. $$\int \mathrm{d}x x e^q(x) = \frac{m_q}{M} N_q$$

$$\int \mathrm{d}x f_1^q(x) = N_q$$

2. $$e(x, k_\perp) = \frac{m}{M} \frac{f_1(x, k_\perp)}{x} + \tilde{e}(x, k_\perp),$$

$$\Rightarrow \int \mathrm{d}x\, x\, \tilde{e}(x) = 0$$

proton:

$$\int \mathrm{d}x x \tilde{e}(x) = -0.011 << \frac{m_q}{M} = \frac{0.31}{0.98}$$

pion:

$$\int \mathrm{d}x x \tilde{e}(x) = -0.010 << \frac{m_q}{M} = \frac{0.39}{0.139}$$

genuine twist-3 PDFs satisfy QCD sum rules

Proton mass decomposition [X.D. Ji, NPB (2020) 115181]

1. $$M = M_q^{\rm LF} + M_g^{\rm LF} + M_a^{\rm LF}$$

2. $$M_q^{\rm LF} = (a+b)M/2$$

$$M_g^{\rm LF} = (1-a)M/2$$

$$M_a^{\rm LF} = (1-b)M/2$$

$a = \sum_q \int dx x f_1^q(x)$ quark kinetic motions

$b = \sum_q \sigma_q (m_q/M)$ scalar charge

$$\sigma_q = \int_{-1}^{1} dx e_q(x,\mu)$$

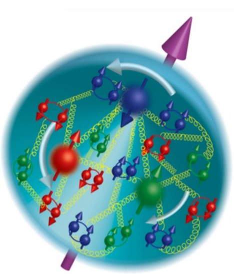
3. Without gluons $a \to 1$

With one gluon

Proton

$a = 0.80$

$b = 2.38$



$$M_q^{\rm LF} = 1.59 \text{ GeV}$$

$$M_g^{\rm LF} = 0.10 \text{ GeV}$$

$$M_a^{\rm LF} = -0.69 \text{ GeV}$$

Pion

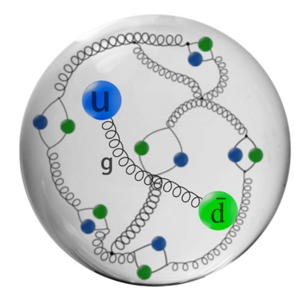
$a = 0.78$

$b = 14.55$

$$M_q^{\rm LF} = 1.065 \text{ GeV}$$

$$M_g^{\rm LF} = 0.0153 \text{ GeV}$$

$$M_a^{\rm LF} = -0.942 \text{ GeV}$$



Outlook

- TMDs carry a wealth of information about parton structure of hadrons.
- Higher twist TMDs contain more structure information, but are more complex.
- Twist-3 observables need dynamical gluons.
- BLFQ can obtain twist-2 and genuine twist-3 TMDs systematically.
- At initial scale, twist-3 TMDs are larger than twist-2 TMDs.
- Further study: Cross section, T-odd TMDs, twist-3 Qiu-Stermen functions, ……

Thank You !